LDPC Coded Combined with Space-Time Codes in Cooperative MIMO System

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Abstract—**This paper proposes a method of LDPC coded combined with space-time codes using in the cooperative MIMO system. An algorithm is introduced that the joint iterative detection algorithm based on Fincke-pohst (sphere detecting) maximum a-posteriori (MAP). theoretical analysis and derivation shown that the performance of LDPC cascading space-time codes (STBC) system is slightly better than that of LDPC coded cascading vertical layered space-time codes (VBLAST) in cooperative MIMO systems. And the FP-MAP detection algorithm compares with the detection algorithm----zero forcing sort plus interference cancellation(ZF-OSIC) and the recursive least squares decision feedback equalization(RLS-DFE) algorithm, its performance is improved by about 0. 6dB and 0. 3dB respectively, its performance is closest to the ML detection.**

Index Terms—**Low-Density Parity-Check Codes; Space-Time Codes; Cooperative MIMO; Sphere Algorithm; Iterative Det-Ection**

I. INTRODUCTION

In multi-input-multi-output (MIMO) systems, spatial diversity can improve the transmission reliability of the system without increasing the transmission time and bandwidth. However, it is not convenient to install multiple antennas on one terminal because of the limitations of volume and power dissipation of mobile station. Hence, the cooperative MIMO system is achieved by the single-antenna terminal to form the virtual multi-antenna array. The single-antenna terminal is combined with the multi-antenna receiving at the receiving base station [1]. Space-time coding [2] is a new coding and signal processing technique in the filed of mobile communications. In this technique, there are multiple antennas at both the transmitting end and the receiving end to be responsible for the transmitting and receiving of signals. The temporal and spatial correlations are introduced between the signals transmitted by different antennas. The two-dimensional temporal and spatial information is utilized for the diversity receiving at the receiving end. Space-time coding integrates the spatial diversity, frequency diversity and temporal diversity together, and improves the communication quality and quantity of the multipath fading channel by taking the communication system as a whole. Applying the space-time coding theory for MIMO to the cooperative MIMO system would lead to the full cooperative diversity of multiple distributed terminals by collaborating.

Low-density parity-check code (LDPC) was proposed by Gallager in 1962 [3]. Because of the strong error-correcting capability, the decoding algorithm with low complexity, and the performance approaching Shannon limit, the LDPC code has been widely used in wireless communication [4]. A scheme of LDPC cascading space-time block codes was proposed in literature [5]. It has been proved effective in MIMO systems. A new distributed VBLAST adaptive DFE algorithm in cooperative MIMO was proposed in literature [6].

In this study, LDPC and cooperative MIMO system with space-time coding are further analyzed. An FP-MAP algorithm based on sphere algorithm is proposed, and the iteration performance of the combined iteration detection is analyzed. Finally, the bit error probability performance of the proposed detection algorithm is compared with that of the ZF-OSIC, RLS-DFE and ML algorithm.

The remainder of this paper is organized as follows: The system model is described in section II, and section III propose iterative detection procedure based on the sphere coding algorithm. The analysis of simulation results is presented in section VI. Section V summarizes the results and concludes the paper.

II. SYSTEM MODEL

Suppose that the number of cooperative relay nodes around the information source node S is n and the number of antennas of basestations at the receiving end is N_R. For the cooperative MIMO, when the time slot is 1, data packets are broadcasted to the neighboring cooperative nodes by S; when the time slot is 2, the cooperative nodes and S constitute an transmitting end with $n+1$ virtual antennas. The space-time coding (taking VBLAST as an example) is utilized for the spatial multiplexing of $n+1$ cooperative nodes. The receiving end with N_R antennas is responsible for multi-user detection to accomplish the decoding and the reconfiguration of the cooperative signal.

