Routing Optimization Based on Taboo Search Algorithm for Logistic Distribution

Yang Hongxue
Beijing Electronic Science & Technology Vocational College, Beijing 100029, China

Xuan Lingling
Zhejiang Institute of Communications, Hangzhou, 311112 China

Abstract—**Along with the widespread application of the electronic commerce in the modern business, the logistic distribution has become increasingly important. More and more enterprises recognize that the logistic distribution plays an important role in the process of production and sales. A good routing for logistic distribution can cut down transport cost and improve efficiency. In order to cut down transport cost and improve efficiency, a routing optimization based on taboo search for logistic distribution is proposed in this paper. Taboo search is a metaheuristic search method to perform local search used for logistic optimization. The taboo search is employed to accelerate convergence and the aspiration criterion is combined with the heuristics algorithm to solve routing optimization. Simulation experimental results demonstrate that the optimal routing in the logistic distribution can be quickly obtained by the taboo search algorithm.**

Index Terms—**Taboo Search; Logistic Distribution; Electronic Commerce; Routing Optimization; Heuristics Algorithm; Aspiration Criterion**

I. Introduction

Logistics is an emerging discipline and distribution is an important element of modern logistics. A good routing for logistic distribution can cut down transport cost and improve efficiency. The efficiency of it has great influence on improving whole efficiency of logistics system and reducing transport cost. The route optimization problem of logistic distribution [1] is a classic combinatorial optimization problem, which is a kind of NPC-hard problems and has high computational complexity. Due to the prosperity of market economy and the speedy development of logistic distribution industry, more and more enterprises recognize that the logistic distribution plays an important role in the process of production and sales of enterprise. The selection of traditional artificial distribution routes needs to cost much time and energy, because it completely depends on the laborers' experiences and wisdom. With the gradual increase of enterprise and business scale and the increasing number of distribution centers and the more complicated distribution routes, the artificial distribution route cannot satisfy the enterprise's business demand, so it is necessary to design routes with computers. In order to solve the problem, many algorithms with intelligence working are developed. These algorithms can find the optimal or sub-optimal solution efficiently. Simulation experiments show that it is a good method to solve the optimization problems for logistics distribution centers. The routing optimization is developed by using aspiration criterion.

In view of the intrinsic characteristics of routing optimization, many searchers study the optimal solution of the linear programming problems, such as the supplier, manufacturer, and logistics center and so on. As we all understand the routing, form the supplier to the manufacturer and the manufacturer to the logistics centers is complicated. Therefore, the routing algorithm has integrated many neighborhood search methods, and has adopted routing decomposition to solve logistic distribution. Our proposed solution is divided into several subsets for routing optimization, each of which is solved by the search algorithm. In order to promote the convergence speed and the global search ability, there are many optimization algorithms for solving distribution routing problem, such as traveling salesman, dynamic programming, saving method, scanning, partition distribution algorithm, scheme evaluation etc. Although these algorithms can solve such problem, there are also some shortcomings, such as combination point messy and being difficult to combine the edge point for saving method, non-optimization optimization problem for scanning. According to the optimization characteristics of physical distribution routing problem, how to construct the heuristic algorithm with simple operation and good optimization performance is well worth further studying. Simulated annealing and taboo search algorithm has been successfully applied to this issue in recent years. But they also have their own problems, for example, local search ability is poor and feasible solution is not very good in the simulated annealing, and taboo search algorithm is heavily reliant on the initial solution. The hybrid algorithm is a research focus currently, which can partially offset the defects.

Taboo search algorithm is a new evolutionary algorithm, which is a new heuristic algorithm through simulating the process of swarms searching for food and is a stochastic heuristic optimization algorithm proposed by Glover [8, 9], a scholar of Italy, for the first time in 1986. Glover makes full use of the similarity between the

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course of swarm searching food and the well known traveling salesman problem, and simulates the course of swarm's searching food to solve this kind of TSP questions, namely, a shortest route between nest and food source is found by the information communication and the mutual cooperation between individual.

The taboo search algorithm is applied to a variety of combinatorial optimization problems, particularly suitable for discrete optimization problem about more points' nondeterministic search in the solution space, such as traveling salesman problem (TSP), quadratic assignment problem (QAP), job-arrangement scheduling problem (JSP) etc. Furthermore, it is widely used in communication network load and water science field. In a word, it is an optimization method on the whole, which has generality and robustness.

