# Beamforming for Downlink Multiuser MIMO Time-Varying Channels Based on Generalized Eigenvector Perturbation

Heejung Yu and Sok-kyu Lee

**A beam design method based on signal-to-leakage-plus-noise ratio (SLNR) has been recently proposed as an effective scheme for multiuser multiple-input multiple-output downlink channels. It is shown that its solution, which maximizes the SLNR at a transmitter, can be simply obtained by the generalized eigenvectors corresponding to the dominant generalized eigenvalues of a pair of covariance matrices of a desired signal and interference leakage plus noise. Under time-varying channels, however, generalized eigendecomposition is required at each time step to design the optimal beam, and its level of complexity is too high to implement in practical systems. To overcome this problem, a predictive beam design method updating the beams according to channel variation is proposed. To this end, the perturbed generalized eigenvectors, which can be obtained by a perturbation theory without any iteration, are used. The performance of the method in terms of SLNR is analyzed and verified using numerical results.**

**Keywords: MU-MIMO, downlink, beamforming, generalized eigendecomposition, perturbation theory, time-varying channels.**

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## I. Introduction

Beamforming for a multiuser multiple-input multiple-output (MU-MIMO) downlink transmission has been adopted in IEEE 802.11ac wireless local area network and 3GPP long-term evolution (LTE) systems for a base station (BS) to serve multiple mobile stations (MSs) with the same frequency and time resources. While single-user MIMO (SU-MIMO) serves only one MS with an exclusively assigned frequency and time slots using multiple antennas on the transmit side and the receive side, that is, point-to-point MIMO transmission, a BS can design beams that avoid interference leakage to undesired MSs and simultaneously support multiple MSs in the same frequency band in MU-MIMO downlink systems. When the number of antennas at an MS is limited by a spatial constraint, an MU-MIMO downlink transmission can improve the system throughput by deploying multiple antennas at a BS. To achieve this throughput gain, the interference caused by a transmission to multiple destinations should be properly controlled. Hence, the throughput of an MU-MIMO downlink dominantly depends on the beamforming scheme.

For the case in which a transmitter and the receivers can obtain perfect channel state information (CSI), it is well known that the dirty paper coding (DPC) scheme, which precancels interference from a transmitter, is an optimal scheme to achieve the capacity of MIMO broadcasting channels [1]-[5]. In spite of its optimality, DPC is hard to implement practically, owing to the computational complexity of such a nonlinear approach. Despite performance loss, several linear beamforming approaches were introduced in [6], [7] to reduce the complexity.

  

The simplest scheme for designing an MU-MIMO downlink beamformer is a zero-forcing (ZF) method, which can eliminate interference leakage to undesired receivers completely with perfect CSI. This ZF approach is the best beam design method in a noise-free environment. When the noise is greater than a certain level, we can relax the zero interference constraint and increase the desired signal power. This is a very similar approach to that used in modifying a ZF receiver design into a minimum mean square error (MMSE) design. Such relaxation is included in [7] under the name "regulated ZF." In [8], the authors proposed a beam design scheme based on signal-to-leakage-plus-noise ratio (SLNR), since it can be considered that SLNR is a very similar performance measure on the transmit side with signal-to-interference-plus-noise ratio (SINR) on the receiver side. It has been shown that a beamforming method based on SLNR performs better than a simple ZF approach. The optimal solution maximizing the SLNR is given by the generalized eigenvectors associated with the dominant generalized eigenvalues of a pair of covariance matrices of the desired signal and interference leakage plus noise.

In quasi-static channel conditions, beam matrices obtained at the beginning of a transmission can be used during the whole transmission period. When the channels vary over time even in one packet, however, the beams should be recalculated to avoid a performance loss. The level of complexity involved in designing beams at each time step is too high for such beams to be implemented since the optimal solution is obtained by generalized eigendecomposition implemented with an iterative algorithm. Therefore, using a noniterative beam updating algorithm is considered a reasonable trade-off between complexity and performance. To develop a noniterative algorithm for the MU-MIMO downlink beam design, a perturbation theory for a generalized eigenvector is employed since it calculates a perturbed generalized eigenvector with the original unperturbed generalized eigenvalues and corresponding eigenvectors as well as a pair of perturbed objective matrices.

The proposed design method is a mixture of generalized eigendecomposition steps and updating steps with a predetermined updating depth. At the last symbol time of the previous uplink phase, we obtain the full eigenvalues and eigenvectors with channel matrices. For the downlink transmission, the beams are calculated using a perturbation approach with the previous eigenvalues and eigenvectors obtained in the previous generalized eigendecomposition step as well as channel variation information.

Additionally, we analyze the performance of the proposed algorithm using perturbed eigenvalues and an autoregressive (AR) channel model. Through numerical results, we verify the analysis and show the effectiveness of the proposed algorithm.

In this paper, we make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For $\mathbf{A}$, $\mathbf{A}^T$ and $\mathbf{A}^H$ indicate its transpose and Hermitian transpose, respectively. For scalar $a$, $a^*$ denotes the complex conjugate. The trace of $\mathbf{A}$ is represented by $Tr(\mathbf{A})$. The matrices $\mathbf{0}$ and $\mathbf{I}$ are respectively the all-zero and identity matrices of an appropriate size. The notation $\mathbf{x}\sim CN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ indicates that $\mathbf{x}$ is a complex Gaussian random vector with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Moreover, $E(\cdot)$ denotes the expectation. For $\mathbf{a}$ and $\mathbf{A}$, $\|\mathbf{a}\|$ and $\|\mathbf{A}\|$ stand for the vector norm and matrix 2-norm, respectively. A variable with a dot over it indicates a first-order derivative.

This paper is organized as follows. The system model and beamforming methods based on SLNR are introduced in section II. In sections III and IV, we describe the channel prediction and a perturbation theory of a generalized eigenvector, respectively. The proposed beam design algorithm and an analysis of the SLNR are discussed in sections V and VI. Next, numerical results are shown in section VII. Finally, we offer some concluding remarks in section VIII.

