Optimal Diversity-Multiplexing Tradeoff of MIMO Multi-way Relay Channel

Yuping Su and Ying Li

A MIMO multi-way relay channel with full data exchange in which K *users exchange messages with each other via the help of a single relay is considered. For the case in which each link is quasi-static Rayleigh fading and the relay is full-duplex, the fundamental diversity-multiplexing tradeoff (DMT) is investigated, and we show that a compress-and-forward relay protocol can achieve the optimal DMT.*

Keywords: Diversity-multiplexing tradeoff, MIMO multi-way relay channel, compress-and-forward, outage probability.

I. Introduction

Recently, relay techniques have attracted increasing attention due to their increasing reliability and throughput in wireless networks. For the slow fading relay channels, the communication capability is described by outage probability, that is, the probability that the transmission rate is not supported by the instantaneous channel capacity. Since it is usually difficult to obtain the accurate outage probability for an arbitrary signal-to-noise ratio (SNR), many works have considered the high SNR regime in which the outage probability approximately shows a diversity-multiplexing tradeoff (DMT) property [1]-[4]. DMT [1] is the tradeoff between reliability and rate, in which the measure of reliability is the diversity gain, which shows how fast the probability of error decreases with an increasing SNR, and the measure of the rate is the multiplexing gain, which describes how fast the actual rate of the system increases with the SNR.

In [4], it is shown that for the full-duplex two-way relay channel (TWRC), in which there are no direct links between two users, the compress-and-forward (CF) relay protocol achieves the optimal DMT. As an extension of the TWRC, a multi-way relay channel (MWRC) in which multiple users exchange their messages through the help of a relay was proposed in [5]. The MWRC model may consist of multiple interfering clusters, and users within the same cluster might wish to exchange messages among themselves. In this letter, we consider a special MWRC with a single cluster, wherein every user wants to receive the messages of all other users. This model is the MWRC with full data exchange. To the best of our knowledge, there is no work focusing on the DMT analysis of the MWRC with full data exchange, and the relay protocol that achieves the optimal DMT is still unknown. When each link is quasi-static, frequency non-selective, and Rayleigh fading and the relay is full-duplex, we derive the fundamental DMT of the MIMO MWRC with full data exchange. To this end, we first give the outer bound of the achievable DMT by using the cut-set theorem and then prove that this outer bound can be achieved by using the CF relay protocol.

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II. Channel Model

The channel model of the MWRC with full data exchange is shown in Fig. 1. In this model, K users exchange messages with each other via the help of a single relay, and there are no direct links among users. User i, $i \in \Phi=\{1, 2, \ldots, K\}$, is equipped with M_i antennas, and the relay is equipped with N antennas. We consider full-duplex communication, that is, all users and the relay can transmit and receive signals at the same time in the same band. For simplicity, we use "MWRC" to denote "MWRC with full data exchange" in the following

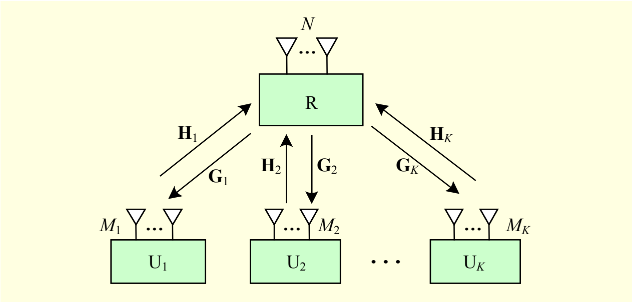

Fig. 1. Multi-way relay channel with full data exchange.

sections.

First, all users transmit their signals to the relay (multiple-access channel phase). The received signal at the relay is

$$\mathbf{Y}_r = \sum_{i=1}^{K} \sqrt{\frac{\text{SNR}}{M_i}} \mathbf{H}_i \mathbf{X}_i + \mathbf{W}_r, \tag{1}$$

where $\mathbf{X}_i \in \mathbb{C}^{M_i \times 1}$, $i \in \Phi$, is the transmitted signal vector at user i and $\mathbf{W}_r \in \mathbb{C}^{N \times 1}$ is the additive noise vector at the relay, whose entries are independent and identically distributed (i.i.d.) complex Gaussian $\mathcal{CN}(0,1)$ random variables. Matrix $\mathbf{H}_i \in \mathbb{C}^{N \times M_i}$, $i \in \Phi$, is the channel matrix from user i to the relay, whose entries are also i.i.d. $\mathcal{CN}(0,1)$ random variables. SNR is the average signal-to-noise ratio at each receiving antenna.

