Effect of Microdiversity and Macrodiversity on Average Bit Error Probability in Gamma-Shadowed Rician Fading Channels

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In this letter, we analyze the error performance of a mobile communication system with microdiversity and macrodiversity reception in gamma-shadowed Rician fading channels for a binary differential phase-shift keying modulation scheme. Analytical expressions for the probability density function (PDF) and moment-generating function (MGF) are derived. The average bit error probability can be calculated by averaging the conditional bit error probability over the PDF or using the MGF-based approach. Numerical results are graphically presented to show the effects of macrodiversity, correlation, number of diversity branches, and severity of both fading and shadowing.

Keywords: Rician fading, gamma shadowing, microdiversity, macrodiversity, average bit error probability.

I. Introduction

In mobile communication systems, the received signal may suffer from both short-term fading, which is the result of multipath propagation, and long-term fading (shadowing), which arises from the multiple scattering conditions. The detrimental effects of multipath fading can be mitigated by using diversity techniques at the single base station (microdiversity). Long-term fading effects can be reduced by macrodiversity techniques which involve the use of a group of geographically distributed base stations in the cell. Rayleigh, Rice, Nakagami-m, and Weibull models are the most frequently used to describe short-term fading in wireless environments. Long-term fading channels are usually modeled as lognormal and gamma. Unfortunately, the use of lognormal distribution to account for shadowing does not lead to a closed-form solution for the probability density function (PDF) of the signal-to-noise ratio (SNR) after microdiversity and macrodiversity combining [1]-[4]. This makes the analysis of system in shadowed fading environment very ponderous. Based on theoretical results and measured data, gamma distribution does the job as well as lognormal [5], [6]. A compound fading model [6]-[8] uses gamma distribution to model the average power to account for shadowing. Such an approach leads to a closed-form expression for the PDF.

In this letter, motivated by the results of propagation measurements in microcellular and picocellular systems [9], and by the fact that gamma distribution can describe shadowing reliably, shadowed fading channels are modeled as Rician-gamma. Useful expression is derived for the PDF of SNR after maximal-ratio combining (MRC) at the micro level and selection combining (SC) at the macro level. By averaging the conditional bit error probability (BEP) over the PDF, the average BEP (ABEP) can be calculated. Unlike the results of previously published papers in this research area, closed-form expression for the moment-generating function (MGF) is obtained. Using the MGF-based approach, the ABEP of several modulation schemes can easily be calculated, eliminating the need for numerical integration. This presents the main contribution of the letter. As an illustration, the ABEP for a binary differential phase-shift keying (BDPSK) modulation scheme is calculated.

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II. System Model

Macrodiversity, which involves the use of two geographically distributed base stations (radio ports) per cell, is considered. Each base station has an L-branch MRC receiver operating over gamma-shadowed Rician fading channels. The instantaneous SNR at the MRC output of the i-th base station is given by $X_i=(E_b/N_0)\Sigma_{i=1}^{L} A_{ij}^2$, i=1, 2, where E_b is the transmitted signal energy per information bit, N_0 is the single-sided power spectral density of the additive white Gaussian noise, and A_{ij} are statistically independent envelopes of the faded signals received on the j-th branch. The average powers at the two base stations are related to the parameters Y_i, i=1, 2. The randomness of the average powers is described as long-term fading. The macrodiversity SC scheme is based on the simple selection of the base station with the larger average power.

III. Average Bit Error Probability

The PDF of the X_i conditioned on the Y_i is expressed as in [3] as

$$f_{X_i}\left(X_i|Y_i\right)=\frac{K+1}{Y_i}\exp\left(-\frac{(K+1)X_i}{Y_i}-KL\right)\left(\frac{(K+1)X_i}{KLY_i}\right)^{\frac{L-1}{2}} \cdot I_{L-1}\left(2\sqrt{\frac{KL(K+1)X_i}{Y_i}}\right), \quad i=1,\,2, \tag{1}$$

where K is the Rician factor defined as the ratio of the signal power in the dominant component over the scattered power, and $I_n(\cdot)$ is the modified Bessel function of the first kind and n-th order.

