

# Discrete Point Cloud Registration using the 3D Normal Distribution Transformation based Newton Iteration

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**Abstract**—The technology of three-dimensional reconstruction based on visual sensor has become an important research aspect. Based on Newton iteration algorithm, the improved 3D normal distribution transformation algorithm” (NI-3DNDT) is put forward, aiming to fix the problem of discrete point cloud registration algorithm in poor astringency and being open to local optimum. The discrete 3d point cloud adopts one order and two order derivative of piecewise smooth functions on surface, divides the point cloud space into Cubic grids, and calculate corresponding value of the mean and covariance matrix. To downgrade algorithm complexity, the Gauss function approximation of the log likelihood function is introduced, the probability density function parameters of 3D normal distribution transformation algorithm is simplified, and the Hessian matrix and gradient vector is solved through translation, rotation relation and Jacobean matrix; to make sure algorithm is converged to one certain point after a small number of iterations, it proposes that Newton iterative algorithm step be improved by employing better line search. Finally, the algorithm is put on simulation experiment and compared with other ways, the result of which proves that the suggested algorithm is able to achieve better registration effect, and Improve accuracy and efficiency.

**Index Terms**—Point Cloud Registration; NI-3DNDT; Newton Iteration; Line Search; Visual Sensor

## I. INTRODUCTION

With the development of computer-aided technology, reverse engineering technology has been widely applied. The point cloud technology uses visual sensor, structured light camera and other equipments to obtain the target data of point cloud [1, 2], and to incorporate point clouds of multiple perspectives into a complete point cloud in different perspectives of translation, rotation matrix of rigid body in space that solved with 3D point cloud. Point cloud registration technology is the core technology of the three-dimensional reconstruction, and it is also the hotspot and focus of research in the computer vision field such as virtual reality, simulation design, heritage

digitization. In addition, a growing number of commercial companies (such as Microsoft and Intel) have launched low-cost point cloud acquisition equipment to develop the subversive model of human-computer interaction. The launch of more interactive design commercial products has opened up a new era.

There are three ways of point cloud registration: instrument registration, manual registration and automatic registration. Automatic registration refers to using algorithms or statistical regularity to compute relationship of translation, rotation matrix of two point clouds. Registration technology has two steps: 1) Pre-match: using a simple one-dimensional features, including curvature [3], normal [4], tagging method [5], etc., or a high-dimensional feature descriptions such as shape [6], contour curve [7], and image rotation [8]. The effect of pre-matching will affect the result of exact match directly; 2) exact match, the most commonly used method is ICP (iterative closest point) algorithm [9, 10].

Iterative closest point (ICP) algorithm based on the quaternary array is an iterative registration algorithm put forward by Besl and McKay in 1992 which firstly utilizes the Newton iteration method or other searching methods to search the corresponding closest point pair in two sets of point cloud, then does the iterative operation on the objective function based on Euclidean distance and finally completes the registration of the three dimensional point cloud data. Many productive results have been achieved with a lot of researches by the domestic and foreign scholars [11]. Almhdie [12] et al. proposed Comprehensive ICP algorithm. The process of looking for corresponding point registration had introduced the concept of search matrix, ensuring the only match by search distance matrix, making the convergence rate of Comprehensive ICP algorithm quicker and the registration results more accurate. Du [13] et al. proposed Affine ICP algorithm, which first estimated the initial position of point cloud by independent component analysis, then registered the point cloud by combining the affine transformation and original ICP. Besl and McKay

[14] proposed point-to-point registration, while this method did not consider the curved shape around the point, which was likely to fall into local minimum value or the relatively small domain of convergence. However, it needed more iterations to converge. Chen and Medioni [15] proposed point-to-plane registration to find out the minimum value of the tangent plane in the current point cloud and the reference point cloud. However, this method was difficult to converge when there was a big gesture initialization error. Pathak [10] proposed an improved ICP algorithm of point-to-plane, which derived uncertainty posture parameters through the least squares method. Chen [16] et al. proposed an automatic registration algorithm based 3D shape modeling. They present the Hong-Tan Iterative Closest Point registration algorithm. This improved ICP algorithm could achieve higher registration accuracy.

Sharp [17] proposed a kind of ICP registration algorithm using Invariant Features (ICPIF) that investigates the use of Euclidean invariant features in a generalization of ICP registration of range images and modified the definition of the distance between two points in the original ICP algorithm by linear weights of the straight-line distance and image eigenvalues. This method is more accurate in the searching of corresponding point pairs.

Du [18] et al. put forward a novel algorithm for affine registration of point sets based on ICP algorithm. It acquired the estimation of the initial position of point cloud through independent component analysis (ICA), then used a combination way of affine transformation and original ICP for registration. A preferable result of affine registration requires better initial registration parameters obtained from ICA. Experiment results show that this proposed algorithm possesses higher accuracy.