The signal model of an LDPC cascading space-time coding system with M transmitting antennas and N receiving antennas ($N \geq M$) could be represented by

$$y = Hs + n \qquad (1)$$

where y is the vector of the receiving signal at some moment, and the channel matrix H is the N*M matrix in complex field. Besides, the matrix element $h_{ij}(i=1,...,N, j=1,...,M)$ denotes the fading coefficients of the channel from transmitting antenna j to receiving antenna i. These elements are statistically independent, and follow the $N(0,1)$ distribution. N-dimensional vector n is a zero mean complex Gaussian white noise. The covariance matrix is $E(nn^H) = \sigma_n^2 I_N$.
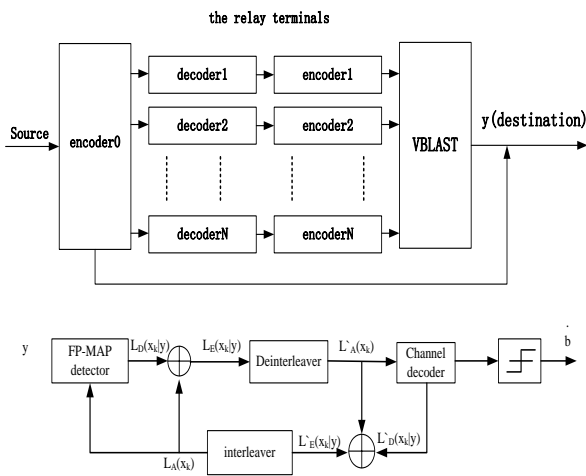
the relay terminals

Figure 1.   Block diagram of the cooperative MIMO system with space-time coding cascaded to LDPC

III.   ITERATIVE DETECTION BASED ON THE SPHERE ALGORITHM

The soft-decision detection algorithm is necessary for the MIMO detector since the LDPC decoder adopts the soft-input soft-output algorithm. It could be known from the receiving end where the detection and collaborative iteration are conducted that the MIMO detector and the channel decoder would exchange information during the iteration process. In the MIMO detection algorithm characterized by soft information input and soft decision output, extrinsic information is obtained by calculating the posteriori information of each bit and subtracting the prior information of the input from it. Then the extrinsic information is used as the input of priori information of the channel encoder / decoder. The posteriori information of the decoder is subtracted from the priori information as the input of the MIMO detector.

1) Calculation of the Output Posteriori Information and Extrinsic Information of the MIMO Detector

Suppose that the transmitting symbol vector is $s = (s_1, s_2, \cdots, s_{N_T})$, where s_i is the 2^M -order modulation symbol, then the posterior likelihood ratio information of each bit in the corresponding bit sequence $x = (x_1, x_2, \cdots, x_{N,M})$ is given by

$$L_D(x_k|y) = \ln \frac{p[x_k = 1|y]}{p[x_k = 0|y]}$$

$$= \ln \frac{\sum_{x \in X_{K,1}} p(y|x)p(x)}{\sum_{x \in X_{K,0}} p(y|x)p(x)}$$

$$= \ln \frac{p(x_k = 1)\sum_{x \in X_{K,1}} p(y|x)\prod_{j \in u_k} p(x_j)}{p(x_k = 0)\sum_{x \in X_{K,0}} p(y|x)\prod_{j \in u_k} p(x_j)} \qquad (2)$$

$$= \ln \frac{p(x_k = 1)}{p(x_k = 0)} + \ln \frac{\sum_{x \in X_{K,1}} p(y|x)\prod_{j \in u_k} p(x_j)}{\sum_{x \in X_{K,0}} p(y|x)\prod_{j \in u_k} p(x_j)}$$

where $X_{k,1} = \{x|x_k = 1\}$ denotes the set of all bit sequences with the k -th bit being 1, $X_{K,0} = \{x|x_k = 0\}$ denotes the set of all bit sequences with the k -th bit being 0, $u_k = \{j|j=1,\cdots,N,M, j \neq k\}$ denotes the set of symbols excluding K, $p(x_j)$ denotes the prior probability information of x_j , and $p(y|x)$ denotes the conditional probability of y being received by the transmitted bit sequence x .

Thus, let $L_A(x_k) = \ln \frac{p(x_k = 1)}{p(x_k = 0)}$ be the priori information of x_k . The priori information refers to the information from the channel decoding.