The prototype of taboo search algorithm (TSA) is a model for searching the shortest route; therefore it has the congenital advantage in route optimization. Now we have lots of successful applicable examples of TSA on the problem of TSP, for example, Swarm-Q [11], MMAS [12] etc. There is the common ground between the problems of logistic distribution routing optimization and TSP, which search the shortest route among all customer points, but the logistic distribution routing optimization also has its characteristic, namely, it has more complicated constraint conditions and optimization objectives. According to the characteristic, this paper researches a route optimization algorithm based on taboo search algorithm, which can avoid algorithm premature and stagnation in the local search process by introducing the chromosome operator and meanwhile, improve the update mode of candidate and the strategy of selecting customer points and enhance the positive feedback effect of TSA so as to promote the convergence speed and the global search ability, which can receive a relatively good practical effect on the logistic distribution routing optimization problems.

II. MATHEMATICAL MODEL OF LOGISTIC DISTRIBUTION

A. Description of Problems

The general distribution route problems can be described as follows: there are L customer points; the quantity demand and locations of each customer point are known; it can use K vehicles at most to arrive at these demand points from the distribution center and each vehicle has a definite deadweight after completing the distribution tasks and returning to the logistic center. It is required that with the arrangement of vehicle's delivery route, the delivery distance is the shortest and at the same time the following constraint conditions can be satisfied; the sum of quantity demand of each route cannot exceed the vehicle's deadweight; the total length of each distribution route cannot exceed the maximum driving distance of each distribution. The demand of each customer point must be satisfied by only one vehicle. The purpose is to get the minimum general cost (i.e. distance, time, etc.)

B. Establishment of Mathematical Model

1) The Definition of Symbol

L: The total number of customer points;

q_i: The quantity demand of customer i, where i=1,2,…,L;

d_{ij}: The distance from customer i to j, specially,when i,j=0, it denotes the distribution center ,for example, $d_{0,3}$ denotes the distance from the distribution center to customer point 3; $d_{2,0}$ denotes the distance from the distribution center to customer point 2; i,j=0,1,2,…,L;

K: The total number of vehicles;

Q_k: The maximum deadweight of vehicle k, where k=1,2,…,K;

D_k: The maximum driving distance of vehicle k, where k=1,2,…,K;

n_k: The total customers number distributed by vehicle k, when n_k=0, it denotes vehicle k does not participate in distribution, k=1,2,…,K;

R_k: A set of customer points distributed by vehicle k. When n_k=0, R_k=Φ; when $n_k \neq 0$, $R_k = \{r_k^1, r_k^2, \cdots, r_k^{n_k}\} \subseteq \{1, 2, \cdots, L\}$, where r_k^i denotes that the order of this customer point in the distribution routes is i, and k=1,2,…,K.

Constraint conditions

According to the previous description of the logistic distribution routing optimization problems, we can abstract the following constraint conditions:

1) The sum of customer points' quantity demand on each route cannot exceed the vehicle's deadweight;

$$\sum_{i=1}^{n_k} q_{r_k^i} \le Q_k,\ n_k \neq 0$$

2) The total length of each distribution route cannot exceed the maximum driving distance of each distribution;

$$\sum_{i=1}^{n_k} d_{r_k^{i-1} r_k^i} + d_{r_k^{n_k} 0} \le D_k,\ n_k \neq 0$$

3) The demand of each customer point must be satisfied by only one vehicle.

$$R_{k1} \cap R_{k2} = \Phi,\ k1 \neq k2$$

4) The distribution routes must cover all customer points:

$$\bigcup_{k=1}^{K} R_k = \{1, 2, \cdots L\}$$

$$0 \le n_k \le L$$

$$\sum_{k=1}^{K} n_k = L$$

2) The Optimization Objective Function

According to the optimization objective of logistic distribution routing optimization problems in the paper, the equation of the optimization objective function is given as follows:

$$\min\left[Z = \sum_{k=1}^{K} \left(\sum_{i=1}^{n_k} d_{r_k^{i-1} r_k^i} + d_{r_k^{n_k} 0} \right) \cdot \mathrm{sgn}(n_k) \right]$$

$$\mathrm{sgn}(n_k) = \begin{cases} 0, n_k \ge 1 \\ 1, Others \end{cases}$$

$$\rho(t+1)=\begin{cases}\max[\lambda\cdot\rho(t),\rho_{\min}] & r=r_{\max}\\ \rho(t) & otherwise\end{cases}$$