## II. System Model and Beamforming Based on SLNR

We consider a downlink transmission from a BS equipped with $N_B$ antennas to $K$ MSs with $N_M$ antennas for each, as shown in Fig. 1. A BS transmits $d$ independent data streams to each MS using a beam matrix with a size of $N_B \times d$. Therefore, this can be expressed as

$$\mathbf{x}[n] = \sum_{k=1}^{K} \mathbf{V}_k[n]\mathbf{s}_k[n], \tag{1}$$

where $\mathbf{V}_k[n]$ and $\mathbf{s}_k[n]$ denote a beam matrix and transmit signal vector for the $k$-th MS at time $n$, respectively. It is assumed that $E\{\mathbf{s}_k[n]\mathbf{s}_k^H[n]\}=(1/d)\mathbf{I}$ and $Tr(\mathbf{V}_k^H[n]\mathbf{V}_k[n])=d$. This transmitted signal passes through the MIMO channel and is received by the MSs. The received signal at the $k$-th MS is given by

$$\begin{aligned}\mathbf{y}_k[n] &= \mathbf{H}_k[n]\mathbf{x}_k[n] \\ &= \mathbf{H}_k[n]\mathbf{V}_k[n]\mathbf{s}_k[n] + \sum_{l=1, l\neq k}^{K} \mathbf{H}_k[n]\mathbf{V}_l[n]\mathbf{s}_l[n] + \mathbf{n}_k[n],\end{aligned} \tag{2}$$

where $\mathbf{n}_k[n]\sim CN(0, \sigma^2\mathbf{I})$ is a noise vector. On the right-hand side (RHS) of the second equality of (2), the first and second terms are the desired signal and interference from the transmit signal to the other MSs, respectively. To maximize the sum rate, we use beams maximizing the SINR given by

$$\mathrm{SINR}_k = \frac{\left\|\mathbf{H}_k[n]\mathbf{V}_k[n]\mathbf{s}_k[n]\right\|^2}{N_M\sigma^2 + \sum_{l\neq k}\left\|\mathbf{H}_k[n]\mathbf{V}_l[n]\mathbf{s}_l[n]\right\|^2}. \tag{3}$$

It is difficult to find a solution maximizing the SINR since the beam matrices are coupled with each other, that is, $\{\mathbf{V}_k[n], k=1,\ldots,K\}$ are included in $\text{SINR}_k$ for all $k=1,\ldots,K$. To simplify the problem, the authors in [8] considered the SLNR instead of the SINR. Instead of investigating the received signal at the $k$-th MS, we examine the transmit signal to the $k$-th MS, that is, $\mathbf{V}_k[n]\mathbf{s}_k[n]$. This signal can reach all receivers and be expressed as

$$\mathbf{H}_k[n]\mathbf{V}_k[n]\mathbf{s}_k[n]+\sum_{l=1,l\neq k}^{K}\mathbf{H}_l[n]\mathbf{V}_k[n]\mathbf{s}_k[n]+\mathbf{n}_k[n]. \quad (4)$$

Here, the second term denotes the interference leakage that reaches undesired receivers. With (4), we can simply consider the simplest ZF beam design with a condition of

$$\sum_{l=1,l\neq k}^{K}\mathbf{H}_l[n]\mathbf{V}_k[n]=\bar{\mathbf{H}}_k[n]\mathbf{V}_k[n]=\mathbf{0}, \quad (5)$$

where

$$\bar{\mathbf{H}}_k[n]=\left[\mathbf{H}_1^H[n],\cdots,\mathbf{H}_{k-1}^H[n],\mathbf{H}_{k+1}^H[n],\cdots,\mathbf{H}_K^H[n]\right]^H. \quad (6)$$

It is well known that a ZF approach is not optimal and that a better performance can be obtained by allowing a certain level of interference depending on the noise power, as in an MMSE approach. As one such approach, a beamforming method maximizing the SLNR was introduced in [8]. In detail, the SLNR in terms of the $k$-th MS is given by

$$\text{SLNR}_k=\frac{\left\|\mathbf{H}_k[n]\mathbf{V}_k[n]\mathbf{s}_k[n]\right\|^2}{N_M\sigma^2+\sum_{l\neq k}\left\|\mathbf{H}_l[n]\mathbf{V}_k[n]\mathbf{s}_k[n]\right\|^2}. \quad (7)$$

When the SLNR is considered as a beam design criterion, the coupling problem does not exist and the closed-form solution maximizing the SLNR is available. The expression for the SLNR can be rewritten with $\bar{\mathbf{H}}_k[n]$ as

$$\text{SLNR}_k=\frac{Tr(\mathbf{V}_k^H[n]\mathbf{H}_k^H[n]\mathbf{H}_k[n]\mathbf{V}_k[n])}{Tr\left(\mathbf{V}_k^H[n]\left(N_M\sigma^2\mathbf{I}+\bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n]\right)\mathbf{V}_k[n]\right)}. \quad (8)$$

The optimal beam design problem is given by

$$\max_{\mathbf{V}_k[n]}\frac{Tr(\mathbf{V}_k^H[n]\mathbf{H}_k^H[n]\mathbf{H}_k[n]\mathbf{V}_k[n])}{Tr\left(\mathbf{V}_k^H[n]\left(N_M\sigma^2\mathbf{I}+\bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n]\right)\mathbf{V}_k[n]\right)}$$
$$s.t.\ \begin{cases} Tr(\mathbf{V}_k^H[n]\mathbf{V}_k[n])=d \\ \mathbf{V}_k^H[n]\mathbf{H}_k^H[n]\mathbf{H}_k[n]\mathbf{V}_k[n] \text{ is diagonal.}\end{cases} \quad (9)$$

The second constraint is needed to decouple multiple transmit data streams at a receiver. The solution is obtained by the generalized eigenvectors associated with the $d$ dominant generalized eigenvalues of a pair of covariance matrices of the desired signal and interference plus noise, that is, $\mathbf{H}_k^H[n]\mathbf{H}_k[n]$ and $\left(N_M\sigma^2\mathbf{I}+\bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n]\right)$. Though the quantity of the SLNR does not directly reflect the sum rate of the MU-MIMO downlink systems, this approach shows a better sum rate than that of a ZF method under various conditions.

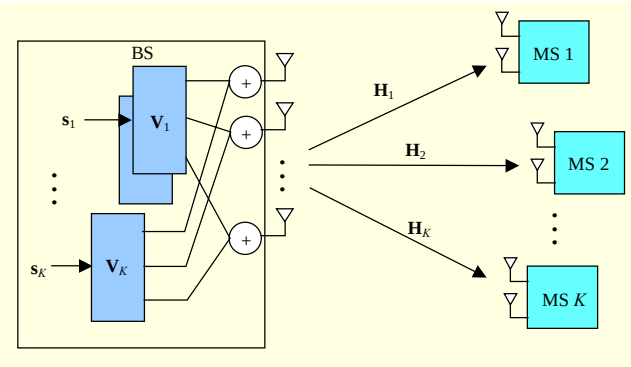


Fig. 1. MU-MIMO downlink model with one BS and $K$ MSs.

## III. Channel Model and Optimal Channel Prediction

In quasi-static and very slowly varying channels, we can determine the downlink beamformers using the CSI obtained in the previous uplink phase. In time-varying channels, however, the transmit beams should be designed with the predicted CSI to avoid a performance loss caused by an outdated CSI.