In view of the problems for the ICP algorithm, Biber and Straßer [19] first proposed the application of normal transform algorithm (NDT) to 2D registration data. This method was of high speed and accurate rate. Biber and Fleck [20] et al. proposed probabilistic framework algorithm based on NDT algorithm, which matched the data points and the probability distribution model. Magnusson [21, 22] proposed the registration that applied the NDT algorithm to 3D point cloud data, and verified that the relative ICP algorithm had higher registration accuracy and efficiency, but the convergence of this algorithm was unknown. The NDT-based 3D point cloud surface was expressed as the number of first derivative and second derivative of piecewise smooth function, and the smooth function could be optimized with numerical optimization algorithm, which could be quickly converged. Moreover, the point in reference registration point cloud was not directly involved in the match, unlike ICP algorithm, which needed a lot of time to search for the last collar. In the case of relative poor initial value, the registration results of 3DNDT algorithm which has improved the efficiency were more accurate.

## II. 3D NORMAL DISTRIBUTION TRANSFORM ALGORITHM

Frequency domain methods utilize the aliasing existing in each low resolution image to reconstruct a high Unlike the traditional algorithm which uses a single point model, the goal of 3D normal distribution transform algorithm is to find the maximum of the matched likelihood function between a point in the current point cloud and the reference point cloud, and it does not use the corresponding feature points to calculate and match in a registration process, so it is faster than the other algorithms. The first step of 3DNDT algorithm is to divide spatial point cloud into cubes grid, and one of the points in the cube is expressed as  $U = \{\vec{u}_1, \dots, \vec{u}_n\}$ , and the mean of each cube:

$$\vec{q} = \frac{1}{n} \sum_i \vec{u}_i \tag{1}$$

Covariance matrix is expressed as:

$$\Sigma = \frac{1}{n-1} \sum_i (\vec{u}_i - \vec{q})(\vec{u}_i - \vec{q})' \tag{2}$$

The probability density function of some point like  $\psi'(a_i)$  in the cube can be expressed as normal distribution function:

$$p(\vec{u}_i) = \varphi \exp\left(-\frac{(\vec{u}_i - \vec{q})' \Sigma^{-1} (\vec{u}_i - \vec{q})}{2}\right) \tag{3}$$

where in  $a_q$  and  $\psi(a_i)$  represent the mean and the covariance matrix of the cube concluding the  $\psi(a_i)$  point respectively. In order to find the gesture of current point cloud when the likelihood function has the maximum value, assume that the transformation function relationship of the space between current point cloud and the reference point cloud is  $\psi'(a_i)$ ,  $a_s$  refers to a reference point that is transformed from the point  $\psi(a_i)$  in current point cloud with the relation of rotation and translation  $\psi'(a_i)$ . As the formula shows below, we can get the R and T relation  $\psi'(a_i)$  when the maximum value of likelihood function is analyzed.

$$\zeta = \prod_{i=1}^n p(\Gamma(\vec{\gamma}, \vec{u}_i)) \tag{4}$$

Take the logarithm of the formula above, we can get:

$$\log \zeta = \sum_{i=1}^n \log(p(\Gamma(\vec{\gamma}, \vec{u}_i))) \tag{5}$$

$$p(\vec{v}_i) = p(\Gamma(\vec{\gamma}, \vec{u}_i)) \tag{6}$$

$$p(\vec{v}_i) = b_1 \exp\left(-\frac{(\vec{v}_i - \vec{q})' \Sigma^{-1} (\vec{v}_i - \vec{q})}{2}\right) + b_2 p_{outlier} \tag{7}$$

$b_1, b_2$  and  $p_{outlier}$  refer to the proportion of expected outliers.

$$\log(p(\vec{v}_i)) = \log(c_1 \exp(-\frac{(\vec{v}_i - \vec{q})^T \Sigma^{-1} (\vec{v}_i - \vec{q})}{2}) + c_2) \quad (8)$$

Likelihood function can be approximated by a Gaussian function:

$$\log(c_1 \exp(-\frac{v^2}{2\sigma^2}) + c_2) = d_1 \exp(-d_2 \frac{v^2}{2\sigma^2}) \quad (9)$$

From  $v=0, v=\sigma$  and  $v=\infty$ , we can get:

$$d_1 = \log(c_1 + c_2) \quad (10)$$

$$d_2 = -2 \log(\frac{\log(c_1 \exp(-1/2) + c_2)}{d_1}) \quad (11)$$

The probability density function of 3DNDT algorithm can be expressed as the following equation approximated by Gaussian function:

$$p'(\vec{v}_i) = -d_1 \exp(-d_2 \frac{(\vec{v}_i - \vec{q})^T \Sigma^{-1} (\vec{v}_i - \vec{q})}{2}) \quad (12)$$