III. Distribution Route Optimization Based on Taboo Search Algorithm

A. Basic Taboo Search Algorithm

The taboo search algorithm (TSA) is a "nature" algorithm inspired by the nature creatures' behavior, which is generated from the behavior study of swarm colony. The biggest characteristic of TSA is the indirect asynchronous contact way with "candidate" as medium of swarms in colony. When the swarms are in action, for example, searching food or finding the route back home, they will leave some chemical substances (these are called "message"). These substances can be felt by the latter swarms in the same colony and influence the latter swarms' action as one kind of signals (it is concretely expressed that the latter coming swarms are much more possible to choose the routes with substances than these without substances.), and the messages left by the latter coming swarms will reinforce the previous candidate and such cycle will continue. In this way, the route chosen by more swarms will be more possible to be chosen by the latter coming swarms and that is because the left information is of high concentration. For the shorter route will be visited by more swarms in a certain time, there will be more left messages, and the route will be more likely to be chosen by other swarms in the next time. This process will not continue until all swarms choose the shortest route.

Vehicles can be replaced by artificial swarms to make distribution on customer points. When the swarm serving in customer point *i* selects the next customer point *j*, it should mainly consider two elements, namely, one is the intimacy between the two customer points *i* and *j*, which is called visibility η_{ij}, the other is the feasibility from *i* to *j* showed in the cycled route scheme so far, called as candidate density τ_{ij}. The probability that the swarm k will transfer from customer *i* to *j* in *t* time is:

$$p_{ij}^{k}(t)=\begin{cases}\dfrac{[\tau_{ij}(t)]^{\alpha}\cdot[\eta_{ij}]^{\beta}}{\sum[\tau_{ik}(t)]^{\alpha}\cdot[\eta_{ik}]^{\beta}} & if\ \ j\in allowed_{k}\\ 0 & otherwise\end{cases}$$

where $allowed_k=\{0,1,\ldots,n-1\}-taboo_k$ denotes the customer point not yet served by the swarm *k*. The visibility is:

$$\eta_{ij}=\frac{1}{d_{ij}}$$

When the next customer point to be served will make the total carrying capacity exceed the vehicle deadweight or make the delivery distance exceed the maximum driving distance for one time, it will return to the distribution center and the artificial swarm will start to continue distribution in replace of the next vehicle. After a complete cycle, the swarm traverses all customer points and completes one distribution. When all swarms finish one cycle, according to the good or bad target function value, the increment of candidate will be calculated and the candidate in the relevant route will be updated. The updating principle is:

$$\tau_{ij}(t+n)=\rho\cdot\tau_{ij}(t)+\Delta\tau_{ij}$$

$$\Delta\tau_{ij}=\sum_{k=1}^{m}\Delta\tau_{ij}^{k}$$

B. Improved Taboo Search Algorithm

The operator is the nuclear content of simulated annealing (SA). We introduce the aspiration criterion, including neighborhood, taboo list and taboo length into taboo search algorithm, in order to promote the convergence speed and the global search ability.

In SA, the primary thought of neighborhood is that the excellent individuals of parent generation can be very close to the global optimal solution and should be inherited in the filial generation and get continuously evolved. Neighborhood can save the excellent individuals of parent generation in the filial generation and avoid the loss of excellent individual in the population caused by taboo list and taboo length, etc.

In TSA, after completing the search of each generation, we reproduce the optimal solution of the present parent generation into the filial generation and make the optimal individual continue to accumulate candidate in filial generation, which can enhance the convergence speed of algorithm.

The taboo list and taboo length of SA are based on the chromosome swap. Therefore, we firstly make the swap of the logistic distribution model before introducing taboo list and taboo length.

Suppose there are L customer points and K distribution vehicles and the swap method in this paper is these L customer points will be labeled from 1 to L separately, which are all natural numbers; when the first vehicle starts from the distribution center, it will be labeled with 0, and other vehicles will be denoted as L+1, L+2, …, L+K-1 respectively. The same vehicle can make distribution for several times, therefore, the vehicle which has made distribution for over 2 times will still be denoted as L+K, L+K+1. When a new vehicle starts from the distribution center or the Swap is completed, that means the previous vehicle's route is over and it should return to the distribution center. Then, one distribution can be denoted as the swap composed by 0 and natural numbers, for example, there are 6 customer points denoted from 1 to 6 separately and 3 of them should be responsible for transportation, then the Swap is:

0, 1, 2, 3, 7, 4, 5, 8, 6

The distribution routes of 3 vehicles are respectively: vehicle1 [0→1→2→3→0], vehicle 2 [0→4→5→0], vehicle 3 [0→6→0].