To investigate the effects of time-varying channels on MU-MIMO downlink transmission, we need an appropriate model for a time-varying channel. The $m$-th order AR model, which is widely used in the literature [9]-[11], is employed. Each element of the channel matrices from a BS to the MSs is modeled by

$$h[n]=\sum_{r=1}^{m}\beta[r]h[n-r]+w[n], \quad (10)$$

where $\beta(r)$, $r=1,\ldots,m$, are the fading coefficients that quantify the channel fading rate and $w[n]\sim CN(0,\sigma_w^2)$ denotes the plant noise of the channel process. We assume that all elements of the channel matrices follow the same process given by (10). The fading coefficients and variance of the plant noise are obtained by solving the Yule-Walker equation with the time-autocorrelation function of the Jakes fading model [11]. The time-autocorrelation is given by

$$\begin{aligned}\text{E}\{h[n]h[n-r]\}&=J_0(2\pi f_D T_s)\\&=\sum_{l=0}^{\infty}\frac{(-1)^l}{2^{2l}l^2}(2\pi f_D T_s r)^{2l},\end{aligned} \quad (11)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind

and $f_D$ and $T_s$ denote the maximum Doppler frequency and symbol duration, respectively.

For transmit beamforming, a BS should predict the channel matrices in the downlink phase based on the previous uplink channels, which are estimated by the uplink pilots, assuming the channel reciprocity.

As an example, we assume that the CSI of previous $m$ symbol time in an uplink phase is perfectly known. With this information, a BS can predict the CSI in a downlink phase based on the AR channel model. The channel model (10) can be rewritten as

$$\mathbf{h}[n] = \mathbf{F}\mathbf{h}[n-1] + \mathbf{w}[n], \quad n = 1, 2, \ldots, \tag{12}$$

where

$$\mathbf{h}[n] = \left[h[n],\ h[n-1], \ldots,\ h[n-m+1]\right]^T, \tag{13}$$

$$\mathbf{w}[n] = [w(n), 0, \ldots, 0]^T, \tag{14}$$

and

$$\mathbf{F} = \begin{bmatrix} \beta[1] & \cdots & \beta[m-1)] & \beta[m] \\ 1 & 0\cdots & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0\cdots & 1 & 0 \end{bmatrix}. \tag{15}$$

With a channel model of (12), the optimal channel prediction is obtained by

$$\hat{\mathbf{h}}[n] = \mathbf{F}^n \mathbf{h}[0], \tag{16}$$

where $\mathbf{h}[0]$ is available since it is obtained by estimating the channel in the previous uplink phase. The channel prediction error vector, $\tilde{\mathbf{h}}[n] = \mathbf{h}[n] - \hat{\mathbf{h}}[n]$, is a Gaussian random vector with zero mean and a covariance of

$$\begin{aligned} \mathbf{P}[n] &= E\left\{\tilde{\mathbf{h}}[n]\tilde{\mathbf{h}}^H[n]\right\} \\ &= \sum_{p=0}^{n-1} \mathbf{F}^p \mathbf{Q}_w (\mathbf{F}^H)^p, \end{aligned} \tag{17}$$

where $\mathbf{Q}_w = E\{\mathbf{w}[n]\mathbf{w}[n]^H\}$ [11]. Therefore, the variance of the channel prediction error, $\sigma_h^2[n]=E\{(h[n]-\hat{h}[n])^*(h[n]-\hat{h}[n])\}$, at time $n$ is given by the (1, 1) component of $\mathbf{P}[n]$.

When $m$=1, that is, the first-order AR channel model, the variance of the channel prediction is expressed by

$$\begin{aligned} \sigma_h^2[n] &= \sum_{p=0}^{n-1} \beta^{2p} \sigma_w^2 \\ &= 1 - \beta^{2n}. \end{aligned} \tag{18}$$

The first equality holds by the definitions of $\mathbf{F}$ and by letting $\beta = \beta[1]$. Since $\sigma_w^2 = 1-\beta^2$ in the first-order AR model, the second equality also holds.

## IV. Perturbation Theory: Generalized Eigenvalue and Eigenvector

In general, a perturbation theory of eigendecomposition makes it possible to calculate new eigenvalues and eigenvectors in the form of a linear combination of the original eigenvalues and eigenvectors as well as the perturbation matrix. This can be used to determine a predictive beam for interference alignment [12]. Similarly, the perturbed generalized eigenvectors can be obtained without any iteration and can be used to update the beam according to the channel variation. Before introducing the details of the proposed algorithm, we must investigate the perturbation theory of the generalized eigenvalues and corresponding eigenvectors with a given pair of objective matrices and their perturbations.

**Theorem 1.** For a symmetric and positive definite matrix pair (A, B) with a size of $N\times N$, generalized eigenvalues and corresponding eigenvectors are given by $\{\lambda_i\}$ and $\{\mathbf{x}_i\}$, that is,

$$\mathbf{A}\mathbf{x}_i = \lambda_i \mathbf{B}\mathbf{x}_i, \tag{19}$$

$$\mathbf{x}_i^H \mathbf{B}_i \mathbf{x}_i = \delta_{i,j} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \tag{20}$$

for $i, j$=1,…, $N$. The perturbed matrices are given by $\mathbf{A}+\tilde{\mathbf{A}}$ and $\mathbf{B}+\tilde{\mathbf{B}}$, where $\left\|\tilde{\mathbf{A}}\right\| << \left\|\mathbf{A}\right\|$ and $\left\|\tilde{\mathbf{B}}\right\| << \left\|\mathbf{B}\right\|$ and the perturbed generalized eigenvalues and eigenvectors are obtained by

$$\lambda_{i,\text{pert}} \approx \lambda_i + \mathbf{x}_i^H \left(\tilde{\mathbf{A}} - \lambda_i \tilde{\mathbf{B}}\right)\mathbf{x}_i \tag{21}$$

and

$$\mathbf{x}_{i,\text{pert}} \approx \mathbf{x}_i \left(1 - \tfrac{1}{2}\mathbf{x}_i^H \tilde{\mathbf{B}} \mathbf{x}_i\right) + \sum_{j=1,\, j\neq i}^{N} \frac{\mathbf{x}_j^H \left(\tilde{\mathbf{A}} - \lambda_i \tilde{\mathbf{B}}\right)\mathbf{x}_i}{\lambda_i - \lambda_j} \mathbf{x}_j, \tag{22}$$

respectively, for $i$ = 1,…, $N$.