Score function  $s(\vec{\gamma})$  can be identified as the sum of probability density function of grid Gaussian of all cubes:

$$H_{ij} = \sum_{k=1}^n d_1 d_2 \exp(-\frac{d_2}{2} \vec{u}_k^T \Sigma_k^{-1} \vec{u}_k) \vec{u}_k^T (-d_2 \vec{u}_k^T \Sigma_k^{-1} \frac{\partial \vec{u}_k}{\partial \gamma_i}) (\vec{u}_k^T \Sigma_k^{-1} \frac{\partial \vec{u}_k}{\partial \gamma_j}) + \vec{u}_k^T \Sigma_k^{-1} \frac{\partial^2 \vec{u}_k}{\partial \gamma_i \partial \gamma_j} + \frac{\partial \vec{u}_k}{\partial \gamma_j} \Sigma_k^{-1} \frac{\partial \vec{u}_k}{\partial \gamma_i} \quad (17)$$

R and T relation  $\phi(0) = -s(\vec{\gamma})$  refers to a point in the current cloud point  $a_i = 0$ , which is transformed into a point  $a_u = 0$ , through Euler angles  $\phi'(0) = -g \Delta \vec{\gamma}$ :

$$\Gamma(\vec{\gamma}, \vec{u}_i) = \begin{bmatrix} \cos \beta \cos \gamma & -\cos \beta \sin \gamma & \sin \beta \\ \cos \alpha \sin \gamma + \sin \alpha \sin \beta \cos \gamma & \cos \alpha \cos \gamma - \sin \alpha \sin \beta \sin \gamma & -\sin \alpha \cos \beta \\ \sin \alpha \sin \gamma - \cos \alpha \sin \beta \cos \gamma & \cos \alpha \sin \beta \sin \gamma + \sin \alpha \cos \gamma & \cos \alpha \cos \beta \end{bmatrix} \vec{u}_i + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \quad (19)$$

so  $\frac{\partial \Gamma}{\partial \gamma_i}$  can be solved by the column vector of Jacobian

matrix  $J$ :

$$J(\gamma_i) = \begin{bmatrix} \frac{\partial \Gamma_1}{\partial \gamma_1} & \frac{\partial \Gamma_1}{\partial \gamma_2} & \frac{\partial \Gamma_1}{\partial \gamma_3} & \frac{\partial \Gamma_1}{\partial \gamma_4} & \frac{\partial \Gamma_1}{\partial \gamma_5} & \frac{\partial \Gamma_1}{\partial \gamma_6} \\ \frac{\partial \Gamma_2}{\partial \gamma_1} & \frac{\partial \Gamma_2}{\partial \gamma_2} & \frac{\partial \Gamma_2}{\partial \gamma_3} & \frac{\partial \Gamma_2}{\partial \gamma_4} & \frac{\partial \Gamma_2}{\partial \gamma_5} & \frac{\partial \Gamma_2}{\partial \gamma_6} \\ \frac{\partial \Gamma_3}{\partial \gamma_1} & \frac{\partial \Gamma_3}{\partial \gamma_2} & \frac{\partial \Gamma_3}{\partial \gamma_3} & \frac{\partial \Gamma_3}{\partial \gamma_4} & \frac{\partial \Gamma_3}{\partial \gamma_5} & \frac{\partial \Gamma_3}{\partial \gamma_6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & k_1 & k_2 \\ 0 & 1 & 0 & k_3 & k_4 & k_5 \\ 0 & 0 & 1 & k_6 & k_7 & k_8 \end{bmatrix} \quad (20)$$

$\frac{\partial^2 \Gamma}{\partial \gamma_i \partial \gamma_j}$  can be calculated by the following matrix:

$$s(\vec{\gamma}) = \sum_{i=1}^n p'(\vec{v}_i) \quad (13)$$

### III. NEWTON ITERATIVE ALGORITHM FOR SOLVING PARAMETER

#### A. Hessian Matrix and the Gradient Vector

Using Newton iterative algorithm to find the solution of parameter  $a_s$  in  $s(\vec{\gamma})$ , work out the formula, wherein  $H$  and  $\vec{g}$  are the Hessian matrix and gradient vector of  $s(\vec{\gamma})$ , respectively.  $\vec{\gamma} \leftarrow \vec{\gamma} + \Delta \vec{\gamma}$ ,  $\Delta \vec{\gamma}$  is the increment of the current pose estimation in each iteration.

$$\vec{v}_k - \vec{q} = \Gamma(\vec{\gamma}, \vec{u}_k) - \vec{q} \quad (14)$$

$$\vec{u}_k \equiv \Gamma(\vec{\gamma}, \vec{u}_k) - \vec{q} \quad (15)$$

The gradient vector  $\vec{g}$  is identified as

$$\vec{g}_i = \frac{\partial s}{\partial \gamma_i} = \sum_{k=1}^n d_1 d_2 \vec{u}_k^T \Sigma_k^{-1} \frac{\partial \vec{u}_k}{\partial \gamma_i} \exp(-\frac{d_2}{2} \vec{u}_k^T \Sigma_k^{-1} \vec{u}_k) \quad (16)$$