Another example:

0, 1, 2, 3, 8, 4, 5, 9, 6

The distribution routes are: vehicle 1 [0→1→2→3→0], vehicle 3 [0→4→5→0], vehicle 1's second distribution [0→6→0].

C. Taboo List

Taboo list is the operation of SA which can enhance the group diversity and avoid algorithm premature and stagnation. Introducing the taboo list into TSA can effectively enhance the search space and avoid the algorithm's falling into the trap of the local optimal solution.

After the search of each generation in TSA, we implement the Swap and taboo list between the optimal solution and the suboptimal solution and the taboo list principle is as follows:

1) Suppose the two groups of Swap are respectively S1 and S2. The length and the start position of taboo list section will be generated randomly at first;

2) Find the taboo list section between S1 and S2 and suppose $S1$: $P_1|P_2|P_3$, $S2$: $Q_1|Q_2|Q_3$, and P_2 and Q_2 are respectively the taboo list sections between S1 and S2; insert Q2 into S1, which is placed in front of P_2, and then get the new Swap $S3$: $P_1|Q_2|P_2|P_3$;

3) In S3, delete the repetitious particle of P_1, P_2, P_3 against Q_2 and then get the taboo list particle S3;

4) Use the same method upon S2, and get the new particle S4;

5) Compare the results of $S1$, $S2$, $S3$, $S4$ and select the two groups of optimal particle to be saved.

Taboo length is an evolution method to enhance the population's diversity. The moderate taboo length can not only maintain the individual diversity in the population but also improve the algorithm efficiency.

In TSA, after the taboo list, the taboo length operation will be executed in the excellent individual in the population, and the operation method is:

1) Generate the taboo length times N randomly;

2) Generate two different natural numbers n_1, $n_2>1$ randomly (The first number will not change in order to make sure the Swap starts from the logistic center.);

3) In the particle S of the optimal individual, exchange the particle in positions number n_1 and n_2 with each other;

4) Repeat the step 2) and 3) for N times and generate the new particle S';

5) Compare the results of S and S' and save the optimal solution.

After improved by the introduced SA, TSA has got improvement in the convergence speed and the global search ability. Then we will improve the candidate's updating way and the strategy of selecting customer point in the following in order to improve the TSA's self-adaptability.

D. Selection of the Candidate Transfer Parameter ρ

According to the basic TSA, ρ is a constant. If ρ is too large, it will relatively reduce the probability of the selected route which has not been searched, which will influence the global search ability; if ρ is too small, it will influence the algorithm's convergence speed. Therefore, we will make some appropriate adjustments on ρ in the improved algorithm. In the initial period of algorithm, we hope to find the sub-optimal solution of the algorithm as soon as possible, so ρ should be relatively large in order to increase the influence of information concentration and promote the algorithm's convergence speed; When the algorithm is in stagnation, we should decrease ρ in order to reduce the influence of candidate on swarm colony and increase the swarm colony's search ability in the solution space and get out of the constraint of the local optimal solution.

$$\rho(t+1)=\begin{cases}\max[\lambda\cdot\rho(t),\rho_{\min}] & r=r_{\max}\\ \rho(t) & otherwise\end{cases}$$

In the equation above, r denotes the cycle times where it has not been in evolution; $r_{\max}$ is a constant; $\lambda\in(0,1)$ is also a constant which controls the attenuation speed ρ; $\rho_{\min}$ is the minimum value of ρ in order to avoid the too small value of ρ will influence the convergence speed. When r gets to a preset value $r_{\max}$, we will reduce ρ and then r will be calculated again; the cycle will not continue until ρ gets to the preset minimum value $\rho_{\min}$.

F. Selection of the Deterministic Search and the Explorative Search

Convergence acceleration is to make evolution as fast as possible so as to get a better solution on the basis of having got the near-optimum solution. Because the TSA is a heuristic algorithm , the unceasing "exploration" is a necessary method to evolve for TSA; it is only the "exploration " that restrict the convergence speed of TSA, for example, when the algorithm gets a sub-optimum solution which can be likely to be further evolved, but the "exploration" scope is very large, which relatively reduces the probability to choose this route for swarms, then the candidate concentration in this route will be gradually attenuate and this route is gradually "forgot".

