Nonlinear Dynamics in High-speed Wireless Networks Congestion Control Model with TCP LogWestwood+ under RED

Xianfan Xu [1], Fan Guo [1], Dawei Ding [1, 2*], Sen Qiu [1], Kun Zhang [1], and Haixu Zhang [1]
1. School of Electronics and Information Engineering, Anhui University, Hefei 230601, China
2. Key Laboratory of Intelligent Computing and Signal Processing of Ministry of Education, Anhui University, Hefei 230039, China
*Corresponding author, Email: xxf@ahu.edu.cn p12202008@ahu.edu.cn, dwding@ahu.edu.cn, qiusn@foxmail.com, zhangkun198756@126.com, 1101549736@qq.com

***Abstract*—In this paper, we proposed a first-order discrete time nonlinear dynamic model of congestion control system with TCP LogWestwood+ (TCPLog) connections and random early detection (RED) gateway in high-speed wireless networks. The model is used to analyze nonlinear dynamics of the TCPLog/RED network and its stability with respect to various RED controller and system parameters. Bifurcation and chaos behaviors are shown to occur when parameters varied. By theoretical derivation, the fixed point and the critical value are obtained, and the nature of the bifurcation is determined. Furthermore, bifurcation diagrams and Lyapunov exponent are exploited to verify the theoretical results. Finally, the bifurcation and chaotic phenomena of the congestion control system are numerically studied with TCPLog connections and RED gateway.**

***Index Terms*—Congestion Control System; TCP LogWestwood+; RED; Bifurcation; Chaos**

I. INTRODUCTION

Transmission Control Protocol (TCP) [1] is widely used to offer reliable, bidirectional, virtual channel between any two hosts on the networks. However, wireless networks are characterized by fairly large propagation delays and increasing available bandwidth. As the growing spread of wireless networks, traditional TCP congestion control protocol is no longer applicable, because it is originally designed for wired networks and unable to react adequately to packet losses not related to congestion [2]. Moreover, TCP becomes inefficient and prone to instability when high bandwidth-delay product (HBDP) occurs [3-4]. Due to the above drawbacks of traditional TCP, many TCP protocol modifications like TCP cubic [5], HS-TCP [6] and BIC TCP [7] have been developed for today’s HBDP environment.

Kliazovich et al. has proposed TCP LogWestwood+ [8], as an enhancement of TCP Westwood+, whose essential algorithm is logarithmic increase, adaptive decrease (LIAD) strategy. LIAD inherits the bandwidth estimation technique in TCP Westwood+ and develops a logarithmic increase function in congestion avoidance phase. By means of LIAD, the congestion window increases rapidly when the current value is small and slowly increases when approaching an estimated maximum, which guarantees better throughput and network utilization than other existing strategies. Also, its small sensitivity with respect to Round Trip Time (RTT) and better intra-protocol fairness of bandwidth allocation are demonstrated [8].

With a sustained explosive growth of wireless network applications and subscribers, Internet congestion occurs when the required resources goes beyond the capacity of the Internet’s communication. Internet congestion may result in loss of information, increasing of delay, and even the collapse of the system. So the problem of congestion control is becoming a hot issue. Internet congestion control is an algorithm to allocate available resources to competing sources efficiently so as to avoid congestion collapse [9]. The congestion control algorithm is a highly complex dynamical model, and it will present nonlinear dynamic behaviors like bifurcation and chaos when the system loses stability, which has attracted a great of attention from the authors. It is shown that TCP Reno/RED system becomes chaotic dynamics with variability of RED parameters [10-12] and its stability has been studied [13-14]. DING et al. found earlier that the TCP/RED fluid-flow model would exhibit a Hopf bifurcation when time delay was increased [15]. Using fluid-flow model, Liu et al. analyzed stability and bifurcation in wireless networks [16] and TCP/AQM networks [17]. With analysis software of the delayed dynamical systems-DDE-BIFTOOL, Fold and Hopf bifurcation have been studied in the delayed Internet TCP-RED congestion control model [18]. So, a deep insight of the system nonlinear dynamics is helpful to understand the nature of congestion control system and to improve its performance.