*Proof.* We redefine the perturbed matrices as

$$\mathbf{A}(\varepsilon) = \mathbf{A} + \varepsilon\bar{\mathbf{A}},$$
$$\mathbf{B}(\varepsilon) = \mathbf{B} + \varepsilon\bar{\mathbf{B}},$$

where $\varepsilon$<<1. If we set $\varepsilon = \|\tilde{\mathbf{A}}\|$, $\bar{\mathbf{A}} = \tilde{\mathbf{A}}/\|\tilde{\mathbf{A}}\|$ and $\bar{\mathbf{B}} = \tilde{\mathbf{B}}/\|\tilde{\mathbf{A}}\|$. The perturbed generalized eigenvalue $\lambda_i(\varepsilon)$ and corresponding generalized eigenvector $\mathbf{x}_i(\varepsilon)$ satisfy the following two conditions:

$$\mathbf{A}(\varepsilon)\mathbf{x}_i(\varepsilon) = \lambda_i(\varepsilon)\mathbf{B}(\varepsilon)\mathbf{x}_i(\varepsilon), \tag{23}$$

$$\mathbf{x}(\varepsilon)_i^H \mathbf{B}(\varepsilon)\mathbf{x}_j(\varepsilon) = \delta_{i,j}, \tag{24}$$

for $i$=1,…, $N$. By the definition of the perturbed matrix pair, (23) can be expressed as

$$\left(\mathbf{A} + \varepsilon\bar{\mathbf{A}}\right)\mathbf{x}_i(\varepsilon) = \lambda_i(\varepsilon)\left(\mathbf{B} + \varepsilon\bar{\mathbf{B}}\right)\mathbf{x}_i(\varepsilon). \tag{25}$$

Assuming that both $\lambda_i(\varepsilon)$ and $\mathbf{x}_i(\varepsilon)$ are differentiable with respect to $\varepsilon$, we have

$$\bar{\mathbf{A}}\mathbf{x}_i(0)+\mathbf{A}\dot{\mathbf{x}}_i(0)=\lambda_i(0)\mathbf{B}\dot{\mathbf{x}}_i(0)+\lambda_i(0)\bar{\mathbf{B}}\mathbf{x}_i(0)+\dot{\lambda}_i(0)\mathbf{B}\mathbf{x}_i(0) \tag{26}$$

by differentiating (24) with respect to $\varepsilon$ and setting $\varepsilon = 0$. Here, $\mathbf{x}_i(0)=\mathbf{x}_i$ and $\lambda_i(0)=\lambda_i$ by definition. Because the original generalized eigenvectors $\mathbf{x}_i$ for $i$=1,…, $N$ can be used as a basis of $N$-dimensional vector space, we can express

$$\dot{\mathbf{x}}_i(0)=\sum_{j=1}^{N}a_i\mathbf{x}_j\ . \tag{27}$$

By substitution, (26) is given by

$$\bar{\mathbf{A}}\mathbf{x}_i+\mathbf{A}\sum_{j=1}^{N}a_i\mathbf{x}_i=\lambda_i\mathbf{B}\sum_{j=1}^{N}a_i\mathbf{x}_i+\lambda_i\bar{\mathbf{B}}\mathbf{x}_i+\dot{\lambda}_i(0)\mathbf{B}\mathbf{x}_i\ . \tag{28}$$

For calculation of $a_j, j\neq i$, we left-multiply (28) with $\mathbf{x}_j^H$, and we have

$$\begin{aligned}&\mathbf{x}_j^H\bar{\mathbf{A}}\mathbf{x}_i+\mathbf{x}_j^H\mathbf{A}\sum_{j=1}^{N}a_i\mathbf{x}_i\\&\quad=\lambda_i\mathbf{x}_j^H\mathbf{B}\sum_{j=1}^{N}a_i\mathbf{x}_i+\lambda_i\mathbf{x}_j^H\bar{\mathbf{B}}\mathbf{x}_i+\dot{\lambda}_i(0)\mathbf{x}_j^H\mathbf{B}\mathbf{x}_i\ .\end{aligned} \tag{29}$$

By using the two equalities of (19) and (20), (29) can be rewritten as

$$\mathbf{x}_j^H\bar{\mathbf{A}}\mathbf{x}_i+a_j\lambda_j=a_j\lambda_i+\lambda_i\mathbf{x}_j^H\bar{\mathbf{B}}\mathbf{x}_i\ . \tag{30}$$

Therefore, we have

$$a_j=\frac{\mathbf{x}_j^H\left(\lambda_i\bar{\mathbf{B}}-\bar{\mathbf{A}}\right)\mathbf{x}_i}{\lambda_j-\lambda_i}\ . \tag{31}$$

On the other hand, we also have

$$\mathbf{x}_i^H\bar{\mathbf{A}}\mathbf{x}_i=\lambda_i\mathbf{x}_i^H\bar{\mathbf{B}}\mathbf{x}_i+\dot{\lambda}_i(0) \tag{32}$$

by left-multiplying (28) with $\mathbf{x}_i^H$. Hence, we can obtain

$$\dot{\lambda}_i(0)=\mathbf{x}_i^H\left(\bar{\mathbf{A}}-\lambda_i\bar{\mathbf{B}}\right)\mathbf{x}_i. \tag{33}$$

Using the Taylor expansion, both $\lambda_i(\varepsilon)$ and $\mathbf{x}_i(\varepsilon)$ can be expressed as

$$\begin{aligned}\lambda_i(\varepsilon)&=\lambda_i(0)+\varepsilon\dot{\lambda}_i(0)+O(\varepsilon^2)\\&=\lambda_i+\varepsilon\mathbf{x}_i^H\left(\bar{\mathbf{A}}-\lambda_i\bar{\mathbf{B}}\right)\mathbf{x}_i+O(\varepsilon^2)\end{aligned} \tag{34}$$

and

$$\begin{aligned}\mathbf{x}_i(\varepsilon)&=\mathbf{x}_i(0)+\varepsilon\dot{\mathbf{x}}_i(0)+O(\varepsilon^2)\\&=\mathbf{x}_i+\varepsilon\sum_{j=1}^{N}a_j\mathbf{x}_j+O(\varepsilon^2).\end{aligned} \tag{35}$$

Since we have $a_j, j\neq i$, we must calculate the remaining $a_i$. To this end, we use the equality of (25) with $i=j$.

$$\left(\mathbf{x}_i+\varepsilon\dot{\mathbf{x}}_i(0)\right)^H\left(\mathbf{B}+\varepsilon\bar{\mathbf{B}}\right)\left(\mathbf{x}_i+\varepsilon\dot{\mathbf{x}}_i(0)\right)=1, \tag{36}$$

$$\mathbf{x}_i^H\mathbf{B}\mathbf{x}_i+\varepsilon\mathbf{x}_i^H\bar{\mathbf{B}}\mathbf{x}_i+\varepsilon\mathbf{x}_i^H\mathbf{B}\dot{\mathbf{x}}_i(0)+\varepsilon\dot{\mathbf{x}}_i(0)^H\mathbf{B}\mathbf{x}_i+O(\varepsilon^2)=1. \tag{37}$$

By using (27) and (20), we have

$$\varepsilon\mathbf{x}_i^H\bar{\mathbf{B}}\mathbf{x}_i+2\varepsilon a_i+O(\varepsilon^2)=0. \tag{38}$$

If we ignore the higher-order terms, we finally have

$$a_i=-\frac{1}{2}\mathbf{x}_i^H\bar{\mathbf{B}}\mathbf{x}_i\ . \tag{39}$$

Combining the above results and ignoring the higher-order terms, we can prove the theorem. □

With the above results on the perturbed generalized eigenvectors, we can develop an algorithm to design the MU-MIMO downlink beamformer based on the SLNR. With the predicted CSI, the perturbed matrices can be obtained. Based on the updating formula given by (22), the transmit beams can be designed. However, the approximation in (22) holds only when the norm of the perturbation matrix is much less than that of the original matrix, that is, the channel variation is not very significant. The performance with approximated beams will be degraded with the channel variation depending on the mobile velocity and duration of the transmission.