The value of $q0$ is also discussed. When $q<q0$, the algorithm adopts the deterministic search and the swarm chooses the shortest route with the probability $q0$ at this moment; when $q\geq q0$, the algorithm adopts the explorative search and the swarm will choose the route randomly with the probability l- $q0$. In the initial iteration of the algorithm, $q0$ chooses the relatively large initial value and make the deterministic search with relatively large probability, which can accelerate the speed to find the local near-optimum route; in the middle range of the algorithm, $q0$ chooses the relatively small initial value and executes the explorative search with relatively large probability, which can enlarge the search space; In the later stage of algorithm, $q0$ returns to the initial value in order to accelerate the convergence speed.

Combining with the improved TSA, the algorithm flow chart of the logistic distribution routing optimization problems based on the improved TSA is shown as follows.

IV. Experiment and Calculation

The literature [13] adopts the improved SA to get the solution of the logistic distribution routing optimization problems, and we will use the examples in this literature to calculate the result and compare their performance.

Example 1: a certain distribution center uses 2 vehicles to make distributions for 8 customers. Suppose the deadweight of vehicle is 8,000kg and the maximum driving distance for each time is 40km. The distances between distribution center and customer and the distances among customers are in the following table (0 denotes the distribution center, 1~8 denote the ID of 8 customer points, respectively.):