Researchers have developed many nonlinear dynamics models of other congestion control algorithms and further studied their stability. DING et al. validated periodic doubling bifurcation by means of one-order discrete model for congestion control system with TCP Westwood [19]. A continuous time model and extensive stability analysis of FAST TCP congestion control mechanism in bufferless Optical Burst Switched Networks (OBS) has

doi:10.4304/jnw.9.7.1714-1720

been provided in [20]. In [21], the authors investigated the linear stability and Hopf bifurcation of the eXplicit Control Protocol (XCP) for the Internet congestion control system. A mathematical model was presented to systematically analyze the characteristics of fast retransmission and recovery in TCP-SACK [22].

In this paper, in order to study the nonlinear dynamics of a high-speed wireless network with TCP LogWestwood+ connections and a RED gateway, first, a deterministic discrete time dynamic feedback model of this congestion control system is introduced. Then, by theoretical calculation, the fixed point, the critical value, and the nature of bifurcations in this system are determined. Finally, choosing different RED parameters as bifurcation parameters, period doubling bifurcation and chaotic behaviors are proved and described by bifurcation diagram and Lvapunov exponent.

The remainder of this paper is organized as follows. Section 2 presents the nonlinear discrete-time model used in the analysis. In Section 3, the fixed point and bifurcations are analyzed. In Section 4, numerical examples illustrating the nonlinear dynamics are shown in the model. Finally, conclusions are drawn in Section 5.

II. Dynamic Model for TCP LogWestwood+ under RED

The congestion control strategy of TCP LogWestwood+ is logarithmic increase, adaptive decrease, targeting adaptation to the high-speed wireless environment. Logarithmic increase means that the actions undertaken in response to the reception of an ACK packet are different from additive increase of standard TCP, leading to an approximately logarithmic increase in absence of loss events. Moreover, the main idea of adaptive decrease is to keep an estimate of the available end-to-end capacity and to exploit such information in order to reduce transmission rate, instead of the blind window halving implemented in TCP as well as in other related algorithms [8].

Suppose a simple network where a single bottleneck link is shared by multiple connections. Assume that all connections are TCP LogWestwood+ connections with the same round-trip propagation delay, which is denoted by d second. These TCP LogWestwood+ flows are connected with two routers that run RED algorithm. The number of connections is N and their packet size is M bit/packet. The capacity of the bottleneck link is denoted by C bit/s.

According to Ref. [10], the congestion control model is defined as follows. The RED controller at the router provides feedback signal p_k (packet drop probability) at period k, which determines the throughput of connections and the queue size q_{k+1} at period $k+1$. The queue size q_{k+1} is used to compute the average queue size $\bar{q}_{k+1}$ according to the exponential averaging rule. Then the packet drop probability p_k is a function of the average queue size $\bar{q}_k$ at period k. These can be expressed as follows:

$$q_{k+1} = G(p_k) \tag{1}$$

$$\bar{q}_{k+1} = A(\bar{q}_k, q_{k+1}) \tag{2}$$

$$p_{k+1} = H(\bar{q}_{k+1}) \tag{3}$$

in which $A(\bar{q}_k, q_{k+1})$ is the averaging function:

$$A(\bar{q}_k, q_{k+1}) = w \cdot q_{k+1} + (1-w) \cdot \bar{q}_k \tag{4}$$

where w is the exponential averaging weight that determines how fast the RED mechanism reacts to a time-varying load. If w is small enough, the average queue size $\bar{q}_k$ will depend on the long-term changing tendency of queue size.

The RED control function $H(\bar{q}_{k+1})$ is given as:

$$p_{k+1} = H(\bar{q}_{k+1}) = \begin{cases} 0, & \bar{q}_{k+1} < q_{\min} \\ 1, & \bar{q}_{k+1} > q_{\max} \\ \dfrac{\bar{q}_{k+1} - q_{\min}}{q_{\max} - q_{\min}} p_{\max}, & otherwise \end{cases} \tag{5}$$

where $q_{\min}$ and $q_{\max}$ are the minimum and maximum critical values of queue size, and $p_{\max}$ is the drop probability when $\bar{q} = q_{\max}$.

The steady state throughput of a TCP LogWestswood+ connection [8] is given by

$$r^{\log west} = \frac{1-p}{2\alpha p T_q} \cdot \left(\sqrt{1 + \frac{4 W_{\max} T_q}{(1-p)RTT}} - 1 \right) \tag{6}$$

where RTT is average round trip time:

$$RTT = d + \frac{q \cdot M}{C} \tag{7}$$

T_q is average queuing time and equal to the difference between RTT and the minimum round trip time $R_{\min}$.

$$T_q = RTT - RTT_{\min} = \frac{q_{ave} \cdot M}{C} = \frac{B \cdot M}{2C} \tag{8}$$

where B is the finite buffer size of RED router and q_{ave} is the average queuing size that approximately equals to $B/2$.

