Final Report

Project Title: Large-Eddy Simulation of Anisotropic MHD Turbulence

Recipient: University of Michigan, 4901 Evergreen Road, Dearborn, MI 48128-1491

Award Number: DE-FG02-03ER46062

Principal Investigator: Dr. Oleg Zikanov, (313) 593-3718, zikanov@umd.umich.edu

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Project objective:

To acquire better understanding of turbulence in flows of liquid metals and other electrically conducting fluids in the presence of steady magnetic fields and to develop an accurate and physically adequate LES (large-eddy simulation) model for such flows.

Summary of achievements:

The scientific objectives formulated in the project proposal have been fully completed. Several new directions were initiated and advanced in the course of work. Particular achievements include a detailed study of transformation of turbulence caused by the imposed magnetic field, development of an LES model that accurately reproduces this transformation, and solution of several fundamental questions of the interaction between the magnetic field and fluid flows. Eight papers have been published in respected peer-reviewed journals, with two more papers currently undergoing review, and one in preparation for submission. A post-doctoral researcher and a graduate student have been trained in the areas of MHD, turbulence research, and computational methods. Close collaboration ties have been established with the MHD research centers in Germany and Belgium.

I. Research Tasks.

1. Study of fundamental properties of homogeneous MHD turbulence (Ref. [1] in the list of papers).

The pseudo-spectral parallel FFT-based code was developed and applied to extensive direct and large-eddy simulations of the MHD turbulence in a rectangular box with periodic boundary conditions. Statistical characteristics of the computed flow fields were evaluated, including spectra of kinetic energy, viscous and magnetic dissipation, and scale-related anisotropy, integral and microscale length scales in each direction, two-point velocity correlations, pdfs of velocity fluctuations, etc. Particular attention was given to the anisotropy of small-scale turbulent fluctuations caused by the magnetic field. The results are illustrated in figures 1 and 2. Figure 1 shows the development of anisotropy in the flow after introduction of the magnetic field. Figure 2 presents probably the most important result of our study. At moderate and small length scales, the dimensional anisotropy (anisotropy of the velocity gradients) is primarily determined by the strength of the applied magnetic field. The effects of the length scale, Reynolds number, and forcing (i.e., the large-scale dynamics) are much weaker. Similar behavior was observed for the anisotropy of the Reynolds stress tensor (inequality of velocity components).

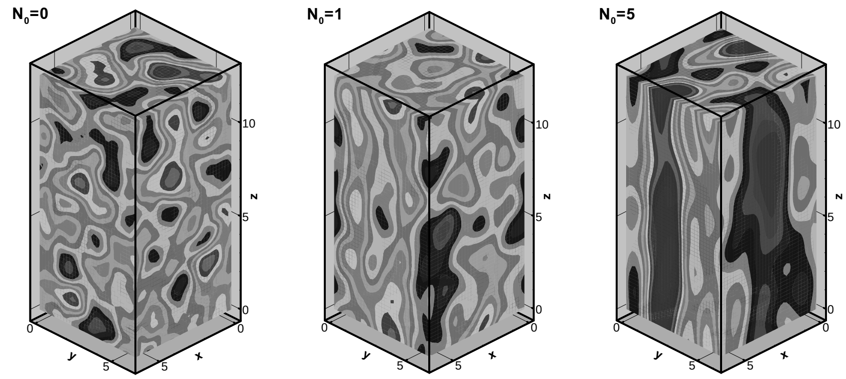

Figure 1. Snapshots of modified pressure field in simulations (DNS with isotropic large-scale forcing at $\mathrm{Re}_\lambda \approx 100$). N_0 is the magnetic interaction parameter that shows the strength of the applied magnetic field.