Let $L_E(x_k|y) = \ln \frac{\sum_{x \in X_{K,1}} p(y|x)\prod_{j \in u_k} p(x_j)}{\sum_{x \in X_{K,0}} p(y|x)\prod_{j \in u_k} p(x_j)}$ be the extrinsic information of x_k . Therefore, the posteriori information of x_k would be

$$L_D(x_k|y) = L_A(x_k) + L_E(x_k|y) \qquad (3)$$

During the first iteration, suppose that the transmitted signal has with equal probability, and the channel is AWGN, $L_A(x_k) = 0$,

$$L_E(x_k|y) = \log \frac{\sum_{s \in S_{x_k=1}} \exp(-\frac{\|y - Hs\|^2}{2\sigma_n^2} + \sum_{j, j \neq k} \log p(x_j))}{\sum_{s \in S_{x_k=0}} \exp(-\frac{\|y - Hs\|^2}{2\sigma_n^2} + \sum_{j, j \neq k} \log p(x_j))} \qquad (4)$$

σ_n^2 is the additive noise covariance. According to the maximum logarithm approximation principle (Max-log approximation), the equation above could be transformed into

$$L_E(x_k|y) \cong \frac{1}{2} \max_{x \in X_{K,1}} \{-\frac{1}{\sigma_n^2}\|y - Hs\|^2 + X_{[k]}^T \cdot L_{A[k]}(x_k|y)\}$$

$$-\frac{1}{2}\max_{x \in X_{k,0}} \{-\frac{1}{\sigma_n^2}\|y - Hs\|^2 + X_{[k]}^T \cdot L_{A[k]}(x_k|y)\} \qquad (5)$$

Here $X_{[k]}$ denotes the sub-vector of the k-th element, and $L_{A[k]}(x_k|y)$ denotes the sub-vector of k-th element.

Solving the Symbol Sequence Vector S that Satisfies the Conditions (Estimation)

The optimal information transmission of the MAP detector could be obtained by testing all possible combinations of symbol sequences. However, the calculation might be very complex in this situation. The complexity could be decreased by decreasing the searching space. Thus, the FP-MAP sphere detection algorithm [7] could be used. It enables a small searching range but an effective space to decrease the complexity.

The maximum likelihood decoding (ML) is to find a signal from all possible transmitted signals that satisfies

$$\hat{s} = \arg\min \|y - Hs\|^2 \qquad (6)$$

That is, the equation (14) is solved to select the minimum s as the estimation of the transmitted signal. where s is an element in the vector set of the transmitted signals. Although the decoding performance of the maximum likelihood decoding algorithm is excellent theoretically, and has a minimum error probability, the complexity of the algorithm, the number of transmitting antennas, and the points of the modulation constellation have an exponential relation. As a result, it is not feasible in applications. The complexity of computation is very high, and it can be viewed as a performance boundary to judge the performance of other decoding algorithms.

The FP-MAP detector is also a MAP detector. The basic idea of sphere decoding is that a sphere with the signal receiving point as the center is preset in the space of signal receiving. Next, the sphere is mapped to an elliptical sphere in the space of signal transmitting, and the signal transmitting point is searched inside the elliptical sphere. Once the point is found, the preset sphere would shrink with the distance between the mapping point of the signal point and the received signal as the radius. Then further searches would be performed in a smaller range. If the number of points included in the region is rather small in relation to the total number of points in the whole grid, the searching time would decrease greatly [8].

It could search for the set of symbols (grid points) in a sphere with a certain radius and estimate the soft information

$L_E(x_k|y)$ by the MAP criterion.

$$\hat{s}_{map} = \arg\max_{s \in S^{n_T}} p(s|y) \qquad (7)$$

By using the Bayesian rule, the MAP algorithm would be transformed into

$$\hat{s}_{map} = \arg\max_{s \in S^{n_T}} \frac{p(y|s)p(s)}{p(y)} = \arg\max_{s \in S^{n_T}} p(y|s)p(s) \qquad (8)$$

Suppose that the symbols are transmitted independently, then there is

$$p(s) = \prod_{k=1}^{n_T} p(s_k) = \exp[\sum_{k=1}^{n_T} \log p(s_k)] \qquad (9)$$