α is window update adjusting weight. We decide that $\alpha = 2$, because the value less than 2 will makes the increase of congestion window too aggressive.

$W_{\max}$ is defined as the window size at which the last packet loss event was detected. TCP Westwood (TCPW) and TCP Westwood+ (TCPW+) show good fairness properties that network resource is shared evenly when all connections are TCPW or TCPW+ flows in the steady-state. TCP LogWestwood+ is superior to TCPW+

on fairness. So consider all network resource of link capacity and router buffer:

$$W_{\max} = \frac{C \cdot d + B \cdot M}{N \cdot M} \quad (9)$$

Consider two edge cases: 1) the smallest packet drop probability p_u results in a queue size of zero at the next period; 2) the largest probability p_l leads to a queue size q_{k+1} of the buffer size B.

Because the aggregate throughput of connections cannot exceed link capacity, p_u can be determined when the bandwidth capacity constraint is satisfied:

$$N \cdot r^{\log west} \cdot M = C. \quad (10)$$

From (6) and (10), the following equation can then be obtained:

$$\frac{N \cdot M(1-p_u)}{2\alpha p_u T_q}\left(\sqrt{1+\frac{4W_{\max}T_q\alpha p_u}{(1-p_u)RTT}}-1\right)=C \quad (11)$$

As $q_{k+1}=0$, if $p_k \ge p_u$. Now RTT is equal to d. The following equation can be derived:

$$p_u = \frac{2MN}{2MN+\alpha Cd}$$

The average queue size $\bar{q}_u$, which satisfies $q_{k+1}=0$, if $\bar{q}_k \ge \bar{q}_u$, is given by

$$\bar{q}_u = \begin{cases} \dfrac{p_u(q_{\max}-q_{\min})}{p_{\max}} + q_{\min} & p_u \le p_{\max} \\ q_{\max}, & \text{otherwise} \end{cases} \quad (12)$$

Thus, if $p_k < p_u$, p_l still satisfies (11):

$$\frac{N \cdot M(1-p_l)}{2\alpha p_l T_q} \times \left(\sqrt{1+\frac{4W_{\max}T_q\alpha p_l}{(1-p_l)\left(d+\dfrac{BM}{C}\right)}}-1\right)=C \quad (13)$$

Then p_l can be obtained:

$$p_l = \frac{W_{\max}N^2M^2 - NM\cdot\delta}{W_{\max}N^2M^2 - NM\cdot\delta + \alpha T_q C\cdot\delta} \quad (14)$$

where $\delta = dC + BM$.

From (9) and (14), p_l is equal to zero. TCP LogWestwood+ algorithm specifies that, for each acknowledgment packet received, the congestion window should be updated according to:

$$W \leftarrow \frac{W_{\max}-W}{\alpha W}$$

This updating strategy is more aggressive and efficient than conservative additive increase of TCP when the congestion window is small, and the congestion window value increases very slowly when it approaches $W_{\max}$. Therefore, the dependency on RTT is reduced and better network utilization can be achieved. Queue size q_{k+1} can be equal to B only when p_l equals to zero.

From (5), the corresponding average queue size $\bar{q}_l$ is:

$$\bar{q}_l = \frac{p_l(q_{\max}-q_{\min})}{p_{\max}} + q_{\min} \quad (15)$$

If $p_l < p_k < p_u$, queue size q_{k+1} satisfies the following equation:

$$\frac{N \cdot M(1-p_k)}{2\alpha p_k T_q}\left(\sqrt{1+\frac{4W_{\max}T_q\alpha p_k}{(1-p_k)\psi}}-1\right)=C \quad (16)$$

where $\psi = d + \dfrac{q_{k+1}\cdot M}{C}$.