The MGF of the MRC output SNR at the i-th port conditioned on the Y_i can be obtained from (1) as

$$M_{X_i}\left(s|Y_i\right)=\int_0^\infty f_{X_i}\left(X_i|Y_i\right)\exp\left(-sX_i\right)dX_i, \quad i=1,\,2. \tag{2}$$

After substituting (1) in (2) and after some straightforward algebraic manipulation, the integral equation can be solved with the aid of [10] resulting in

$$M_{X_i}\left(s|Y_i\right)=\exp(-KL)\sum_{n=0}^{\infty}\left(\frac{K+1}{K+1+sY_i}\right)^{L+n}\frac{(KL)^n}{n!}, \quad i=1,2. \tag{3}$$

In this letter, Y_1 and Y_2 are correlated and identically gamma distributed with the joint PDF given as in [7] as

$$f_{Y_1Y_2}\left(Y_1,Y_2\right)=\frac{\rho^{-\frac{c-1}{2}}\left(Y_1Y_2\right)^{\frac{c-1}{2}}}{\Gamma(c)(1-\rho)Y_0^{c+1}}\exp\left(-\frac{Y_1+Y_2}{Y_0(1-\rho)}\right)I_{c-1}\left(\frac{\sqrt{4\rho Y_1Y_2}}{Y_0(1-\rho)}\right). \tag{4}$$

In (4), ρ is the correlation between Y_1 and Y_2, c is the order of gamma distribution, Y_0 is related to the average power of Y_1 and Y_2, and $\Gamma(\cdot)$ is the gamma function. The existence of correlation is a realistic case because the necessary spatial separation between the ports to ensure that they are not shadowed by the same obstacles should be very large. The severity of gamma shadowing is measured in terms of c. The lower value of c means higher shadowing. The relationship between the parameter c and the standard deviation (σ) of shadowing in dB in the lognormal shadowing is presented in [6]. In practical situations, σ does not exceed 9 dB. For values of σ near 6 dB, shadowing is described as average.

Using the concepts of probability, the expression for the MGF of the SNR after the diversity combining at the micro and macro level can be derived as

$$\begin{aligned} M_X(s) &= \int_0^\infty dY_1\int_0^{Y_1} M_{X_1}\left(s|Y_1\right)f_{Y_1Y_2}\left(Y_1,Y_2\right)\,dY_2 \\ &\quad +\int_0^\infty dY_2\int_0^{Y_2} M_{X_2}\left(s|Y_2\right)f_{Y_1Y_2}\left(Y_1,Y_2\right)\,dY_1 \\ &= 2\int_0^\infty M_{X_1}\left(s|Y_1\right)dY_1\int_0^{Y_1} f_{Y_1Y_2}\left(Y_1,Y_2\right)\,dY_2, \end{aligned} \tag{5}$$

which, by substituting (3) and (4) and by using [10], yields

$$\begin{aligned} M_X(s) &= \frac{2\exp(-KL)}{\Gamma(c)}\sum_{n=0}^{\infty}\sum_{j=0}^{\infty}\frac{\rho^j(KL)^n(K+1)^{c+j}(c+j-1)!}{(sY_0)^{c+j}(1-\rho)^j n!\,j!\,\Gamma(j+c)} \\ &\quad \cdot\left\{\Gamma(c+j)\psi\left(c+j,c+j-L-n+1;\,\frac{\alpha}{s}\right)-\sum_{l=0}^{c+j-1}\frac{\Gamma(c+j+l)}{l!\left(Y_0(1-\rho)\right)^l}\right. \\ &\quad \left.\cdot\left(\frac{s}{K+1}\right)^{-l}\psi\left(c+j+l,\,c+j+l-L-n+1;\,\frac{2\alpha}{s}\right)\right\}, \end{aligned} \tag{6}$$

where $\psi(a, b; x)$ is the confluent hypergeometric function and $\alpha=(K+1)/(Y_0(1-\rho))$.

Following the same procedure as for the MGF, the PDF of the output SNR can be written as

$$\begin{aligned} f_X(X) &= \frac{4\exp(-KL)}{\Gamma(c)}\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(K+1)^{\frac{L+c+i+j}{2}}(KL)^i\,Y_0^{-\frac{c+L+i+j}{2}} \\ &\quad \cdot\frac{\rho^j(c+j-1)!\,X^{\frac{i+j+c+L-2}{2}}}{(1-\rho)^{\frac{L-c+i+j}{2}}\,i!\,j!\,\Gamma(i+L)\Gamma(j+c)} \\ &\quad \cdot\left\{K_{c-L-i+j}\left(2\sqrt{\alpha X}\right)-\sum_{l=0}^{c+j-1}\frac{1}{2^{\frac{c-L-i+j+l}{2}}\,l!}(\alpha X)^{\frac{l}{2}}K_{c-L-i+j+l}\left(2\sqrt{2\alpha X}\right)\right\}, \end{aligned} \tag{7}$$

where $K_n(\cdot)$ is the modified Bessel function of the second kind and n-th order. Closed-form expressions for the MGF and PDF are derived for the case where c is an integer. This is not a major restriction for the following reasons: first, the channel may be sometimes characterized or measured to an accuracy corresponding to whole integer arithmetic; second, if the channel is described by noninteger value of *c*, then interpolation between the upper bounding and lower bounding integer values of *c* may be used.