## V. Beam Updating Algorithm in Time-Varying Channels

For updating MU-MIMO downlink beams with predicted CSI in time-varying channels, the beam design scheme considering a channel prediction error should be investigated. In [8], the method used for the beam design was introduced when the variance of the channel estimation error was given. The predicted CSI is expressed as

$$\hat{\mathbf{H}}_k[n]=\mathbf{H}_k[n]+\tilde{\mathbf{H}}_k[n], \tag{40}$$

where $\tilde{\mathbf{H}}_k[n]$ denotes the channel prediction error matrix of which each element is assumed to be independent and identically distributed complex Gaussian with zero mean and a variance of $\sigma_h^2[n]$. Hence, we can also define the extended channel matrix as

$$\bar{\hat{\mathbf{H}}}_k[n]=\bar{\mathbf{H}}_k[n]+\bar{\tilde{\mathbf{H}}}_k[n], \tag{41}$$

where

$$\bar{\hat{\mathbf{H}}}_k[n]=\left[\hat{\mathbf{H}}_1^H[n],\cdots,\hat{\mathbf{H}}_{k-1}^H[n],\hat{\mathbf{H}}_{k+1}^H[n],\cdots,\hat{\mathbf{H}}_K^H[n]\right]^H$$

and

$$\bar{\tilde{\mathbf{H}}}_k[n]=\left[\tilde{\mathbf{H}}_1^H[n],\cdots,\tilde{\mathbf{H}}_{k-1}^H[n],\tilde{\mathbf{H}}_{k+1}^H[n],\cdots,\tilde{\mathbf{H}}_K^H[n]\right]^H.$$

The SNLR in terms of the $k$-th transmit signal is given by

$$\mathrm{SLNR}_k[n] = \frac{Tr\left(\mathbf{V}_k^H[n]\left(\hat{\mathbf{H}}_k^H[n]\hat{\mathbf{H}}_k[n] + N_M\sigma_h^2\mathbf{I}\right)\mathbf{V}_k[n]\right)}{Tr\left(\mathbf{V}_k^H[n]\left(N_M\sigma^2\mathbf{I} + \bar{\hat{\mathbf{H}}}_k^H[n]\bar{\hat{\mathbf{H}}}_k[n] + (K-1)N_M\sigma_h^2\mathbf{I}\right)\mathbf{V}_k[n]\right)}. \quad (42)$$

Therefore, the optimal beam matrix is obtained by the generalized eigenvectors of a pair of matrices $\left(\hat{\mathbf{H}}_k^H[n]\hat{\mathbf{H}}_k[n] + N_M\sigma_h^2[n]\mathbf{I}\right)$ and $\left(N_M\left(\sigma^2 + (K-1)\sigma_h^2[n]\right)\mathbf{I} + \bar{\hat{\mathbf{H}}}_k^H[n]\bar{\hat{\mathbf{H}}}_k[n]\right)$.

At the last symbol time of an uplink phase, a BS calculates $\mathbf{P}[0] = \mathbf{H}_k^H[0]\mathbf{H}_k[0]$ and $\mathbf{R}[0] = \left(N_M\sigma^2\mathbf{I} + \bar{\mathbf{H}}_k^H[0]\bar{\mathbf{H}}_k[0]\right)$. Here, we set the time index $n$=0 at the last symbol time in an uplink phase. With the above matrices, the generalized eigenvalues $\{\lambda_i\}$ and eigenvectors $\{\mathbf{x}_i\}$ can be obtained by a generalized eigendecomposition. Here, we assume that $\lambda_i \geq \lambda_j$ for $1 \geq i > j \geq N$. In the $k$-th downlink symbol at time step $n=m$, the predicted CSI and prediction error variance can be obtained using the prediction algorithm in Section III. A BS obtains $\mathbf{P}[m] = \left(\hat{\mathbf{H}}_k^H[m]\hat{\mathbf{H}}_k[m] + N_M\sigma_h^2[m]\mathbf{I}\right)$ and $\mathbf{R}[m] = \left(N_M\left(\sigma^2 + (K-1)\sigma_h^2[m]\right)\mathbf{I} + \bar{\hat{\mathbf{H}}}_k^H[m]\bar{\hat{\mathbf{H}}}_k[m]\right)$. Using these, the perturbation matrices from time step 0 to time step $m$ are defined by $\tilde{\mathbf{P}}[m] = \mathbf{P}[m] - \mathbf{P}[0]$ and $\tilde{\mathbf{R}}[m] = \mathbf{R}[m] - \mathbf{R}[0]$ for $m$=1,…, $T_d$, where $T_d$ stands for the duration of the downlink phase. With these perturbation terms, we determine the updated beams, that is, the dominant $d$ generalized eigenvectors $\mathbf{x}_{i,\mathrm{pert}}[m]$ for $i$=1,…, $d$, with the first-order approximation using Theorem 1. This approximation is valid only when the channel variation is not significant, that is, $\|\tilde{\mathbf{P}}[m]\| << 1$ and $\|\tilde{\mathbf{R}}[m]\| << 1$. If the norms of the perturbation terms are not sufficiently small, the error in the approximated beams creates an additional interference leakage, which causes SLNR and rate losses in MU-MIMO downlink systems.

## VI. Analysis of SLNR for Predictive Beam Design

In the above section, we proposed an efficient beam design algorithm based on the perturbation theory of generalized eigenvectors in a time-varying channel. To evaluate the performance of the proposed scheme, we analyze the SLNR of the proposed predictive beamforming method. For a simple analysis, we assume a single stream transmission with $d$=1.