Then, the relay processes the received signal to generate a new message and broadcasts it to all users (broadcast channel [BC] phase). The received signal at the i-th user is

$$\mathbf{Y}_i = \sqrt{\frac{\text{SNR}}{N}} \mathbf{G}_i \mathbf{X}_r + \mathbf{W}_i, \quad i \in \Phi, \tag{2}$$

where $\mathbf{X}_r \in \mathbb{C}^{N \times 1}$ is the transmitted signal vector at the relay and $\mathbf{W}_i \in \mathbb{C}^{M_i \times 1}$, $i \in \Phi$, is the additive noise vector at user i with i.i.d. $\mathcal{CN}(0,1)$ entries. Matrix $\mathbf{G}_i \in \mathbb{C}^{M_i \times N}$, $i \in \Phi$, is the channel matrix from the relay to user i, whose entries are also i.i.d. $\mathcal{CN}(0,1)$ random variables. All the links are assumed to be quasi-static and frequency non-selective fading, that is, each channel matrix remains constant within a block of L symbol, where L is the codeword length of users.

For each user, we have the power constraint, $Tr(E[\mathbf{X}_i\mathbf{X}_i^H]) \le M_i$, $i \in \Phi$, where $Tr(\cdot)$ and $(\cdot)^H$ denote the trace and conjugate transpose of a matrix, respectively, and $E[\cdot]$ denotes the expectation of a random variable. The power constraint at the relay is $Tr(E[\mathbf{X}_r\mathbf{X}_r^H]) \le N$.

We assume that channel state information (CSI) is available at all the receivers, that is, all the channel matrices $\mathbf{H}_i$, $i \in \Phi$, are known by the relay, and $\mathbf{G}_i$ is known by user i. We also assume that matrices $\mathbf{H}_i$, $i \in \Phi$, are known by all the users at the end of each round of information exchange. Moreover, we assume that the block length L is sufficiently long so that the error probability is dominated by the channel outage probability.

III. DMT of MIMO MWRC

In this section, we derive the fundamental DMT of the MIMO MWRC and show that the CF relay protocol can achieve the optimal DMT performance. Before proceeding, we first give some definitions, as in [1].

Let $\{\mathcal{C}(\text{SNR})\}$ be a sequence of codebooks, where, for each SNR, the corresponding code $\mathcal{C}(\text{SNR})$ consists of $2^{LR_1(\text{SNR})} \times \cdots \times 2^{LR_K(\text{SNR})}$ codewords and the code rate for user i is $R_i(\text{SNR})$, $i \in \Phi$. For this sequence of codebooks, user i is known to have the multiplexing gain of r_i if

$$\lim_{\text{SNR}\to\infty} \frac{R_i(\text{SNR})}{\log \text{SNR}} = r_i.$$

Let $P_e^i(\text{SNR})$ be the error probability of user i. The diversity gain of user i is d_i if

$$\lim_{\text{SNR}\to\infty} \frac{P_e^i(\text{SNR})}{\log \text{SNR}} = -d_i.$$

We use "$\doteq$" to denote exponential equality, that is, $f(\text{SNR}) \doteq \text{SNR}^b$ denotes $\lim_{\text{SNR}\to\infty} \frac{\log f(\text{SNR})}{\log \text{SNR}} = b$, and $\dot{\le}, \dot{\ge}$ are similarly defined.

When K=2, the MWRC becomes a TWRC, where each user only needs to decode the other user's message. Apparently, each user in such a TWRC model can have different diversity gain requirements [4]. However, for the MWRC with $K \ge 3$, the case is different. In fact, for any user $l \in \Phi$, the message transmitting from the other (K–1) users to that user is a multiple-access relay channel and user l needs to jointly decode (K–1) users' messages. The joint decoding requirement at user l makes it necessary to assume that users in set $\Phi \setminus \{l\}$ have the same diversity gain [2], where $\Phi \setminus \{l\}$ denotes the set of Φ by removing the element l. Since the selection of user l is arbitrary, that is, for each $l \in \Phi$, users in set $\Phi \setminus \{l\}$ should have the same diversity gain requirement, which further implies that all K users should have the same diversity gain, that is, $d_i = d$ for all $i \in \Phi$. As a result, we assume that all the users have the same diversity gain in the following DMT analysis of the MWRC.