Hessian matrix is  $H_{ij} = \frac{\partial^2 s}{\partial \gamma_i \partial \gamma_j}$

$$\vec{\gamma} = [t_x, t_y, t_z, \alpha, \beta, \gamma]^T \quad (18)$$

$$H_{ij} = \begin{bmatrix} \frac{\partial^2 \Gamma}{\partial \gamma_1^2} & \frac{\partial^2 \Gamma}{\partial \gamma_1 \partial \gamma_2} & \dots & \dots & \dots & \frac{\partial^2 \Gamma}{\partial \gamma_1 \partial \gamma_6} \\ \frac{\partial^2 \Gamma}{\partial \gamma_2 \partial \gamma_1} & \ddots & \dots & \dots & \dots & \vdots \\ \vdots & \dots & \ddots & \dots & \dots & \vdots \\ \vdots & \dots & \dots & \ddots & \dots & \vdots \\ \vdots & \dots & \dots & \dots & \ddots & \vdots \\ \frac{\partial^2 \Gamma}{\partial \gamma_6 \partial \gamma_1} & \dots & \dots & \dots & \dots & \frac{\partial^2 \Gamma}{\partial \gamma_6^2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_9 & k_{10} & k_{11} \\ 0 & 0 & 0 & k_{12} & k_{13} & k_{14} \\ 0 & 0 & 0 & k_{15} & k_{16} & k_{17} \end{bmatrix} \quad (21)$$

**B. Step Update Based on Linear Search**

For step update problems of Newton iteration algorithm, we can use linear search algorithm. When the linear search algorithm satisfies the sufficient descent and curvature conditions, with a given continuously differentiable function  $\phi$ , incoming 3DNDT algorithm fraction equation into the formula:  $\phi(a_t) = -s(\vec{\gamma})$ ,  $\phi'(a_t) = -g\Delta\vec{\gamma}$ .

For a continuously differentiable function  $f$ , and the down trend  $p$  of one point  $x$  of the  $f$ , we identify the function as:

$$\phi(a) \equiv f(x+ap) \quad (22)$$

$f \in [-\infty, \infty)$ ,  $a > 0$ , so the function  $\psi(a)$  can be identified as:

$$\psi(a) \equiv \phi(a) - \phi(0) - \mu\phi'(0)a \quad (23)$$

$$\psi'(a) = \phi'(a) - \mu\phi'(0) \quad (24)$$

For a search range  $I[a_l, a_u]$ , there is  $a_t \in [a_l, a_u]$ , for the update search interval  $I[a_t^+, a_u^+]$ , exists  $a_t^+ \in [a_t^+, a_u^+]$ , so there exists:

$$\psi(a_t) = \phi(a_t) - \phi(0) - \mu\phi'(0)a_t \quad (25)$$

$$\psi'(a_t) = \phi'(a_t) - \mu\phi'(0) \quad (26)$$

Definition  $a_c$  is the minimum of cubic interpolation of  $\psi(a_t)$ ,  $\psi(a_t)$ ,  $\psi'(a_t)$  and  $\psi'(a_t)$ ,  $a_q$  is the minimum of cubic interpolation of  $\psi(a_t)$ ,  $\psi(a_t)$  and  $\psi'(a_t)$ ,  $a_s$  is the minimum of cubic interpolation of  $\psi(a_t)$ ,  $\psi'(a_t)$  and  $\psi'(a_t)$ . The renewal of  $a_t^+$  can be divided into the following four kinds of circumstances:

1) When  $\psi(a_t) > \psi(a_l)$ :

$$a_t^+ = \begin{cases} a_c, & \text{if } |a_c - a_t| < |a_q - a_t| \\ 0.5(a_c + a_q), & \text{otherwise} \end{cases} \quad (27)$$

2) When  $\psi(a_t) \leq \psi(a_l)$  and  $\psi'(a_t)\psi'(a_l) < 0$ :

$$a_t^+ = \begin{cases} a_c, & \text{if } |a_c - a_t| \geq |a_s - a_t| \\ a_s, & \text{otherwise} \end{cases} \quad (28)$$

3) When  $\psi(a_t) \leq \psi(a_l)$ ,  $\psi'(a_t)\psi'(a_l) \geq 0$  and  $|\psi'(a_t)| \leq |\psi'(a_l)|$ :

$$a_t^+ = \begin{cases} a_c, & \text{if } |a_c - a_t| \geq |a_s - a_t| \\ a_s, & \text{otherwise} \end{cases} \quad (29)$$

$$a_t^+ = \begin{cases} \min\{a_t + 0.66(a_u - a_t), a_t^+\}, & \text{if } a_t > a_l \\ \max\{a_t + 0.66(a_u - a_t), a_t^+\}, & \text{otherwise} \end{cases} \quad (30)$$

4) When  $\psi(a_t) \leq \psi(a_l)$ ,  $\psi'(a_t)\psi'(a_l) \geq 0$  and  $|\psi'(a_t)| > |\psi'(a_l)|$ , then  $a_t^+$  is the minimum of cubic interpolation of  $\psi(a_u)$ ,  $\psi(a_t)$ ,  $\psi'(a_u)$  and  $\psi'(a_t)$ .