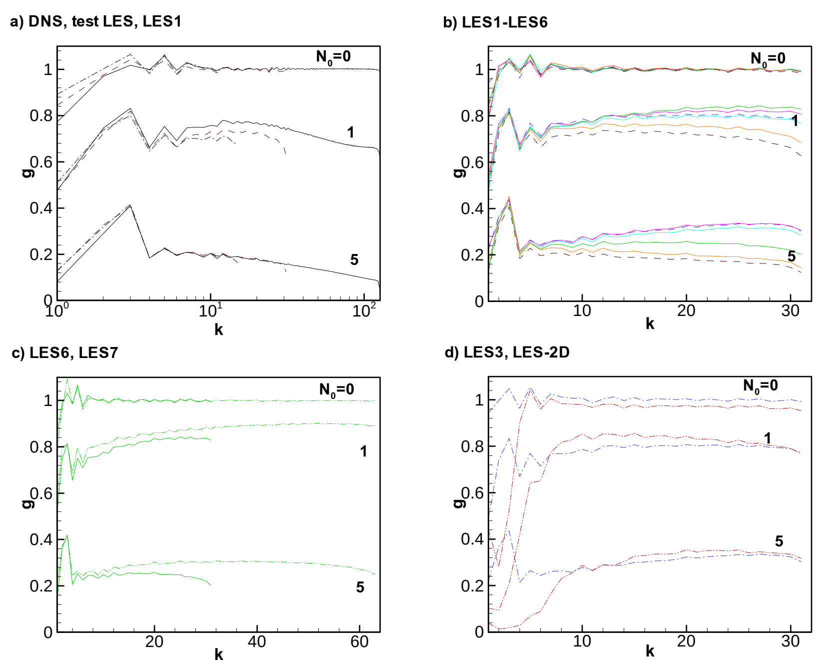

Figure 2. Measure of scale-dependent dimensional anisotropy $g(k) = (3\tau/2)\mu(k)/E(k)$, which is equal to 1 in an isotropic flow and 0 in a two-dimensional flow). *(a)*, DNS and test LES at the same parameters; *(b)*, LES at different values of Reynolds number; *(c)*, LES with $\mathrm{Re}_\lambda \approx 300$ at different numerical resolution; *(d)*, LES at $\mathrm{Re}_\lambda \approx 150$ with isotropic and quasi-2D forcing.

2. Development of LES (Large-Eddy Simulation) model (Refs. [1] and [7]).
Comparison between the DNS and LES results obtained for forced and decaying homogeneous MHD turbulence was used for *a-posteriori* testing of different models. The following models were identified based on our previous results

1. Dynamic Smagorinsky model (classical version developed for non-magnetic flows)
2. Dynamic Smagorinsky model with 3 dynamically determined constants
3. Standard (non-dynamic) Smagorinsky model with the Smagorinsky constant C_S modified to account for the anisotropy

All three models were found to be sufficiently accurate and capable of adjustment to the transformation of the small-scale turbulence caused by the magnetic field. This was in contrast with the over-dissipative action of the standard Smagorinsky model with constant C_S. No distinction could be made between the levels of accuracy of the three models. On the other hand, the model 3 presents an important advantage of much smaller (about ½) amount of required computations. In the model, the numerically expensive dynamic adjustment of C_S is replaced by a simple formula

$$C_S = C_{S0} G, \qquad (1)$$

where C_{S0} is the value of the Smagorinsky constant in an isotropic flow without magnetic field and G is defined as

$$G = \left\langle \left(\partial v / \partial z \right)^2 \right\rangle \Big/ 2 \left\langle \left(\partial v / \partial x_\perp \right)^2 \right\rangle, \qquad (2)$$

where brackets stand for volume averaging and $x_\perp$ is the coordinate across the magnetic field. We found that, because of its near scale-invariance, the MHD-related anisotropy is accurately estimated by this coefficient.

Detailed investigation showed that the model (1) is accurate in the statistically steady flows. This is illustrated in figure 3, which shows that the values of C_S obtained in the dynamic model simulations closely follow (1) in a wide range of Reynolds numbers, filter widths, and magnetic interaction parameters. Some loss of accuracy occurs in transitional flows, where the variation of C_S is delayed in comparison with the variation of G.

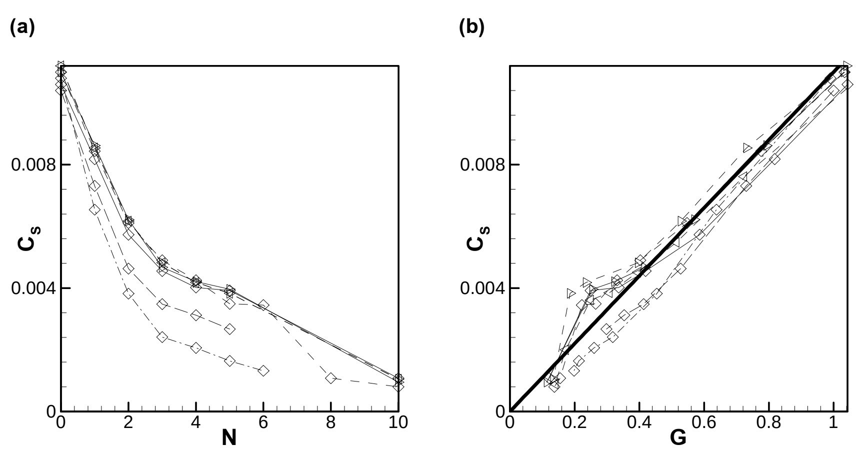

Figure 3: Volume- and time-averaged Smagorinsky constant C_S as a function of the magnetic interaction parameter N and the global anisotropy coefficient G. Symbols are for different Re and numerical resolutions. Bold line in *(b)* is for the linear relation (1).