Let function $d_{M,N}(r)$ or $r_{M,N}(d)$ be the optimal DMT for a point-to-point $M \times N$ MIMO channel [1]. For any set $\mathcal{S} \subseteq \Phi$, we define $M_{\mathcal{S}} = \sum_{i \in \mathcal{S}} M_i$, and $r_{\mathcal{S}}$ is similarly defined.

Theorem 1. Let each user have the same diversity gain, d. For any set $\mathcal{S} \subset \Phi$ and arbitrary element $l \in \Phi$, the optimal

DMT of the MWRC with full-duplex is characterized by

$$\mathcal{R}(d)=\{(r_1,r_2,...,r_K):0\le r_{\mathcal{S}}\le r_{M_{\mathcal{S}},N}(d),\ \mathcal{S}\subset\Phi,\ \text{and}\ 0\le r_{\Phi\setminus\{l\}}\le r_{N,M_l}(d),\ l\in\Phi\},\tag{3}$$

which is achieved by the CF relay protocol.

Proof. We first prove that $\mathcal{R}(d)$ in (3) is an outer bound of the achievable DMT for the MWRC, and then we prove that this outer bound can be achieved by the CF relay protocol.

Outer bound. Given a proper subset $\mathcal{S}\subset\Phi$, we consider the information flow from the users in $\mathcal{S}$ to the remaining users. Applying the cut-set theorem in [6] on the cut formed by set $\mathcal{S}$ and the relay, we know that the achievable sum multiplexing gain of users in set $\mathcal{S}$ is upper bounded by the optimal multiplexing gain of an $M_{\mathcal{S}}\times N$ MIMO system when its diversity gain is *d*. Thus, we have $r_{\mathcal{S}}\le r_{M_{\mathcal{S}},N}(d)$.

For the BC phase transmission, we choose one user *l* from the set Φ. We assume that a genie provides all the messages of the *K* users to the users in set $\Phi\setminus\{l\}$ and the relay. Hence, the relay only needs to transmit those messages of users in set $\Phi\setminus\{l\}$ to user *l*. By using the cut-set bound on the cut formed by the relay and user *l*, we know that the achievable sum multiplexing gain of users in set $\Phi\setminus\{l\}$ is upper bounded by the optimal multiplexing gain of an $N\times M_l$ MIMO system when its diversity gain is *d*, that is, $r_{\Phi\setminus\{l\}}\le r_{N,M_l}(d)$.

Combining the results obtained above and the fact that multiplexing gain is nonnegative, we prove that $\mathcal{R}(d)$ in (3) is an outer bound of the achievable DMT for the MIMO MWRC.

Achievability. Now we prove that the DMT outer bound $\mathcal{R}(d)$ can be achieved by using a CF relay strategy. We assume that all users and the relay perform block Markov encoding, and each receiver obtains other users' messages through joint decoding after receiving the message from the relay, as in [5]. In the coding scheme, the relay quantizes the received signal and transmits the corresponding channel codeword to all users. Each receiver decodes other users' messages based on its received signal and its own message through joint typicality decoding. Using this coding scheme, the achievable rate region of the MWRC follows Theorem 1 of [5].

$$R_{\mathcal{S}}\le\min\Big\{I(\mathbf{X}(\mathcal{S});\hat{\mathbf{Y}}_{\mathrm{r}}\,|\,\mathbf{X}(\mathcal{S}^c),\mathbf{H}(\Phi)),\ \Big[\min_{k\in\mathcal{S}^c}\Big\{I(\mathbf{X}_{\mathrm{r}};\mathbf{Y}_k\,|\,\mathbf{G}_k)-I(\mathbf{Y}_{\mathrm{r}};\hat{\mathbf{Y}}_{\mathrm{r}}\,|\,\mathbf{X}(\Phi),\mathbf{H}(\Phi))\Big\}\Big]^+\Big\}\tag{4}$$

for all $\mathcal{S}\subset\Phi$, where $\mathbf{X}(\Phi)=(\mathbf{X}_1,...,\mathbf{X}_K)$, $[x]^+=\max\{x,0\}$, $\mathcal{S}^c$ is the complement of $\mathcal{S}$ in the set of Φ, and $\hat{\mathbf{Y}}_{\mathrm{r}}$ is the quantized signal at the relay.