First, we evaluate the SLNR at the given time step in a downlink phase, assuming perfect channel prediction. The SLNR at time $n$ is expressed as

$$\begin{aligned}\mathrm{SLNR}_k[n] &= \frac{\left\|\mathbf{H}_k[n]\mathbf{x}_{1,\mathrm{pert}}[n]\right\|^2}{N_M\sigma^2 + \left\|\bar{\mathbf{H}}_k[n]\mathbf{x}_{1,\mathrm{pert}}[n]\right\|^2} \\ &= \frac{\mathbf{x}_{1,\mathrm{pert}}^H[n]\left(\mathbf{H}_k^H[n]\mathbf{H}_k[n]\right)\mathbf{x}_{1,\mathrm{pert}}[n]}{N_M\sigma^2 + \mathbf{x}_{1,\mathrm{pert}}^H[n]\bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n]\mathbf{x}_{1,\mathrm{pert}}[n]} \\ &= \frac{\mathbf{x}_{1,\mathrm{pert}}^H[n]\left(\mathbf{H}_k^H[n]\mathbf{H}_k[n]\right)\mathbf{x}_{1,\mathrm{pert}}[n]}{\mathbf{x}_{1,\mathrm{pert}}^H[n]\left(N_M\sigma^2\mathbf{I} + \bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n]\right)\mathbf{x}_{1,\mathrm{pert}}[n]} \\ &\approx \lambda_{1,\mathrm{pert}}[n]. \end{aligned} \quad (43)$$

The last equality holds because $\left(\mathbf{H}_k^H[n]\mathbf{H}_k[n]\right)\mathbf{x}_{1,\mathrm{pert}}[n] \approx \lambda_{1,\mathrm{pert}}[n]\left(N_M\sigma^2\mathbf{I} + \bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n]\right)\mathbf{x}_{1,\mathrm{pert}}[n]$. We can obtain the perturbed eigenvalue with (21) in Theorem 1. Accordingly, we have

$$\begin{aligned}\lambda_{1,\mathrm{pert}}[n] \approx \lambda_1 + \mathbf{x}_1^H\{&(\mathbf{H}_k^H[n]\mathbf{H}_k[n] - \mathbf{H}_k^H[0]\mathbf{H}_k[0]) \\ &- \lambda_1(\bar{\mathbf{H}}_k^H[n]\bar{\mathbf{H}}_k[n] - \bar{\mathbf{H}}_k^H[0]\bar{\mathbf{H}}_k[0])\}\mathbf{x}_1. \end{aligned} \quad (44)$$

Therefore, when the channel information is given, we can calculate the approximate SLNR at each time step.

For an analysis of the SLNR under more practical conditions employing channel prediction, we use the first-order AR channel model since it makes the closed-form expression for the variance of channel prediction error possible, as shown in section III. For accurate modeling of time-varying channels, the higher-order AR model can be used. Since the extension of the higher-order channel model is possible without difficulty, we only consider the first-order channel model in this paper. As shown in section III, the predicted channels at the $n$-th symbol time in a downlink phase are given by

$$\hat{\mathbf{H}}_k[n] = \beta^n\mathbf{H}_k[0] \quad (45)$$

and

$$\bar{\hat{\mathbf{H}}}_k[n] = \beta^n\bar{\mathbf{H}}_k[0] \quad (46)$$

with an assumption that all MIMO channel components have the same fading coefficient $\beta$. With the above relation and channel prediction error variance given by (18), we can simply compute the perturbation matrices as

$$\tilde{\mathbf{P}}[n] = (\beta^{2n} - 1)\mathbf{H}_k^H[0]\mathbf{H}_k[0] + N_M(1 - \beta^{2n})\mathbf{I} \quad (47)$$

and

$$\tilde{\mathbf{R}}[n] = N_M(K-1)(1 - \beta^{2n})\mathbf{I} + (\beta^{2n} - 1)\bar{\mathbf{H}}_k^H[0]\bar{\mathbf{H}}_k[0]. \quad (48)$$

Hence, the SLNR, which is obtained by the perturbed dominant eigenvalue, is given by

$$
\begin{aligned}
SLNR[n] &\approx \lambda_{1,\text{pert}}[n] \\
&\approx \lambda_1 + \mathbf{x}_1^H \Big\{ \big( (\beta^{2n}-1)\mathbf{H}_k^H[0]\mathbf{H}_k[0] + N_M(1-\beta^{2n})\mathbf{I} \big) \\
&\quad - \lambda_1 \big( N_M(K-1)(1-\beta^{2n})\mathbf{I} + (\beta^{2n}-1)\bar{\mathbf{H}}_k^H[0]\bar{\mathbf{H}}_k[0] \big) \Big\} \mathbf{x}_1 \\
&= \lambda_1 + \mathbf{x}_1^H \Big\{ \big( (\beta^{2n}-1)\mathbf{H}_k^H[0]\mathbf{H}_k[0] + N_M(1-\beta^{2n})\mathbf{I} \big) \\
&\quad - \lambda_1 \big( N_M(K-1)(1-\beta^{2n})\mathbf{I} + (\beta^{2n}-1)\bar{\mathbf{H}}_k^H[0]\bar{\mathbf{H}}_k[0] \big) \\
&\quad - \lambda_1 \big( N_M(\beta^{2n}-1)\sigma^2 + N_M(1-\beta^{2n})\sigma^2 \big)\mathbf{I} \Big\} \mathbf{x}_1 \\
&= \lambda_1 + \Big\{ N_M(1-\beta^{2n}) \big( 1 - \lambda_1(1-\beta^{2n})(K-1+\sigma^2) \big) \Big\}. \qquad (49)
\end{aligned}
$$

On the RHS of the third equality, we add $N_M(\beta^{2n}-1)\sigma^2 + N_M(1-\beta^{2n})\sigma^2 = 0$. The last equality holds by the definition of a generalized eigenvalue and the corresponding eigenvector $\mathbf{x}_1^H \left( \mathbf{H}_k^H[0]\mathbf{H}_k[0] \right) \mathbf{x}_1 = \lambda_1 \mathbf{x}_1^H \left( \bar{\mathbf{H}}_k^H[0]\bar{\mathbf{H}}_k[0] + N_M\sigma^2\mathbf{I} \right) \mathbf{x}_1$. As shown in the last equality of (49), the SLNR can be calculated using only the unperturbed eigenvalue, that is, the SLNR at the first time step, fading coefficient, and noise variance.

The analytic results in this section will be verified by numerical simulations in the following section.

## VII. Numerical Results

In this section, we evaluate the performance, including the interference level, SLNR value, and spectral efficiency of MU-MIMO downlink beamforming systems to verify the proposed algorithm. To show the effectiveness of the proposed method, we consider two extreme beam design methods as conventional approaches. As the first conventional approach, we consider the perfect beam design in which the beams at each time step are obtained by full computation of a dominant generalized eigenvector at every time step with a new CSI. Though this method shows the best performance, its computational complexity is too high to implement practically. The other conventional approach is to use the beamformers obtained with the CSI in the previous uplink phase, that is, $\{\mathbf{V}_k[0]\}$, for all downlink phases at time step $n$=1, 2,…, $T_d$. Under quasi-static channel conditions in which the channel is fixed during a total frame with the uplink and downlink phases, this approach does not cause any performance loss. In realistic systems, the assumption of quasi-static channels is not valid, and the mismatch between beams and current CSI bring about greater interference leakage and loss in the sum rate performance. Therefore, we can see that the perfect beam design and non-updating approach show the upper and lower bounds of the performance, such as the interference leakage, SLNR, and sum rate.