Suppose that in AWGN channel, the MAP criterion in equation (6) could be rewritten as another optimization problem, thus turning the problem into minimization of the bit error probability.

$$\hat{s}_{map} = \arg\min_{s \in S_{n_T}}[\|y - Hs\|^2 - \sum_{k=1}^{n_T} p(s_k)] \qquad (10)$$

The goal of an FP-MAP detector is to find the approximation which would provide correct soft information $L_E(x_k|y)$, rather than finding the value of $\hat{s}_{map}$ [9] [10]. The criterion above is used for generating a hypersphere $\sqrt{d^2 + \sigma_n^2 \sum_{k=1}^{n_T} \log p(s_k)}$, namely:

$$d^2 + \sigma_n^2 \sum_{k=1}^{n_T} \log p(s_k) \geq \|y - Hs\|^2 \qquad (11)$$

3) After the de-interleaving of the extrinsic information $L_E(x_k|y)$, it is used as the priori information of the bit encoding $L_A^{'}(x_k)$ and transmitted to the BP decoder (the information of the detector is transferred to the decoder).

Suppose $N(w)$ denotes the set of nodes adjacent to the checking nodes (vertices) in the bipartite graph w. When $k \in N(w)$, it means that the checking nodes k and w share a common edge. Suppose $N(w)/k$ denotes the set of all adjacent nodes excluding the node k. Suppose $q_{w \to k}(x)$, $x \in \{0,1\}$ denotes the information transmitted from variable node w to checking node k. Besides, the probability of the variable node w is 0 or 1, and all checking nodes adjacent to variable node w do not include node k. Similarly, suppose $r_{k \to w}(x)$ and $x \in \{0,1\}$ denote the information transmitted from the checking node k to the variable node w, and all variable nodes adjacent to checking node k do not include node w. Then according to the LLR principle, the information transmitted from variable node to checking node and the information transmitted from checking node to variable node are defined as follows, respectively:

$$\lambda_{w \to k}(x_n) = \log(\frac{q_{w \to k}(1)}{q_{w \to k}(0)})$$
$$\Lambda_{k \to w}(x_n) = \log(\frac{r_{k \to w}(1)}{r_{k \to w}(0)}) \qquad (12)$$

Here $n = \{1, 2, \cdots, m\}$. In the initialization phase, each variable node is configured with a corresponding log-likelihood value. Considering the consistency of the description of the iteration system at the receiving end, the log-likelihood value could be obtained by the MIMO

detector, represented as $L_A^{'}(x_k)$. Thus, suppose $L_A^{'}(x_k) = \lambda_{w \to k}(x_n)$ and $\Lambda_{k \to w}(x_n) = 0$.

Therefore, the updating information (which is the transmitted information from any checking node k to any variable node w connected to it, where $w \in N(k)$) of the checking node would be given as

$$\Lambda_{k \to w}(x_n) = 2 \tanh^{-1}(\prod_{w^{'} \in N(k)/w} \tanh(\frac{\lambda_{w^{'} \to k}(x_n)}{2})) \quad (13)$$

The updating (which is the information transmitted from any variable node n to any checking node m adjacent with it) of variable node is represented as

$$\lambda_{w \to k}(x_n) = L_A^{'}(x_k) + \sum_{k^{'} \in N(w)/k} \Lambda_{k^{'} \to w}(x_n) \quad (14)$$

The obtained total information (reliability) of any variable node n by the end of the current iteration is given by

$$\lambda_w(x_n) = L_A^{'}(x_k) + \sum_{k \in N(w)} \Lambda_{k \to w}(x_n) \quad (15)$$

The pre-decision is based on equation (15). If $\lambda_w(x_n) \geq 0$, then the bit $x_n = 0$ is transmitted. Otherwise, the bit $x_n = 1$ is transmitted. Then, $X = \{x_1, x_2, \cdots x_N\}$ is the decision vector of the decoding at this time. If $XH^T = 0$, then the decoding is successful. Otherwise, equation (13) would be solved again to continue the iteration. If it reaches the maximum iteration number, $XH^T \neq 0$, then the iteration is stopped, and the decoding fails.