We assume that each user and the relay use Gaussian codebooks, that is, $\mathbf{X}_i\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_{M_i})$ and $\mathbf{X}_{\mathrm{r}}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_N)$, where $\mathbf{I}_N$ is an $N\times N$ identity matrix. Let $\hat{\mathbf{Y}}_{\mathrm{r}}=\mathbf{Y}_{\mathrm{r}}+\mathbf{Q}$, where $\mathbf{Q}\sim\mathcal{CN}(\mathbf{0},\mathbf{I}_N)$ is the quantization noise. Thus, the mutual information in (4) is computed as follows:

$$I(\mathbf{X}(\mathcal{S});\hat{\mathbf{Y}}_{\mathrm{r}}\,|\,\mathbf{X}(\mathcal{S}^c),\mathbf{H}(\Phi))=\log\left|\mathbf{I}_N+\frac{\mathrm{SNR}}{2}\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H\right|\triangleq I_{\mathcal{S},R},\tag{5}$$

$$I(\mathbf{X}_{\mathrm{r}};\mathbf{Y}_k\,|\,\mathbf{G}_k)=\log\left|\mathbf{I}_{M_k}+\frac{\mathrm{SNR}}{N}\mathbf{G}_k\mathbf{G}_k^H\right|\triangleq I_{R,k},\tag{6}$$

$$I(\mathbf{Y}_{\mathrm{r}};\hat{\mathbf{Y}}_{\mathrm{r}}\,|\,\mathbf{X}(\Phi),\mathbf{H}(\Phi))=\log|2\mathbf{I}_N|=N,\tag{7}$$

where $\mathbf{H}_{\mathcal{S}}=\left[\frac{\mathbf{H}_{l_1}}{\sqrt{M_{l_1}}}\cdots\frac{\mathbf{H}_{l_{|\mathcal{S}|}}}{\sqrt{M_{l_{|\mathcal{S}|}}}}\right]$ with $l_i\in\mathcal{S},\ i\in\{1,2,...,|\mathcal{S}|\}$ and $|\mathcal{S}|$ denotes the cardinality of set $\mathcal{S}$.

For any set $\mathcal{S}\subset\Phi$, if the real transmission sum rate $R_{\mathcal{S}}$ is larger than the achievable sum rate in (4), the channel is in outage, and we denote this outage probability as $P_{\mathrm{out}}(\mathcal{S})$. Based on (4)-(7), we have

$$\begin{aligned}P_{\mathrm{out}}(\mathcal{S})&\triangleq\Pr\Big\{R_{\mathcal{S}}>\min\Big\{I_{\mathcal{S},R},\min_{k\in\mathcal{S}^c}\big[I_{R,k}-N\big]^+\Big\}\Big\}\\&\le\Pr\big\{R_{\mathcal{S}}>I_{\mathcal{S},R}\big\}+\sum_{k\in\mathcal{S}^c}\Pr\Big\{R_{\mathcal{S}}>\big[I_{R,k}-N\big]^+\Big\},\end{aligned}\tag{8}$$

where (8) is obtained by using the union bound. Next, we compute $\Pr\{R_{\mathcal{S}}>I_{\mathcal{S},R}\}$ and $\Pr\{R_{\mathcal{S}}>[I_{R,k}-N]^+\}$ in (8). Let $R_{\mathcal{S}}=r_{\mathcal{S}}\log\mathrm{SNR}$, and let $\lambda_{\mathcal{S},j}=\mathrm{SNR}^{-\alpha_{\mathcal{S},j}}$, $j\in\{1,...,M_{\mathcal{S}}^*\}$ with $M_{\mathcal{S}}^*=\min\{M_{\mathcal{S}},N\}$ be the nonzero eigenvalues of $\mathbf{H}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^H$ in (5). We compute $\Pr\{R_{\mathcal{S}}>I_{\mathcal{S},R}\}$ in (8) as