According to the theory and methods of optimization,  $a_c$ ,  $a_q$  and  $a_s$  can be defined respectively as:

$$a_c = a_t + \frac{(a_t - a_l)(w - \psi'(a_t) - z)}{\psi'(a_t) - \psi'(a_l) + 2w} \quad (31)$$

$$a_q = a_t - \frac{0.5\psi'(a_t)(a_t - a_l)}{\psi'(a_t) - (\psi(a_t) - \psi(a_l)) / (a_t - a_l)} \quad (32)$$

$$a_s = a_t - \frac{(a_t - a_l)(w - \psi'(a_t) - z)}{\psi'(a_t)(\psi'(a_t) - \psi'(a_l))} \quad (33)$$

$$w = \sqrt{z^2 - \psi'(a_t)\psi'(a_l)} \quad (34)$$

$$z = \frac{3(\psi(a_t) - \psi(a_l)) - \psi'(a_t) - \psi'(a_l)}{a_t - a_l} \quad (35)$$

It is concluded,  $\vec{\gamma} \leftarrow \vec{\gamma} + \Delta\vec{\gamma}a_t^+$ , the renewal of algorithm is determined by the following conditions in search interval:

- 1) When  $\psi(a_t) > \psi(a_l)$ , then  $a_t^+ = a_l$ ,  $a_u^+ = a_t$ , the algorithm continue to search.
- 2) When  $\psi(a_t) \leq \psi(a_l)$ ,  $\psi'(a_t)(a_t - a_l) > 0$ , then  $a_t^+ = a_t$ ,  $a_u^+ = a_u$ , the algorithm continue to search.

3) When  $\psi(a_i) \leq \psi(a_i)$ ,  $\psi'(a_i)(a_i - a_i) < 0$ , then  $a_i^+ = a_i, a_u^+ = a_i$ , the algorithm continue to search.

4) Under other circumstances, the algorithm terminates the search.

IV. EXPERIMENTAL RESULTS AND ANALYSIS

Programs in this paper are all written in C++ language, and run in the Intel Core i7-3820QM 2.70GHZ, 8.0GB of memory. As shown in figure 1, the experiment adopts different types of point cloud, and it contains the scan data of 360-degree in different perspectives in the same room. There are 110,000 data points in each point cloud. (The data were downloaded from <http://kos.informatik.uni-osnabrueck.de/3Dscans>)

A. Example 1

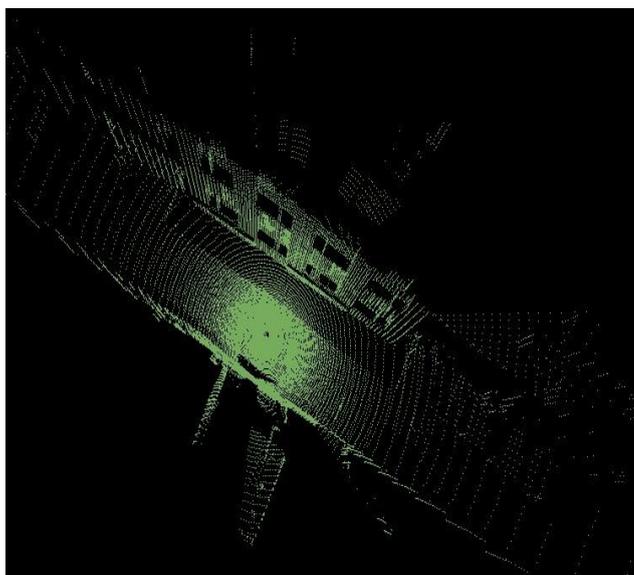
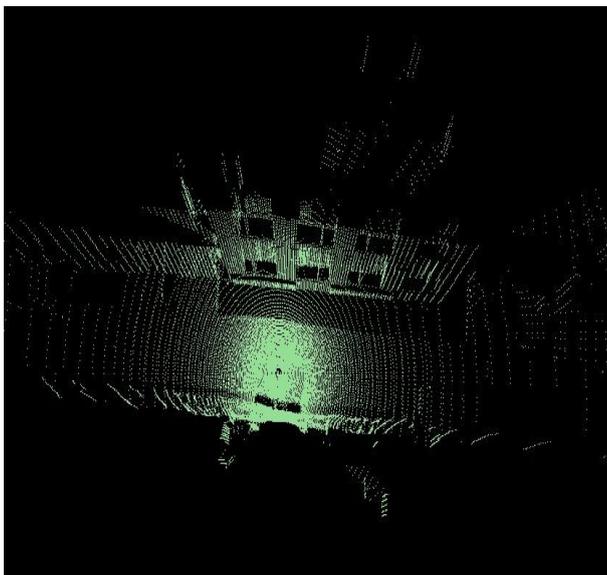
Take the algorithm in Doctor Magnusson's paper to test in point cloud a1 and a2: The resolution of NI-

3DNDT is  $r=1$ ,  $p_{outlier} = 0.55$ ,  $c_1 = 10 * (1 - p_{outlier}) = 4.5$ ,  $c_2 = \frac{p_{outlier}}{r^3} = 0.55$ .