Queue size q_{k+1} is given by:

$$q_{k+1} = \frac{W_{\max}MN^2(1-p_k)}{\alpha T_q C p_k + MN(1-p_k)} - \frac{dC}{M} \quad (17)$$

From above analysis, it can be derived that

$$G(p_k) = \begin{cases} 0, & p_k \ge p_u \\ B, & p_k \le p_1 \\ \dfrac{W_{\max}MN^2(1-p_k)}{\alpha T_q C p_k + MN(1-p_k)} - \dfrac{dC}{M}, & otherwise \end{cases} \quad (18)$$

From (1)-(3) and (18), we can obtain the nonlinear one-order discrete-time dynamic model of TCP LogWestwood+ under RED:

$$\bar{q}_{k+1} = A(\bar{q}_k, q_{k+1}) = \begin{cases} (1-w)\bar{q}_k & \bar{q}_k > \bar{q}_u \\ (1-w)\bar{q}_k + wB, & \bar{q}_k < \bar{q}_l \\ (1-w)\bar{q}_k + w \times \left(\dfrac{W_{\max}MN^2(1-p_k)}{\alpha T_q C p_k + MN(1-p_k)} - \dfrac{dC}{M}\right), & \text{otherwise} \end{cases} \quad (19)$$

where $p_k = p_{max} \cdot ((\bar{q}_k - q_{\min})/(q_{\max}-q_{\min}))$.

III. Fixed Point and Bifurcation

A. Fixed Point of the System

To derive the fixed point of (19), the authors first denote

$$\bar{q}_{k+1} = g(\bar{q}_k, \rho) \quad (20)$$

where ρ is the system parameter such as exponential averaging weight w. The fixed point of mapping $g(\cdot)$ is an average queue size q^* such that $q^* = g\left(q^*, \rho\right)$. If the parameters are properly configured, the fixed point should remain between $q_{\min}$ and $q_{\max}$. Solve $q^* = g\left(q^*, \rho\right)$ as follows:

$$q^* = (1-w)q^* + w\left(\frac{W_{\max}MN^2(1-p_k)}{\alpha T_q C p_k + MN(1-p_k)} - \frac{dC}{M}\right) \tag{21}$$

where $p_k = p_{max} \cdot ((q^* - q_{\min})/(q_{\max} - q_{\min}))$, then denote $v = p_{max}/(q_{\max} - q_{\min})$.

The fixed point of the system, which is the positive real number solution of the follow equation, can be obtained.

$$ax^2 + bx + c = 0 \tag{22}$$

$$a = \alpha T_q C v - MNv \tag{23}$$

$$b = \eta + \beta - \gamma v + \lambda v \tag{24}$$

$$c = \gamma + \gamma q_{\min} v - \beta q_{\min}/M - \lambda - \lambda q_{\min} v \tag{25}$$

$$\beta = dC^2 \alpha T_q v / M \tag{26}$$

$$\gamma = dCN \tag{27}$$

$$\eta = MN + MNq_{\min} v - \alpha T_q C q_{\min} v \tag{28}$$

$$\lambda = W_{\max} MN^2 \tag{29}$$

Hence the representation of q^* is:

$$q^* = \frac{-b + \sqrt{\Delta}}{2a} \tag{30}$$

where $\Delta = b^2 - 4ac$.

B. Bifurcation Analysis

The associated eigenvalue of (20) is showed as follows:

$$\left.\frac{\partial g\left(\bar{q}_k, \rho\right)}{\partial \bar{q}_k}\right|_{\bar{q}_k = q^*} = 1 - w - w\frac{\varepsilon}{\left(\sigma q^* + \eta\right)^2} \tag{31}$$

where $\varepsilon = W_{\max} MN^2 \alpha T_q C v$ and $\sigma = \alpha T_q C v - MNv$.

The linear stability criterion is $\left|\partial g\left(\bar{q}_k, \rho\right)/\partial \bar{q}_k\right|_{\bar{q}_k = q^*} < 1$ or

$$\left|1 - w - w\frac{\varepsilon}{\left(\sigma q^* + \eta\right)^2}\right| < 1 \tag{32}$$

If parameter settings violate the above linear stability criterion, the fixed point q^* losses stability and periodic doubling bifurcation occurs. In order to describe such bifurcation, one should choose a bifurcation parameter. Several parameters can be chosen as bifurcation parameter, such as the exponential average weight w, the number of TCP connections N, the propagation delay d and $p_{\max}$.