The ABEP of BDPSK modulation signaling can be calculated using the MGF-based approach

$$\bar{P}_{be} = 0.5 M_X(1) \tag{8}$$

or by averaging the conditional BEP over the PDF of X

$$\bar{P}_{be} = \int_0^\infty f_X(X) \frac{1}{2} \exp(-X)\, dX. \tag{9}$$

Closed-form expression for the MGF converges rapidly (no more than fifteen terms are necessary to achieve accuracy at the fourth significant digit) with a significant speedup factor compared to numerical integration techniques used to evaluate the integral of (9).

IV. Numerical Results

A family of curves for the PDF of the output SNR for different fading and shadowing severity is plotted in Fig. 1. It reveals that decreasing severity of fading and shadowing increases the chance of taking on a larger output SNR, and therefore results in better performance of the system.

Figure 2 shows the effect of different numbers of diversity branches at the micro level and correlation between average powers on the ABEP, where BDPSK modulation is considered. It is evident that the error performance is improved as the number of diversity branches increases. Increasing the correlation coefficient leads to a deterioration of system performance. When the correlation is very strong, for example,

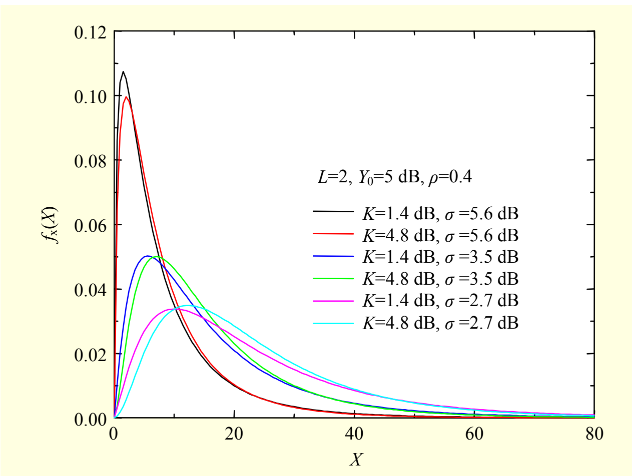

Fig. 1. PDF of the output SNR for different fading and shadowing severity.

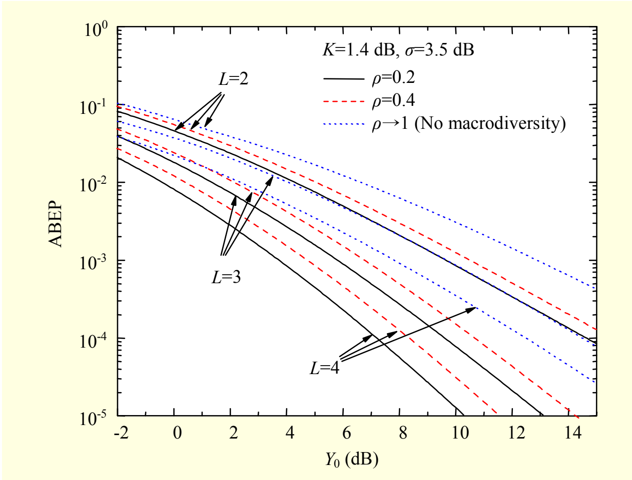

Fig. 2. ABEP of BDPSK versus average power Y_0 for different numbers of branches and correlation.

($\rho \rightarrow 1$), the performance coincides with that having no macrodiversity. It is obvious that the combination of microdiversity and macrodiversity is more effective in providing performance improvement than microdiversity only. For example, for K=1.4 dB, σ=3.5 dB, and ρ=0.2, at an ABEP of 10^{-3}, the macrodiversity gain for L=2 is about 3.3 dB, for L=3, 3.8 dB, and for L=4, 4.1 dB. The macrodiversity gain here is defined as the reduction in the Y_0 compared with the case having no macrodiversity. System performance enhancement due to use of diversity at the macro level is much higher for a larger number of diversity branches at the micro level.

V. Conclusion

In this letter, a system with microdiversity and macrodiversity reception in gamma-shadowed Rician fading channels has been considered. The microdiversity scheme is based on MRC and the macrodiversity scheme is based on SC. Closed-form expressions for the PDF and MGF of the output SNR have been derived. Numerical results for the PDF and ABEP for BDPSK modulation format have been graphically presented. They demonstrate that system performance improves with an increase of K, L, and Y_0, while an increase of ρ and σ leads to deterioration of system performance. Improvement obtained through macrodiversity has also been shown.

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