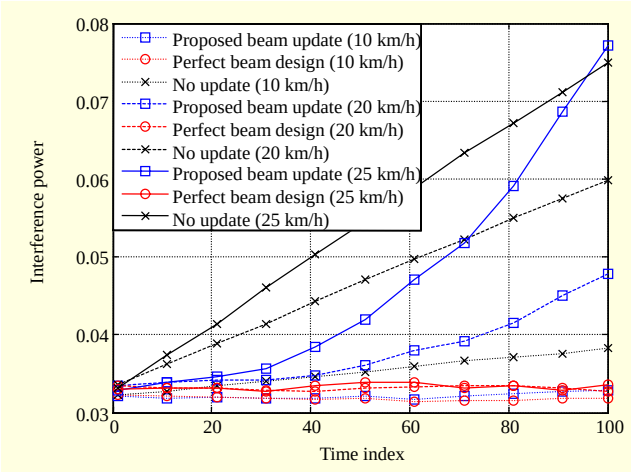


Fig. 2. Interference power along downlink symbols at one MS at different speeds (perfect CSI, $N_B$=3, $N_M$=1, $K$=3, $d$=1, SNR=12 dB).

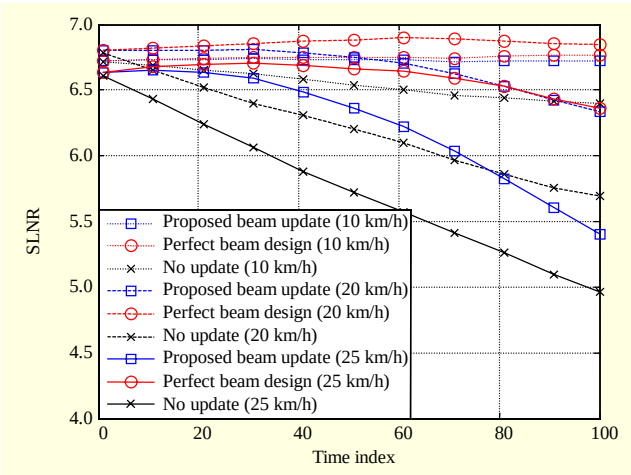


Fig. 3. SLNR along downlink symbols with different speeds (perfect CSI, $N_B$=3, $N_M$=1, $K$=3, $d$=1, SNR=12 dB).

To realize time-varying channels, we use the first-order AR model since it makes a theoretical analysis of the SLNR performance possible. In this paper, for a simple analysis, we assume that all downlink channels from a BS to the MSs have the same fading coefficient. For the numerical simulations, we assume a 2.0-GHz carrier frequency and a symbol duration of 66.7 μs, which is the OFDM symbol duration of 3GPP LTE. The number of symbols in the downlink phase is 100, that is, $T_d$ = 100. The results are obtained by averaging the values from 5,000 independent channel realizations.

Figures 2, 3, and 4 show the interference power, SLNR, and spectral efficiency against the downlink time index using three different approaches, respectively. In these results, we assume that a BS knows the CSI of all downlink channels perfectly. As expected, the non-updating scheme, which fixes beams during the entire downlink period, has the worst performance since it

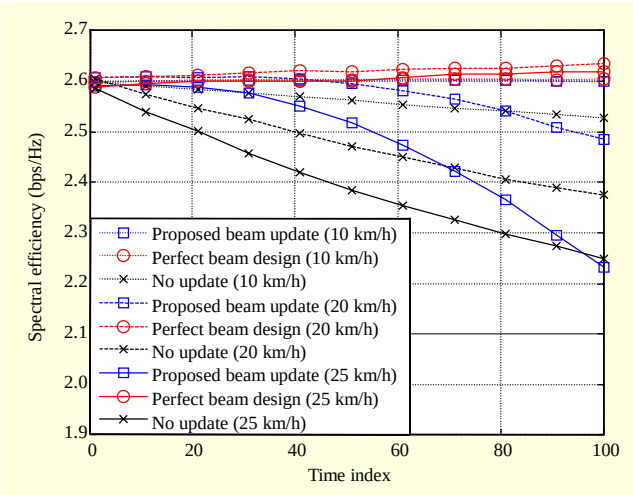


Fig. 4. Spectral efficiency along downlink symbols at one MS with different speeds (perfect CSI, $N_B$=3, $N_M$=1, $K$=3, $d$=1, SNR=12 dB).

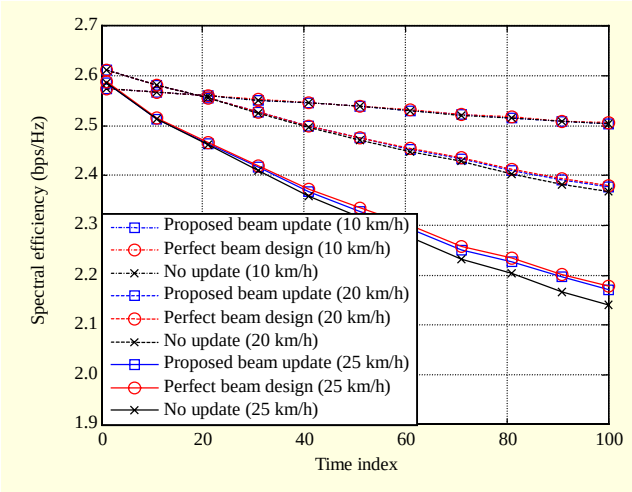


Fig. 5. Spectral efficiency along downlink symbols at one MS with different speeds (predicted CSI based on first-order AR model, $N_B$=3, $N_M$=1, $K$=3, $d$=1, SNR=12 dB).

cannot reflect the channel variations. Its performance loss increases with the mobile speed. The perfect design method recalculating the generalized eigenvector at every time step shows the best performance, which is almost the same regardless of the mobile speed since it can perfectly track the variations of the channels. The performance of the proposed beam updating method is in between those of the non-updating and perfect designs. The performance loss of the proposed design from the perfect design scheme increases with the time index and mobile speed since the norm of perturbation matrices increases with them. As we can see in Theorem 1, the inaccuracy of a perturbed generalized eigenvector is directly related to the amount of perturbations. In an extreme case with a mobile speed of 30 km/h at time index $n$=100, the performance of the proposed scheme is worse than the non-updating approach. Therefore, we can see that the proposed method should be used in a case with a low mobile speed and short downlink duration, that is, small $f_dT_s \times T_d$. When the mobile speed is less than 10 km/h, the proposed method shows nearly the same performance as that of the perfect design approach, even with very low complexity.