First, to explain this bifurcation, we choose the exponential average weight w as bifurcation parameter. From (32), it is known that the eigenvalue is a linearly decreasing function of w, so the critical value of w is a value which can satisfy the following equation:

$$1 - w - w\frac{\varepsilon}{\left(\sigma q^* + \eta\right)^2} = -1 \tag{33}$$

The critical value of w can be obtained:

$$w_c = \frac{2}{1 + \frac{\varepsilon}{\left(\sigma q^* + \eta\right)^2}} \tag{34}$$

The fixed point will become unstable and a period doubling bifurcation will occur when the exponential average weight w is increased to pass the critical value w_c. A period doubling bifurcation has two types: supercritical and subcritical. A supercritical bifurcation leads to a steady oscillatory behavior near the fixed point, while a subcritical bifurcation results in divergent oscillations. To determine the nature of the bifurcation, the second and the third derivatives of $g(\cdot)$ are computed:

$$\left.\frac{\partial g^2\left(\bar{q}_k, \rho\right)}{\partial \bar{q}_k^2}\right|_{\bar{q}_k = q^*} = \frac{2\sigma w \varepsilon}{\left(\sigma q^* + \eta\right)^3} \tag{35}$$

$$\left.\frac{\partial g^3\left(\bar{q}_k, \rho\right)}{\partial \bar{q}_k^3}\right|_{\bar{q}_k = q^*} = -\frac{6\sigma^2 w \varepsilon}{\left(\sigma q^* + \eta\right)^4} \tag{36}$$

The quantity

$$S = \frac{1}{2}\left(\frac{\partial g^2}{\partial \bar{q}_k^2}\right)^2 + \frac{1}{3}\left(\frac{\partial g^3}{\partial \bar{q}_k^3}\right) \tag{37}$$

determines the nature of the period doubling bifurcation. A positive S indicates that the bifurcation is supercritical and a negative S implies a subcritical bifurcation. For (19),

$$S = \frac{2\sigma^2 w \varepsilon}{\left(\sigma q^* + \eta\right)^4}\left(\frac{w\varepsilon}{\left(\sigma q^* + \eta\right)^2} - 1\right) \tag{38}$$

We should choose parameters properly and keep the value of S positive, because a subcritical bifurcation directly leads to unexpected oscillation in router queues.

IV. Numerical Examples

In this section, by choosing the exponential average weight w, drop probability $p_{\max}$, the number of TCP connections N and the propagation delay d as bifurcation parameter respectively, the stability of the system is numerically studied and the analysis in Section 3 is validated. Furthermore, a bifurcation diagram shows qualitative changes in the nature and the number of fixed points of a dynamic system with varied parameters.

A. Exponential Averaging Weight

First, we study the effect of exponential averaging weight w. The remaining system parameters are set as follows:

$$q_{\max} = 750\,packet, q_{\min} = 250\,packet, p_{\max} = 0.3,$$
$$C = 15Mbit/s, B = 3750\,packets, M = 4000bit,$$
$$d = 0.1s, N = 100.$$

From (30), it is drawn that the fixed point of the system $q^* = 417.94$. The critical value of w is $w_c = 0.3816$ and $S = 0.00004387$ at w_c by computing (34) and (38). The bifurcation diagram with w varying from 0.3 to 0.5 is plotted in Fig.1, in which we can see that the system is stable and these plots have a fixed point for small $w < w_c$. When w increases to w_c, the system loses its stability and a supercritical period doubling bifurcation emerges as $S > 0$. This oscillatory behavior in the system is caused by inherent nonlinearity. Increasing w results in more complex behavior such as chaotic phenomenon. A negative Lyapunov exponent indicates that the system is local stable. Since a positive Lyapunov exponent can be used to judge chaos behavior, the Lyapunov exponent corresponding to bifurcation scenario of Fig. 1 is also plotted in Fig. 2. From Fig. 2 one can see that in the chaos region there exist a large number of periodic orbits.