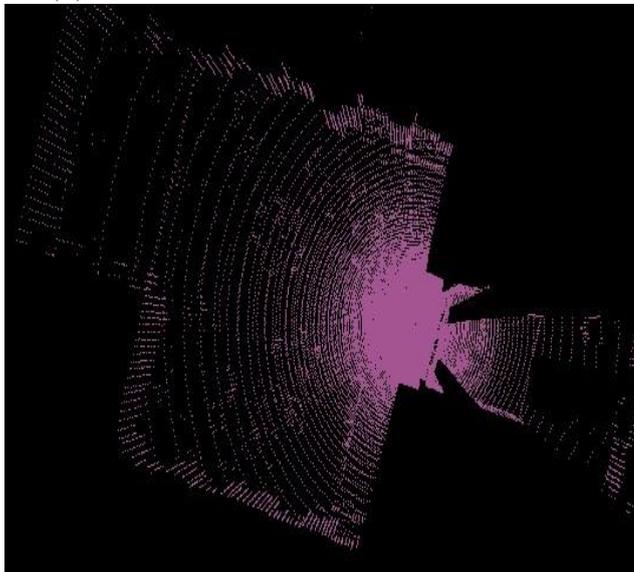
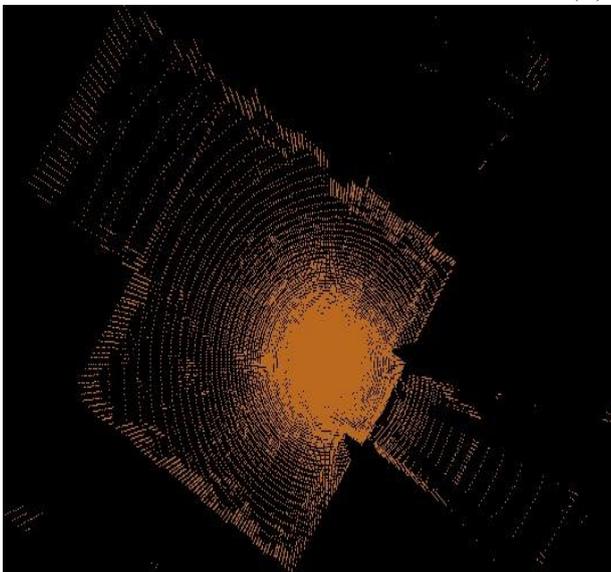
According to the Newton iterative initialization parameter  $\phi(0) = -s(\vec{\gamma})$ ,  $a_i = 0$ ,  $a_u = 0$  and  $\phi'(0) = -g\Delta\vec{\gamma}$ , the translation, rotation matrix can be calculated:

0.793983	-0.447965	0.0110171	2.27322
0.447847	0.794034	0.0116674	0.0730304
-0.0150726	-0.00549647	0.999871	0.0523041
0	0	0	1

To test the improved algorithm NI-3DNDT which is proposed in this paper, the translation, rotation matrix can be calculated:



(a1) Scene 1 (a2)



(a3) Scene 2 (a4)

Figure 1. Different types of 3D point cloud data

0.893912    -0.448124    0.0103027    2.48265  
 0.448013    0.893954    0.0114286    0.0755997  
 -0.0143315    -0.0056004    0.999882    0.0527146  
 0            0            0            1

The error distance of the two kinds of detection algorithm are 0.17511 and 0.14543 respectively, and the running time of them are 1350ms and 1160ms respectively. Therefore, with the detection algorithms in this paper, we can get better results. Besides, it has a high convergence speed and accuracy registration. Figure 2 and figure 3 are the comparison of 3DNDT algorithm and the algorithm which is proposed in this paper in registration time and errors when the three-dimensional spatial resolution from 0.1 to 4.5. As shown in figure 2, when the resolution range is from 0.7 to 1.5, NI-3DNDT obtains a good effect of registration, registration algorithm convergence rate is also significantly faster than the improved algorithm (about 50ms). As shown in figure 3, detection time of these two kinds of registration algorithm increases with the improvement of detection resolution.