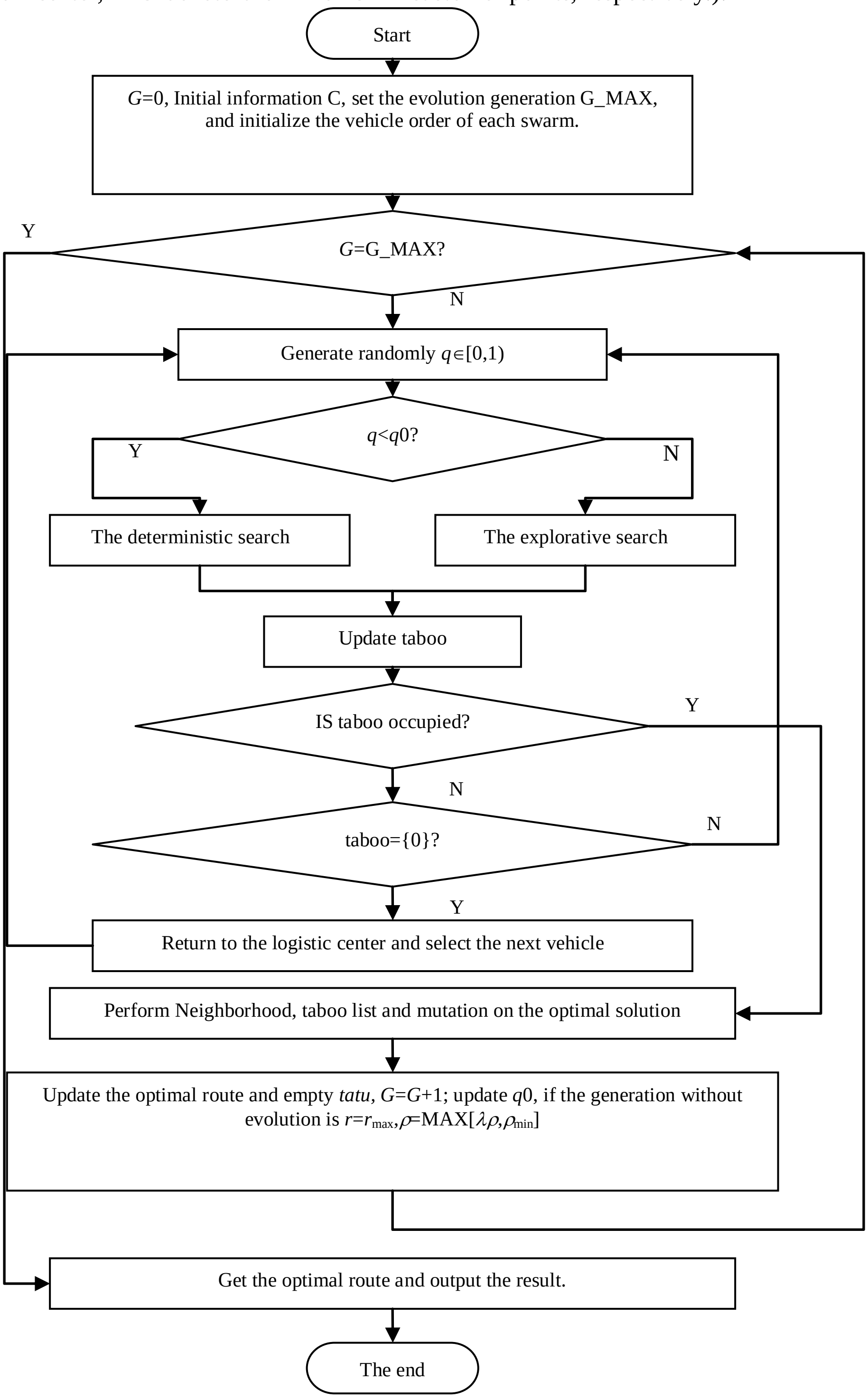

Figure 1. The algorithm flow chart

TABLE I. THE DISTANCES AMONG THE DISTRIBUTION CENTER OR CUSTOMERS(UNIT: KM)

	0	1	2	3	4	5	6	7	8
0	0	4	6	7.5	9	20	10	16	8
1	4	0	6.5	4	10	5	7.5	11	10
2	6	6.5	0	7.5	10	10	7.5	7.5	7.5
3	7.5	4	7.5	0	10	5	9	9	15
4	9	10	10	10	0	10	7.5	7.5	10
5	20	5	10	5	10	0	7	9	7.5
6	10	7.5	7.5	9	7.5	7	0	7	10
7	16	11	7.5	9	7.5	9	7	0	10
8	8	10	7.5	15	10	7.5	10	10	0

TABLE II. THE COORDINATES OF CUSTOMERS' POSITION AND THE QUANTITY DEMAND

D	X axes coordinate(km)	Y axes coordinate(km)	quantity demand(T)	ID	X axes coordinate(km)	Y axes coordinate(km)	quantity demand(T)
1	12.8	8.5	0.1	11	6.7	16.9	0.9
2	18.4	3.4	0.4	12	14.8	2.6	1.3
3	15.4	16.6	1.2	13	1.8	8.7	1.3
4	18.9	15.2	1.5	14	17.1	11.0	1.9
5	15.5	11.6	0.8	15	7.4	1.0	1.7
6	3.9	10.6	1.3	16	0.2	2.8	1.1
7	10.6	7.6	1.7	17	11.9	19.8	1.5
8	8.6	8.4	0.6	18	13.2	15.1	1.6
9	12.5	2.1	1.2	19	6.4	5.6	1.7
10	13.8	5.2	0.4	20	9.6	14.8	1.5

The results of 10 times calculation are as follows:

TABLE III. THE CALCULATION RESULTS AND THE CONCRETE SCHEME

ORDER	1	2	3	4	5	6	7	8	9	10
The total distance	113.0	109.6	110.2	111.7	110.4	111.2	109.1	109.6	107.8	110.4

The average calculation result for ten times is 110.3083km, which is higher than the average result 122.0km in literature[6] and 112.5km in literature[14] and the optimal solution is 107.84km. The corresponding concrete scheme is:

0→4→3→17→11→20→0

0→8→19→15→16→13→6→0

0→5→14→2→12→9→10→7→1→0

0→18→0

Comparing with the optimal solution 108.6 km in literature [14], it has also got improved. The optimal result figure of this paper is shown as follows:

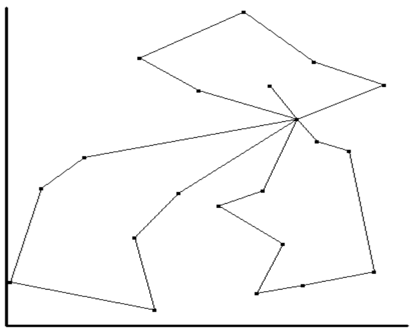

Figure 2. The optimal trajectory result

V. CONCLUSION

According to the characteristics of the logistic distribution routing optimization problems, an optimization route algorithm based on taboo search algorithm is proposed. Through introducing aspiration criterion, this algorithm can avoid algorithm premature and stagnation in local search process and meanwhile, improve the candidate's updating way and the strategy of selecting customer point and enhance the positive feedback effect of taboo search algorithm so as to promote the convergence speed and the global search ability. The experimental results show that the improved taboo search algorithm can get the optimal solution or the optimal approximate solution of the logistic distribution routing optimization fast and effectively. The research in this paper has a certain reference value for the study of taboo search algorithm and logistic distribution routing optimization problems.

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