While the soft decision decoding is performed by the BP decoder, the extrinsic information of the bit encoding $L_E^{'}(x_k | y)$ is provided. After the extrinsic information is interleaved, it is used as the updated priori information $L_A^{'}(x_k)$ of the bit encoding, and is transmitted to the detector (the information of decoder is transmitted to the detector).

That is to say, all codewords would generate soft extrinsic information $L_E^{'}(x_k | y) = L^k(Q_n) - L_A^{'}(x_k)$ at the same time and one iteration is completed. It should be noted that the extrinsic information here is obtained by subtracting the priori information from the complete soft information. Here the priori information comes from the MIMO detector.

IV. ANALYSIS OF SIMULATION RESULTS

Suppose that the channel is AWGN, and the information of the source node is processed by LDPC coding. A checking matrix consisting of 6 rows and 3 columns is randomly generated with the Mackay construction method, and the number of overlapping non-zero elements between any two columns cannot be larger than 1. The code length is 512, and the BPSK modulation is adopted. The coded cooperative method is utilized for the relay nodes and the BP decoding is

performed. The initial radius of the FP sphere algorithm is set as $d_0 = 2.83$, and the interleaving degree is 512. The cooperative MIMO system consisting of 2 transmitting antennas, 2 relay nodes and 2 receiving antennas is used.

A. Performance Analysis of Iteration

First, the iteration is performed extrinsically for one time, and BP decoding is performed for several times. Then the iteration performance is compared (Figure 2). Figure 3 shows that the performance is relatively stable when the times of BP decoding reaches 12. Then suppose the times of extrinsic iteration is 2, 3, 4 and 5, respectively, and the times of stable BP decoding iterations (12 times) is used. The bit error rates are compared when internal times=1, internal times =2, internal times=3, internal times=4 and internal times=5, and the result is shown in Figure3.
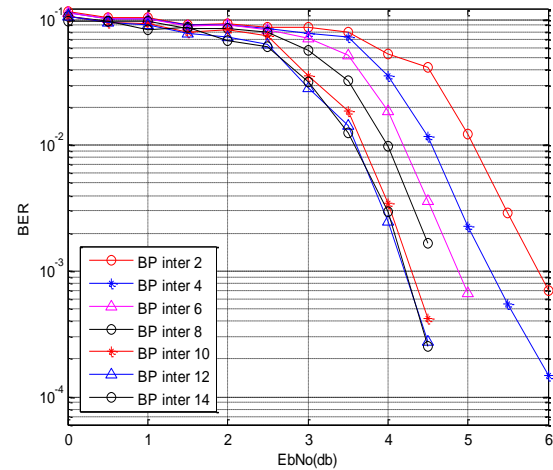

Figure 2. BER performance comparison of BP decoding at different internal iterations with one external iterative

Figure 2 shows that when the extrinsic iteration is done for once, the greater the times of BP iterations is, the better the performance, and the performance reaches a saturation level when the iteration times is 12. then next step, the external iteration times is increased when the times of BP decoding iteration is set to be 12 to study the overall system performance.
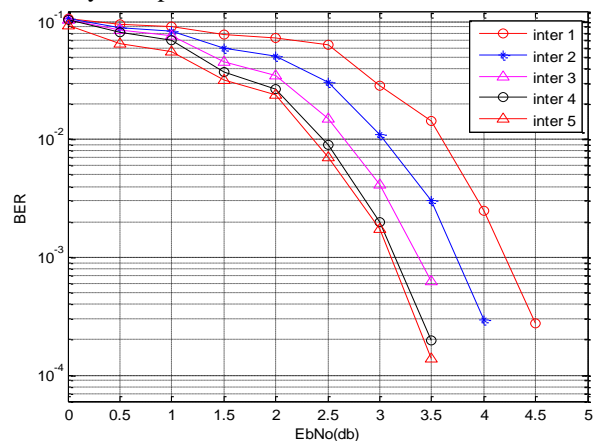

Figure 3. BER cure of comparison of the internal iteration times with external iteration times=12

Figure 3 shows that the performance of the system is improved by increasing the number of extrinsic iterations. It is a 0. 5dB improvement of the performance compared with iteration for only once, and it is a 0. 3dB improvement when the iteration time =3 compared with the iteration times=2. However, the edge performance gain is decreasing with the increasing of iteration number. If the increase is too much, the correlation of soft information transmitted between detector and the decoder is increasing, while the obtained extrinsic information is decreasing. Thus, the improvement of the system performance is reducing, and the processing time delay and decoding complexity would increase.