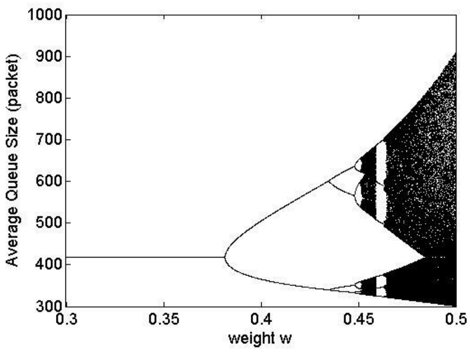

Figure 1. Bifurcation diagram of average queue size with respect to the exponential averaging weight

B. Drop Probability

In this subsection, the drop probability $p_{\max}$'s effect on the stability and behavior of system is studied. First, w is set as $w = 0.25$, then the value of $p_{\max}$ is changed while other parameters are the same as those in above subsection:

$$q_{\max} = 750\,packets, q_{\min} = 250\,packets,$$
$$C = 15Mbit/s, B = 3750\,packets,$$
$$M = 4000bit, d = 0.1s, N = 100, w = 0.25.$$

Fig. 3 is the bifurcation diagram with $p_{\max}$ changing from 0.3 to 0.82, from which one can see the similar nonlinear behavior, period doubling bifurcation leading to chaos orbits as increasing $p_{\max}$. Note that the plot of fixed point is not a horizontal as that in Fig. 1, because it is varying with $p_{\max}$.

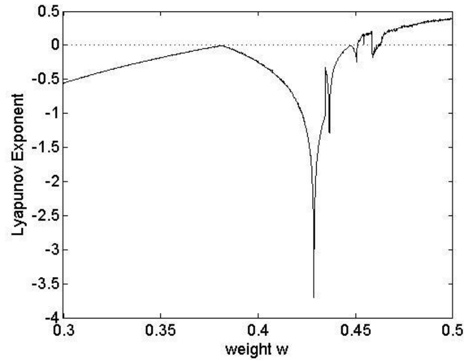

Figure 2. Lyapunov exponent for average queue size with respect to the exponential averaging weight

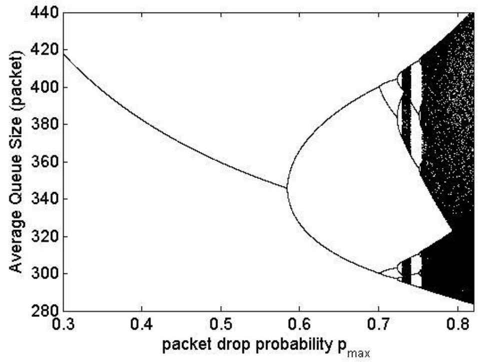

Figure 3. Bifurcation diagram of average queue size with $p_{\max}$

C. Number of Connections

Unlike the parameter studied above, the number of TCP connections and the propagation delay of the networks cannot be controlled by a network manager. So knowing the effect of these parameters on system stability and behaviors is significant for setting the RED control parameters in practice. In this subsection, the effect of the number of TCP LogWestwood+ connections is discussed and the propagation delay's influence is investigated in next subsection.

We choose the number of TCP LogWestwood+ N as bifurcation parameter and set other parameters as follows:

$$q_{\max} = 750\,packet, q_{\min} = 250\,packet,$$
$$C = 15Mbit/s, B = 3750\,packets, M = 4000bit,$$
$$d = 0.1, w = 0.25, p_{\max} = 0.3$$

Here, N is varied from 30 to 60, the bifurcation diagram in Fig. 4 shows that the system stabilizes as the number of connections increases. This result agrees with the general result that a larger number of users tend to stabilize the system [23-24].

D. Propagation Delay

At last, the bifurcation diagram is plotted in Fig. 5 with d varying from 0.15 to 0.23. Other parameters are set as follows:

$q_{\max} = 750\ packets, q_{\min} = 250\ packets, p_{\max} = 0.3,$

$C = 15 Mbit/s, B = 3750\ packets, M = 4000\ bit,$

$N = 100, w = 0.25$

It shows that the system is stable when round trip propagation delay d is small, and finally becomes chaotic with d increasing. This phenomenon also agrees with the result that smaller delay tends to keep the system stable [23-24].

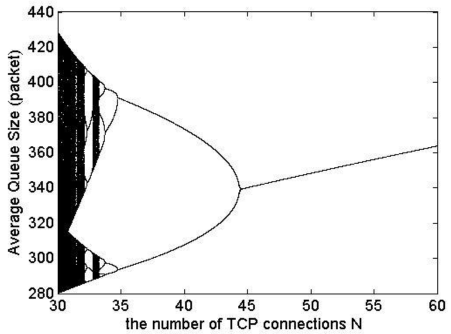

Figure 4. Bifurcation diagram of average queue size with respect to TCP connections