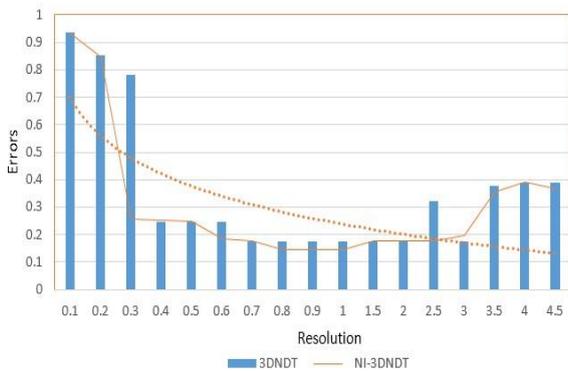


Figure 2. Algorithm detects errors comparison chart based on Scene one

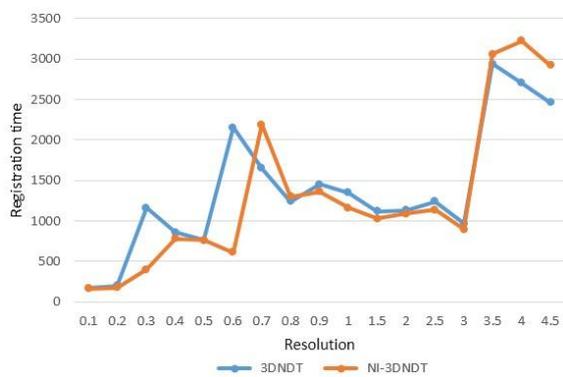


Figure 3. Algorithm matching time comparison chart based on Scene one

**B. Example 2**

To test the two registration algorithm in another scenario, put the algorithm of Doctor Magnusson’s paper in point cloud a3 and a4: The resolution of 3DNDT is  $r = 0.6$ , other experimental conditions are the same as the experimental one, and the translation, rotation matrix can be calculated:

0.881566    -0.191123    0.0006638    0.1313989  
 0.191124    0.881564    -0.001689    -0.347889  
 -0.000329    -0.001783    0.999998    0.0006301  
 0            0            0            1

To test the improved algorithm NI-3DNDT which is proposed in this paper, the translation, rotation matrix can be calculated:

0.984268    -0.176681    -0.000009    -0.032258  
 0.176681    0.984276    -0.001173    -0.325616  
 0.000296    0.0011387    0.999989    -0.000732  
 0            0            0            1

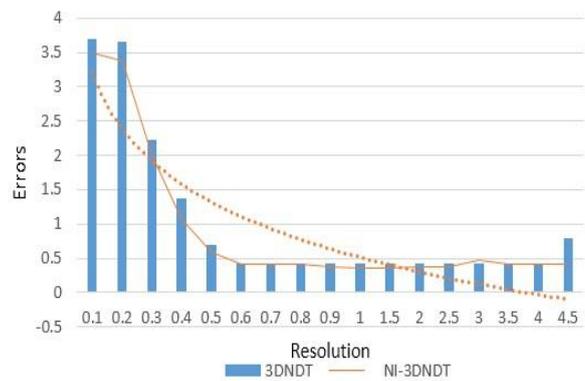


Figure 4. Algorithm detects errors comparison chart based on Scene two

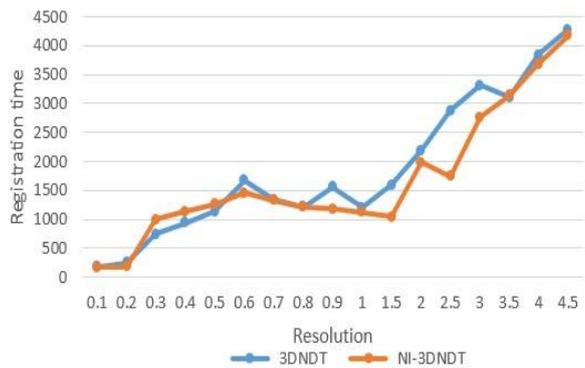


Figure 5. Algorithm matching time comparison chart based on Scene two

Under the circumstance that the resolution is  $r = 0.6$ , the error distance of the two kinds of detection algorithm are 0.418381 and 0.349911 respectively, and their running time are 1210 ms and 1120 ms respectively. Registration algorithm proposed in this paper is better. As shown in figure 4, when the resolution range is from 0.6 to 1.0, NI-3DNDT obtains a good effect of registration. Registration algorithm convergence rate, about 140ms, is also significantly faster than the improved algorithm. Compared with scenario one, NI-3DNDT algorithm is still obtained good effect of registration when the resolution is extreme. Therefore, we should set different detection resolutions and step lengths according to the type of different scenarios.

## V. CONCLUSION

3DNDT algorithm, as a new algorithm of point cloud registration, is put forward in recent years. And it has attracted more and more attention from the researchers because of its high matching precision and convergence speed. NI-3DNDT algorithm proposed in this paper is simplified in the terms of the parameters of the probability density function and improved in view of the linear search algorithm. Besides, it updates step length of Newton iteration algorithm dynamically, accelerates the convergence speed of Newton iteration algorithm, and improves the matching precision. Experiments show that the algorithm has more effective stability on the point cloud in different kinds of scenarios. The algorithm has a good performance under the condition of appropriate spatial resolution, and it has certain advantages in the matching accuracy and matching speed compared with existed algorithms. Therefore this algorithm has higher application value and theoretical reference value.