3. Large-eddy simulations of decaying homogeneous turbulence (Ref. [11]).
The developed LES model was recently applied to investigate the turbulence decaying under the combined impact of viscosity and magnetic field. The main purpose was to consider the possible reasons for significant discrepancy between the rates of the energy decay recorded in earlier experiments. More specifically, the measurements showed entirely different values of the exponent of the asymptotic power laws of the decay, generally considered one of the most important characteristics of turbulence.

Our computations have shown that the main reason of the controversy is that no universal power law behavior should be expected in the MHD case. The rate varies strongly depending on the flow parameters and the stage of the decay process. Moreover, different decay rates are observed for parallel and perpendicular velocity components.

4. Large-eddy simulation of turbulence in MHD channel flow (Ref. [9]).
The periodic box geometry considered in the first stage of the project has proven to be uniquely suitable for understanding the fundamental features of the MHD flow transformation. Turbulence in practical applications of MHD, however, typically occurs in the presence of solid walls and mean shear. This is true, for example, for continuous steel casing, crystal growth, or lithium cooling blankets for fusion reactors. The simplest well defined model flow that includes these features is a pressure-driven channel flow in the presence of imposed magnetic field.

In our work, the focus was on the configuration with the magnetic field in the spanwise (parallel to the wall and perpendicular to the flow) magnetic field. Two directions were pursued: further verification of the LES models and investigation of the flow transformation under the impact of the magnetic field. We found that the dynamic Smagorinsky model remains accurate. In general, this model can be recommended for future simulations of practical MHD flows. The modified non-dynamic model (1)-(2) can also be used although it has poorer accuracy in the areas of strong mean shear.

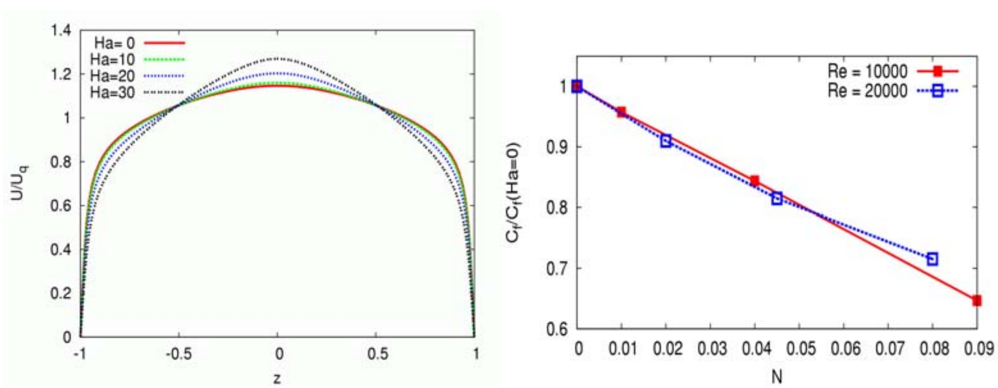

Figure 4. Transformation of the channel flow under the impact of spanwise magnetic field. On the left: mean flow profiles at Re=10000 and different values of the Hartmann number, which shows the strength of the magnetic field. On the right: viscous drag as a function of the magnetic interaction parameter $N=Ha^2/Re$.

The flow transformation under the impact of the magnetic field is illustrated in figure 4. The key effect is the suppression of the turbulent momentum transfer in the wall-normal direction. As a result, the mean flow profile becomes steeper, the centerline velocity grows, and the viscous drag decreases.

5. Analysis of two-dimensional coherent structures in the presence of a magnetic field (Refs. [4], [5]).

In the course of the project, it was realized that development of models of the MHD turbulence requires better understanding of the behavior of coherent flow structures (strained vortices and vortex sheets) in the presence of a strong magnetic field. The traditional picture, according to which the structures grow along the magnetic field lines until they become nearly two-dimensional, is based on linearized models and can be misleading. The reason is that the coherent structures are prone to three-dimensional instabilities and subsequent non-linear breakdown. The effect becomes potentially dangerous in the quasi-two-dimensional state when the suppression of unstable perturbations by the magnetic field weakens.