$$\begin{aligned}\Pr\big\{R_{\mathcal{S}}>I_{\mathcal{S},R}\big\}&=\Pr\{r_{\mathcal{S}}\log\mathrm{SNR}>\log\prod_{j=1}^{M_{\mathcal{S}}^*}(1+\frac{\mathrm{SNR}}{2}\lambda_{\mathcal{S},j})\}\\&\doteq\Pr\{r_{\mathcal{S}}\log\mathrm{SNR}>\log\prod_{j=1}^{M_{\mathcal{S}}^*}\mathrm{SNR}^{(1-\alpha_{\mathcal{S},j})^+}\}\\&\doteq\Pr\{r_{\mathcal{S}}\log\mathrm{SNR}>\log\mathrm{SNR}^{s(\boldsymbol{\alpha}_{\mathcal{S}})}\}\\&\doteq\mathrm{SNR}^{-d_{M_{\mathcal{S}},N}(r_{\mathcal{S}})},\end{aligned}$$

where $s(\boldsymbol{\alpha}_{\mathcal{S}})\triangleq\sum_{j=1}^{M_{\mathcal{S}}^*}(1-\alpha_{\mathcal{S},j})^+$ for any $\mathcal{S}\subset\Phi$, and we use the result of Theorem 4 from [1] for the last step. Similarly, we have

$$\Pr\Big\{R_{\mathcal{S}}>\big[I_{R,k}-N\big]^+\Big\}\doteq\mathrm{SNR}^{-d_{N,M_k}(r_{\mathcal{S}})}.$$

Thus, (8) becomes

$$\begin{aligned}P_{\mathrm{out}}(\mathcal{S})&\dot{\le}\mathrm{SNR}^{-d_{M_{\mathcal{S}},N}(r_{\mathcal{S}})}+\sum_{k\in\mathcal{S}^c}\mathrm{SNR}^{-d_{N,M_k}(r_{\mathcal{S}})}\\&\doteq\mathrm{SNR}^{-d_{M_{\mathcal{S}},N}(r_{\mathcal{S}})}+\mathrm{SNR}^{-d_{N,\widetilde{M}_{\mathcal{S}^c}}(r_{\mathcal{S}})},\end{aligned}\tag{9}$$

where $\widetilde{M}_{\mathcal{S}^c}=\min\{M_k:k\in\mathcal{S}^c\}$. Using the union bound and (9), the overall outage probability of the MWRC with CF relay

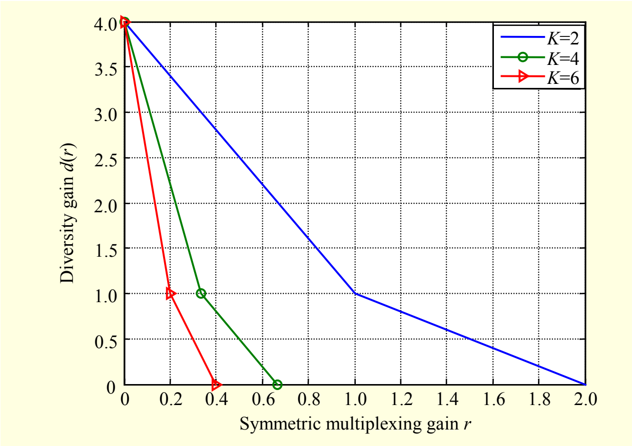

Fig. 2. Symmetric DMT of MWRC with full-duplex relay.

protocol is upper bounded as

$$P_{\text{out}} \le \sum_{\mathcal{S}:\mathcal{S}\subset\Phi} P_{\text{out}}(\mathcal{S}) \,\dot{\le} \sum_{\mathcal{S}:\mathcal{S}\subset\Phi} \left\{ \text{SNR}^{-d_{M_{\mathcal{S}},N}(r_{\mathcal{S}})} + \text{SNR}^{-d_{N,\widetilde{M}_{\mathcal{S}^c}}(r_{\mathcal{S}})} \right\}. \quad (10)$$