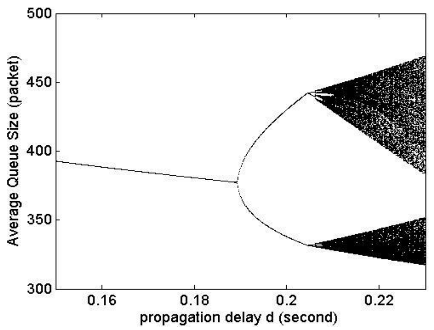

Figure 5. Bifurcation diagram of average queue size with respect to propagation delay

Comparing with [25], our nonlinear one-order discrete-time dynamic model of TCP LogWestwood+ under RED is more complete and accurate, while Chen et al. just model the specific behaviour of TCP Westwood in congestion avoidance phase. Furthermore, by choosing different system and control parameters as bifurcation parameter, the relation between the system stability and every bifurcation parameter is described clearly, which is meaningful for setting parameters in practice. However, only the system stability with different round trip propagation delay is taken into account in [25].

V. CONCLUSIONS

In the paper, the nonlinear dynamics of congestion control system of TCP LogWestwood+ with RED gateway is discussed. First, a simple one-order discrete time model is built by theoretical derivation. Then, the fixed point of the system and the nature of bifurcations are determined by the model. Moreover, by adopting bifurcation diagrams and Lyapunov exponent, the nonlinear behaviors including periodic doubling bifurcation and chaotic phenomena are illustrated when varying different system parameters.

ACKNOWLEDGMENT

The authors would like to thank the reviewers of this paper for their carefully reading of the manuscript as well as their many constructive comments. This paper is supported by NSFC-Guangdong Joint Fund (No: U1201255), the National Natural Science Foundation of China (No: 61201227), Young-backbone Teacher Project of Anhui University, and the Doctoral Scientific Research Foundation of Anhui University.

REFERENCES

[1] J. B. Postel, Transmission control protocol, *RFC 783 (September)* (1981).

[2] Afanasyev A, Tilley N, Reiher P, et al. Host-to host congestion control for TCP. *Communication Surveys & Tutorials, IEEE*, 2010, 12(3) pp. 304-342.

[3] Hong Y, Yang O W W. Robust Explicit Congestion Controller Design for High Bandwidth-Delay Product Network: AH∞ Approach//*Communications (ICC), 2010 IEEE International Conference on. IEEE*, 2010 pp. 1-5.

[4] Jung H, Kim S, Yeom H Y, et al. Adaptive delay-based congestion control for high bandwidth-delay product networks//*INFOCOM, 2011 Proceedings IEEE. IEEE*, 2011 pp. 2885-2893.

[5] Tomita N, Valaee S. Data uploading time estimation for CUBIC TCP in long distance networks. *Computer Networks*, 2012, 56(11) pp. 2677-2689.

[6] Charbonneau N, Vokkarane V M. Performance modeling of HS-RR-TCP over load-balanced optical burst-switched (OBS) networks. *Optical Switching and Networking*, 2011, 8(2) pp. 116-128.

[7] Xu L, Harfoush K, Rhee I. Binary increase congestion control (BIC) for fast long-distance networks//*INFOCOM 2004. Twenty-third AnnualJoint Conference of the IEEE Computer and Communications Societies. IEEE*, 2004, 4 pp. 2514-2524.

[8] Kliazovich D, Granelli F, Miorandi D. Logarithmic window increase for TCP Westwood+ for improvement in high speed, long distance networks. *Computer Networks*, 2008, 52(12) pp. 2395-2410.

[9] Dong T, Liao X, Huang T. Dynamics of a congestion control model in a wireless access network. *Nonlinear Analysis: Real World Applications*, 2012.

[10] Ranjan P, Abed E H, La R J. Nonlinear instabilities in TCP-RED//*Complex Dynamics in Communication Networks. Springer Berlin Heidelberg*, 2005 pp. 279-308.

[11] Liu M, Zhang H, Trajkovic L. Stroboscopic model and bifurcations in TCP/RED//*Circuits and Systems, 2005. ISCAS 2005. IEEE International Symposium on. IEEE*, 2005 pp. 2060-2063.

[12] Liu M, Marciello A, di Bernardo M, et al. Discontinuity-induced bifurcations in TCP/RED communication algorithms//*Circuits and Systems, 2006. ISCAS 2006. Proceedings. 2006 IEEE International Symposium on. IEEE*, 2006 pp. 4.