Figure 5 shows the spectral efficiencies with the same conditions as the above results, except for the use of channel prediction based on the first-order AR channel model. When the channel prediction is also included, the performance gaps among the three approaches decrease. The channel prediction error is dominant over the inaccuracy of the beams, and the tracking capability of the channel variation in the beam design is thus rather insignificant. However, if we can reduce the channel prediction error, spectral efficiencies of the three approaches can be close to those in the case of the perfect CSI, as shown in Fig. 4. Although we use the first-order AR channel

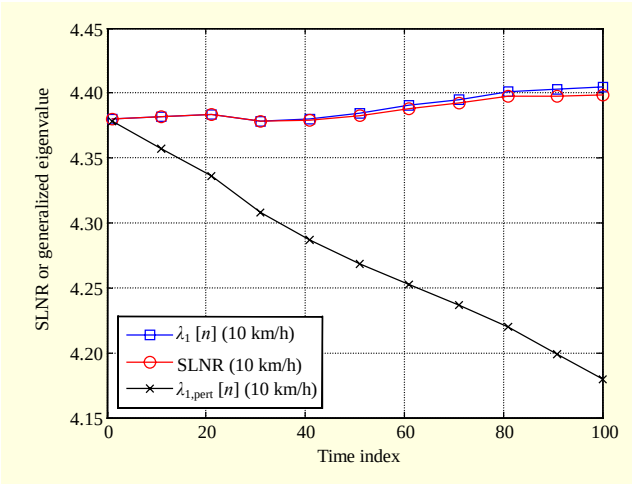


Fig. 6. SLNR and generalized eigenvalues along downlink symbols at one MS with different speeds (perfect CSI, $N_B$=3, $N_M$=1, $K$=3, $d$=1, SNR=10 dB).

model for a simple analysis in this paper, we can use the higher-order channel model. A closed-form analysis is not possible with the higher-order model, but it can provide a more accurate model of the Rayleigh fading channels and a better channel prediction performance [11].

To verify the analysis in section VI, we show the SLNR and generalized eigenvalues in Figs. 6 and 7, where $\lambda_1[n]$ and $\lambda_{1,\mathrm{pert}}[n]$ in the legend are the exact generalized eigenvalues obtained by the given channel matrices, and the perturbed value obtained by (21). The SLNR is calculated using the channel matrices and the updated beams, that is, the perturbed eigenvectors given by (22). Figure 6 shows that the perturbed eigenvalue $\lambda_{1,\mathrm{pert}}[n]$ can provide an estimate of the SLNR with 95% accuracy. The estimation error comes from the inaccuracy

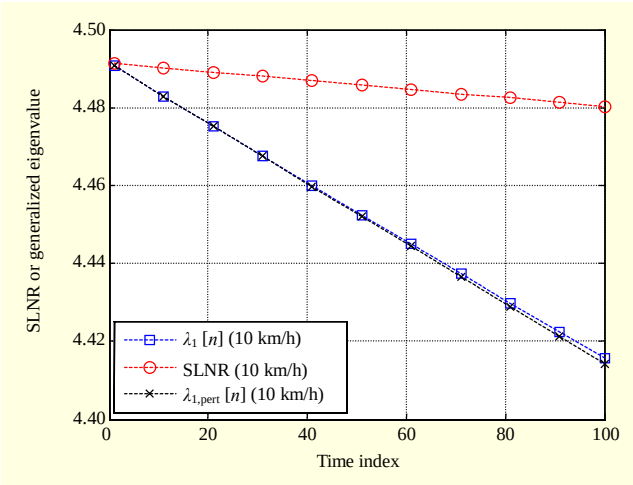


Fig. 7. SLNR and generalized eigenvalues along downlink symbols at one MS with different speeds (predicted CSI based on first-order AR model, $N_B$=3, $N_M$=1, $K$=3, $d$=1, SNR=10 dB).

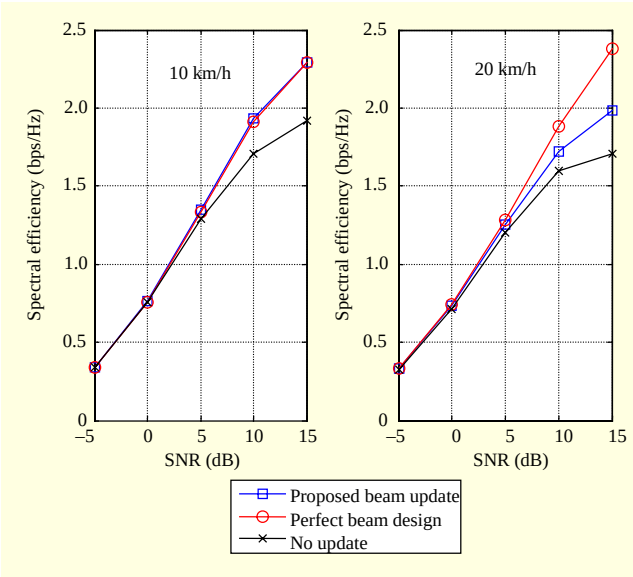


Fig. 8. Spectral efficiencies of three different beam design methods along SNR at the last downlink symbol ($n$=$T_d$ =100, predicted CSI based on first-order AR model, $N_B$=4, $N_M$=2, $K$=3, $d$=1, mobile speed=10 km/h and 20 km/h).

of both a perturbed eigenvector and perturbed eigenvalue. In Fig. 7, $\lambda_1[n]$ and $\lambda_{1,\text{pert}}[n]$ are calculated using the predicted channels, which are almost the same. Owing to a channel prediction error, the eigenvalue is less than the exact SLNR values with up to a 5% difference. In cases with low mobility, the perturbed eigenvalue can provide the SLNR estimate, which we can use to estimate the performance level of MU-MIMO downlink systems.

Finally, in Fig. 8, we show the spectral efficiencies against the SNR at the last symbol time of the downlink phase with different system configurations, including the number of antennas. It can be seen that the proposed algorithm can provide significant gain over the non-updating method with low complexity.

## VIII. Conclusion

We proposed an efficient beam design algorithm for an MU-MIMO downlink channel. The algorithm computed the beamforming matrices that maximize the SLNR using the perturbed generalized eigenvectors without any complicated operations or iterations. Owing to the nature of the downlink beamforming system, we introduced a channel prediction based on the AR channel model, which we used to design the beams. The SLNR performance of the proposed beamformer was analyzed using the perturbed generalized eigenvalues. Through numerical simulations, we showed the effectiveness of the proposed beam design and verified our analytical results.

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