B. Analysis of Bit Error Rate Performance

In the cooperative MIMO system with space-time coding cascaded to LDPC, with two transmitting antennas, two relay nodes and two receiving nodes. BPSK modulation is used. LDPC is cascaded to STBC. Then the bit error rate performance of the FP-MAP algorithm is analyzed and compared with that of the ZF-OSIC, RLS-DFE [11] [12] and ML algorithm.
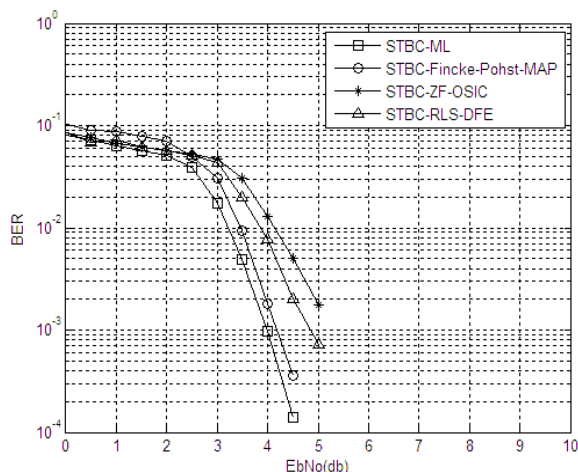

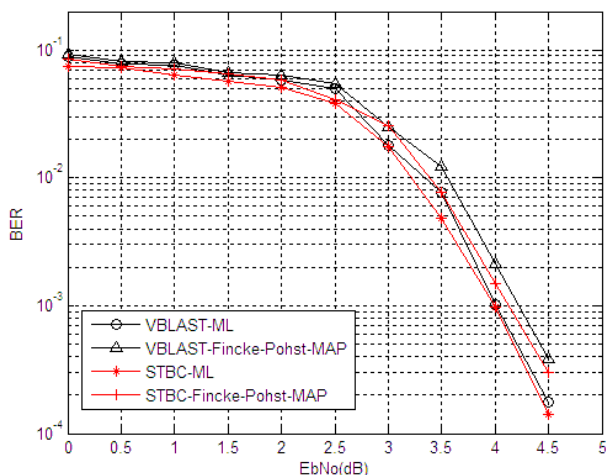Figure 4.   Comparison of the BER cure of the FP-MAP algorithm with ZF-OSIC, ML, and RLS-DFE

Figure 5.   Performance Comparison between ML Algorithm and FP-MAP Algorithm with LDPC Cascaded with Two Types of Space-Time Coding

According to Figure 4, the bit error rate performance of FP-MAP algorithm is close to ML algorithm. At BER=10^{-2}, the BER performance of LDPC that cascading to STBC is improved by about 0.6dB and 0.3dB respectively compared with ZF-OSIC and RLS-DFE algorithms in the Cooperative MIMO System. Because STBC code can obtain high diversity gain, it can be good to reduce the bit error rate. According to Figure 5, the STBC cooperative MIMO system using LDPC code outperforms the VBALST cooperative MIMO system using LDPC code.

V.   CONCLUSIONS

In this paper, space-time coding cascaded to LDPC is introduced in the cooperative MIMO system. By simulating, it shows that the bit error rate performance of the STBC cooperative MIMO system using LDPC code outperforms the VBLAST system using LDPC code slightly. Besides, the performance of the iteration detection which combines the detector at the receiving end and the BP decoder is better than the RLS-DFE algorithm, ZF-OSIC algorithm and the FP-MAP algorithm. Its performance is close to the that of the ML algorithm.

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