[13] Chen X, Wong S C, Tse C K, et al. Stability study of the TCP-RED system using detrended fluctuation analysis//*Circuits and Systems, 2008. ISCAS 2008. IEEE International Symposium on. IEEE,* 2008 pp. 324-327.
[14] Chen X, Wong S C, Tse C K, et al. Stability analysis of RED gateway with multiple TCP Reno connections//*Circuits and Systems, 2007. ISCAS 2007. IEEE International Symposium on. IEEE,* 2007 pp. 1429-1432.
[15] DING D, ZHU J, LUO X. Hopf bifurcation analysis in a fluid flow model of Internet congestion control algorithm. *Nonlinear Analysis: Real World Applications,* 2009, 10(2) pp. 824-839.
[16] Feng Liu, Hua O. Wang, Zhi-Hong Guan. Stability Analysis and Control of bifurcation in a TCP Fluid Flow Model of Wireless Networks //*WCICA, 2012 Proceedings IEEE. IEEE,* 2012 pp. 1026-1030.
[17] Feng Liu, Zhi-Hong Guan, Guangxi Zhu, et al. Stability Analysis and Impulsive Control of Bifurcation and Chaos in Fluid Flow Model for TCP/AQM Networks// *WCICA, 2010 Proceedings IEEE. IEEE,* 2010 pp. 1257-1261.
[18] Lijun P, Li Z, Ruimin W. Co-dimension 1 bifurcations of the delayed Internet TCP-RED model//*Multimedia Information Networking and Security (MINES), 2010 International Conference on. IEEE,* 2010 pp. 565-569.
[19] DING D, ZHU J, LUO X, et al. Nonlinear dynamic in Internet congestion control model with TCP Westwood under RED. *The Journal of China Universities of Posts and Telecommunications,* 2009, 16(4) pp. 53-58.
[20] Shihada B, El-Ferik S, Ho P H. FAST TCP over optical burst switched networks: Modeling and stability analysis. *Optical Switching and Networking,* 2012.
[21] Liu F, Wang H O, Guan Z H. Hopf bifurcation control in the XCP for the Internet congestion control system. *Nonlinear Analysis: Real World Applications,* 2012, 13(3) pp. 1466-1479.
[22] Sheu T L, Wu L W. An analytical model of fast retransmission and recovery in TCP-SACK. *Performance Evaluation,* 2007, 64(6) pp. 524-546.
[23] Hollot C V, Misra V, Towsley D, et al. A control theoretic analysis of RED//INFOCOM 2001. *Twentieth Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE. IEEE*, 2001, 3 pp. 1510-1519.
[24] Low S H, Paganini F, Wang J, et al. Dynamics of TCP/RED and a scalable control//*INFOCOM 2002. Twenty-First Annual Joint Conference of the IEEE Computer and Communications Societies. Proceedings. IEEE. IEEE,* 2002, 1 pp. 239-248.
[25] Chen J, Paganini F, Sanadidi M Y, et al. Fluid-flow analysis of TCP Westwood with RED. *Computer Networks*, 2006, 50(9) pp. 1302-1326.

Xianfan Xu was born in Hefei, Anhui, China on December, 1957. He received his B.S. degree in radio engineering from the Anhui University in 1981. He is currently an Associate Professor at the Anhui University. He has been in charge of several research programs supported by Ministry of Science and Technology and Anhui Province. His research interests include high frequency communication and electronic measurement.

Fan Guo is a master candidate in communication and information system of Anhui University. Her current research interests include congestion control, nonlinear dynamics and chaos.

Dawei Ding received his PhD degree in circuits and systems from the Shanghai Jiaotong University in 2008. He is currently a professor at the Anhui University, China. He has published more than 10 papers in the corresponding areas. Currently, he is in charge of a National Natural Science Foundation (No.61201227) and a research program supported by NSFC-Guangdong Joint Fund (No.U1201255). His research interests include network communication, congestion control and applications of compressed sensing in communication and network.

Sen Qiu is a master candidate in communication and information system of Anhui University. His current research interests include congestion control and bifurcation control.

Kun Zhang received his M.S. degree in communication and information system from Anhui University in 2013. His current research interests include network communication and cloud computing.

Haixu Zhang is studying for an undergraduate degree in Anhui University. His research interests include computer networks and signal processing.