To better understand the instabilities and their suppression by the magnetic field we conducted theoretical and computational analysis of several idealized flow systems: inviscid flow in a tri-axial ellipsoid, isolated elliptical vortex, and isolated vortex sheet. The results indicate that even in the conditions of very strong magnetic field, there exist coherent structures, which are unstable and likely to lead to localized three-dimensional bursts. In this sense, a purely two-dimensional turbulence must be considered as an asymptotic solution, unlikely to be achieved in real MHD flows.

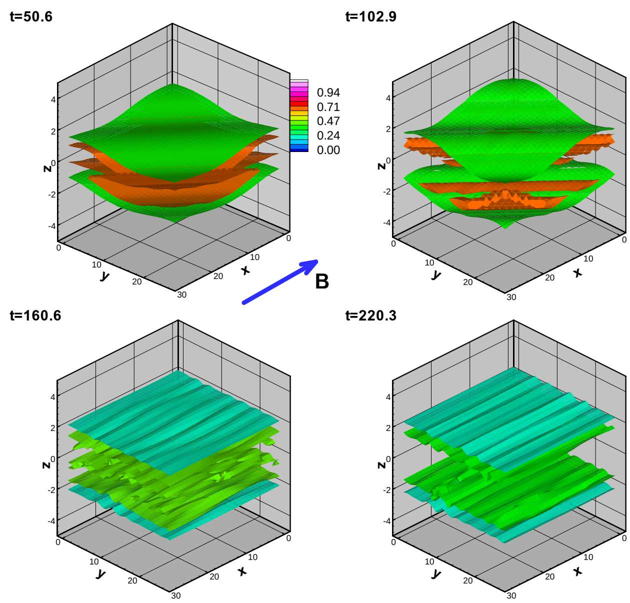

Figure 5. A vortex sheet loosing its stability to oblique rolls. Isolines of streamwise perturbation velocity are shown for growing rolls, their 3D secondary instability, and suppression of turbulence by the magnetic field.

Further investigations were conducted to analyze the loss of stability by plane vortex sheets. It was found that at sufficiently strong magnetic field, the two-dimensional Kelvin-Helmholz rolls are replaced as the fastest growing perturbations by oblique rolls. Further growth of the magnetic field does not change the shape of the rolls with exception of the increasing oblique angle. The rolls become increasingly aligned with the flow.   Stages of the transition to turbulence caused by growth and breakdown of the oblique rolls are illustrated in figure 5. DNS of the transition showed that the initial perturbations in the form of superposition of two symmetric oblique waves are particularly effective in inducing three-dimensionality and turbulence in the flow.

6. Intermittency in the presence of magnetic field (Ref. [10]).

One interesting result of our investigation of homogeneous turbulence and coherent structures (tasks 1 and 5 above) was detection of the novel flow regime characterized by long periods of nearly steady, two-dimensional behavior interrupted by violent three-dimensional bursts. Without reference to specific flow geometry, this large-scale intermittency evolves according to the following scenario. Under the action of the magnetic field, an initially 3D flow evolves into a pattern of nearly 2D structures. The flow gradients along the magnetic field are very weak in this state, so the Joule dissipation decreases to nearly zero. If, however, the 2D state is not a stable attractor of the Navier-Stokes equations and the magnetic field is not strong enough to completely suppress 3D instabilities, perturbations grow and destroy the 2D structures. The flow enters a 3D turbulent state and the process repeats itself.

The intermittency scenario has far-reaching implications for specific flows and for magnetohydrodynamics in general. The flows acquire properties unforeseeable under statistical equilibrium assumptions for MHD turbulence, whereby the flow is either nearly isotropic, statistically steady anisotropic, or 2D depending on the strength of the magnetic field. In particular, the scalar transport, and, as demonstrated in figure 5, friction drag are heavily affected.

We conducted a detailed investigation of the phenomenon using the configuration of a channel with spanwise magnetic field. Direct and Large-eddy simulations showed that the intermittency is a persistent feature of the flow in a wide range of the Reynolds and Hartmann numbers.