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## REFERENCE

- [1] Chen S Y, Li Y F, Zhang J W. Vision Processing for Real time 3D Data Acquisition Based on Coded Structured Light. *IEEE Transactions on Image Processing*, 2008, 17(2) pp. 167-176.
- [2] Chen S Y, Zhang J H, Li Y F, A Hierarchical Model Incorporating Segmented Regions and Pixel Descriptors for Video Background Subtraction. *IEEE Transactions on Industrial Informatics*, 2012, 8(1) pp. 118-227.
- [3] Wang S, Wang Y, Jin M, et al, Conformal geometry and its applications on 3D shape matching, recognition, and stitching. *Medical Engineering & Physics*, 2007, 29(7) pp. 1209-1220.
- [4] Kalle Å, Johan K, Olof E, Anders E, Fredrik K. Automatic feature point correspondences and shape analysis with missing data and outliers using MDL. *Image Analysis*, 2007, 4522(5) pp. 21-30.
- [5] Luo X, Zhong Y X, Li R J. Data registration in 3D scanning systems. *Journal of Tsinghua University (Science and Technique)*, 2004, 44(8) pp. 1104-1106.
- [6] Frome A, Huber D, Kolluri R, et al. Recognizing objects in range data using regional point descriptors // *In Proceedings of the European Conference on Computer Vision, Prague, Czech Republic*, 2004 pp. 224-237.
- [7] Yang R, Allen P. Registering integrating and building cad models from range data // *In: IEEE International Conference on Robotics and Automation, Leuven, Belgium*, 1998 pp. 3115-3120.
- [8] Johnson A, Hebert M. Using spin images for efficient object recognition in cluttered 3D scenes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1999, 21(5) pp. 674-686.
- [9] Herrmann M, Otsteinu M, Otto M. Fast and robust point cloud matching based on EM-ICP repositioning // *10th*

*International Symposium on Electronics and Telecommunications, Timisoara, Rumania*, 2012 pp. 99-103.

- [10] Pathak K, Birk A, Vaškevičius N, Poppinga J. Fast Registration Based on Noisy Planes With Unknown Correspondences for 3-D Mapping. *IEEE Transactions on Robotics*, 2010, 26(3) pp. 424-441.
- [11] Xin W, Pu J X. Point cloud integration base on distances between points and their neighborhood centroids. *Journal of Image and Graphics*, 2011, 16(5) pp. 886-891.
- [12] Almhdie A, Léger C, Deriche M, et al. 3D registration using a new implementation of the ICP algorithm based on a comprehensive lookup matrix: Application to medical imaging. *Pattern Recognition Letters*, 2007, 28(12) pp. 1523-1533.
- [13] Du S Y, Zheng N N, Meng G F, Yuan Z J. Affine registration of point sets using ICP and ICA. *IEEE Signal Processing Letters*, 2008, 15 pp. 689-692
- [14] Besl P J, McKay N D. A method for registration of 3-D shapes. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1992, 14(2) pp. 239-256.
- [15] Chen Y, Medioni G. Object modelling by registration of multiple range images. *Image and vision Computing*, 1992, 10(3) pp. 145-155.
- [16] Chen J, Wu X J, Michael Y W, et al. 3D shape modeling using a self-developed hand-held 3D laser scanner and an efficient HT-ICP point cloud registration algorithm. *Optics and Laser Technology*, 2013, 45(1) pp. 414-423.
- [17] Sharp G. C, Lee S. W, Wehe D. K. ICP registration using invariant features. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 2002, 24(1) pp. 90-102.
- [18] Du S. Y, Zheng N. N, Meng G. F, Yuan Z. J. Affine Registration of Point Sets Using ICP and ICA. *IEEE Signal Processing Letters*, 2008, 15 pp. 689-692.
- [19] Biber P, Straßer W. The normal distributions transform: A new approach to laser scan matching // *In Proceedings of the IEEE International Conference on Intelligent Robots and Systems, Las Vegas, USA*, 2003 pp. 2743-2748.
- [20] Biber P, Fleck S, Strasser W. A Probabilistic Framework for Robust and Accurate Matching of Point Clouds. *Lecture Notes in Computer Science*, 2004, 3175 pp. 480-487.
- [21] Magnusson M, Andreasson H, Nuchter A, Lilienthal A J. Appearance-Based Loop Detection from 3D Laser Data Using the Normal Distributions Transform // *IEEE International Conference on Robotics and Automation, Osaka, Japan*, 2009 pp. 23-28.
- [22] Magnusson M. The Three-Dimensional Normal-Distributions Transform-an Efficient Representation for Registration, Surface Analysis, and Loop Detection. Sweden: *Orebro University*, 2009.



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