According to (10) and the definition of diversity gain, we know that the achievable diversity gain d satisfies $0 \le d \le d_{M_{\mathcal{S}},N}(r_{\mathcal{S}})$ and $0 \le d \le d_{N,\widetilde{M}_{\mathcal{S}^c}}(r_{\mathcal{S}})$ for any $\mathcal{S} \subset \Phi$, so the equivalent multiplexing gain region for a given value of diversity gain d with the CF relay protocol is

$$\mathcal{R}_{\text{CF}}(d) = \{(r_1, r_2, \ldots, r_K) : 0 \le r_{\mathcal{S}} \le r_{M_{\mathcal{S}},N}(d),\ \mathcal{S} \subset \Phi \text{ and } 0 \le r_{\mathcal{S}} \le r_{N,\widetilde{M}_{\mathcal{S}^c}}(d),\ \mathcal{S} \subset \Phi\}. \quad (11)$$

Now, we will show that $\mathcal{R}_{\text{CF}}(d) = \mathcal{R}(d)$. Let $\mathcal{A}=\{\mathcal{S} : |\mathcal{S}|=K-1,\ \mathcal{S} \subset \Phi\}$ and $\mathcal{B}=\{\mathcal{S} : 0<|\mathcal{S}|<K-1,\ \mathcal{S}\subset\Phi\}$. Obviously, $\mathcal{A} \cup \mathcal{B}$ consists of all the possible non-empty proper subsets of Φ. For set $\mathcal{S} = \Phi \setminus \{l\} \in \mathcal{A},\ l \in \Phi$, we have $\widetilde{M}_{\mathcal{S}^c} = M_l$, and the second constraint in (11) becomes

$$0 \le r_{\Phi\setminus\{l\}} \le r_{N,M_l}(d),\ l \in \Phi. \quad (12)$$

To complete the proof, we only need to show that (12) implies

$$0 \le r_{\mathcal{S}} \le r_{N,\widetilde{M}_{\mathcal{S}^c}}(d),\ \mathcal{S} \in \mathcal{B}. \quad (13)$$

Given $\mathcal{S} \in \mathcal{B}$ and for any $l \in \mathcal{S}^c$, it is clear that $\mathcal{S} \subset \Phi \setminus \{l\}$ and $r_{\mathcal{S}} \le r_{\Phi\setminus\{l\}}$. Using (12) for any $l \in \mathcal{S}^c$ and the fact that $r_{\mathcal{S}} \le r_{\Phi\setminus\{l\}}$, we have

$$r_{\mathcal{S}} \le r_{\Phi\setminus\{l\}} \le r_{N,M_l}(d),\ l \in \mathcal{S}^c. \quad (14)$$

Thus, we have $r_{\mathcal{S}} \le \min\{r_{N,M_l}(d) : l \in \mathcal{S}^c\} = r_{N,\widetilde{M}_{\mathcal{S}^c}}(d)$, which is exactly (13). Therefore, $\mathcal{R}_{\text{CF}}(d) = \mathcal{R}(d)$ and the theorem has been proven. □

Let $r_i = r$ for all $i \in \Phi$. For $M_i = N = 2,\ i \in \Phi$, the symmetric DMT of MWRC is illustrated in Fig. 2. It is shown that the symmetric multiplexing gain r becomes smaller as the number of users increases, and this is due to the fact that there is increasing interference when the number of users increases.

For K=2, the MWRC model becomes a TWRC with $\Phi = \{1, 2\}$. The set $\mathcal{S}$ in $\mathcal{R}(d)$ for the first constraint can be $\{1\}$ and $\{2\}$, so the corresponding constraint becomes $0 \le r_1 \le r_{M_1,N}(d)$ and $0 \le r_2 \le r_{M_2,N}(d)$. Similarly, the second constraint in $\mathcal{R}(d)$ becomes $0 \le r_2 \le r_{N,M_1}(d)$ and $0 \le r_1 \le r_{N,M_2}(d)$. Combining the results above, we obtain the achievable DMT of the TWRC as

$$0 \le r_i \le r_{\widetilde{M}_\Phi,N}(d),\ i \in \Phi,$$

where $\widetilde{M}_\Phi = \min\{M_i : i \in \Phi\}$. The equivalent achievable diversity gain is $0 \le d \le d_{\widetilde{M}_\Phi,N}(r_i),\ i \in \Phi$. It can be seen that the result obtained here is the same as that obtained in Theorem 3.4 of [4] for the case that both users have the same diversity gain. Therefore, the result of TWRC obtained in [4] can be seen as a special case of the result obtained in this letter.

IV. Conclusion

For the MIMO MWRC with full-duplex relay, we established its fundamental DMT and showed that the CF relay protocol can achieve the optimal DMT.

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