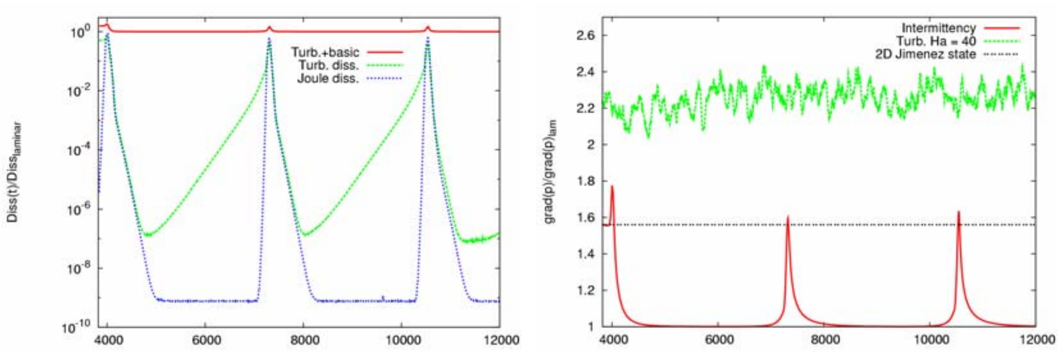

Figure 6. Large-scale intermittency in a channel flow with spanwise magnetic field. Re=8000, Ha=80. On the left, rates of viscous and magnetic (Joule) dissipation are shown as functions of time. Long periods of nearly zero Joule dissipation correspond to nearly two-dimensional flow, while short periods of strong Joule dissipation show three-dimensional bursts. On the right, pressure gradient needed to drive the flow of the same flow rate, i.e. the viscous friction coefficient, is shown for the intermittent regime and, for comparison, for 3D turbulent flow at Ha=40 and for a purely 2D turbulence analyzed by J. Jimenez (1990).

7. Transition to turbulence in MHD channel flow (Refs. [3], [8]).
Another feature of many practical MHD flows is that the flows are strongly suppressed by the magnetic field. As a result, they often appear in a weakly turbulent or transitional state. The question of instability and transition to turbulence is, therefore, even more important in the MHD case than in the non-magnetic hydrodynamics.

We investigated the transition to turbulence in a channel flow with spanwise magnetic field. As the other parallel flows in ducts and channels, this flow experiences transition to turbulence at much lower Reynolds number than predicted by the linear stability theory. Our study has confirmed validity of a non-linear transition mechanism based on transient amplification and subsequent breakdown of certain optimal perturbations. Albeit not new in principal and observed earlier for other flows, this mechanism takes an unusual form in our case since the magnetic field preferentially suppresses the streamwise-independent perturbations, the usual culprits of the transition. As a result, oblique modes such as those shown in figure 7, become optimal and the transition occurs via nonlinear interaction between these modes and their reflections with respect to the channel axis.

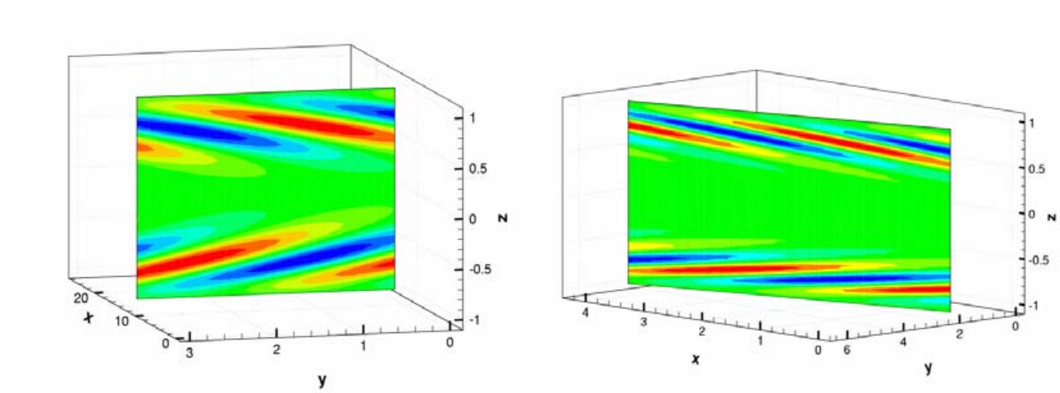

Figure 7. Optimal perturbations in the channel flow with spanwise magnetic field. Isolines of streamwise velocity are shown in the planes perpendicular to the perturbation's wavenumber vector for Re=5000 and Ha=10 (left) and Ha=50 (right).

8. Theoretical study of Lorentz force flowmeter (Ref. [2]).
A Lorentz force flowmeter is a device for the contactless measurement of flow rates in electrically conducting fluids. It is based on the measurement of a force on a magnet system that acts upon the flow. Although simple in principle, the flowmeter is still far from being ready for practical applications because of several serious unresolved questions. The most important of them is the effect of flow internal properties and their transformation under the impact of the magnetic field on the measured sugnal.

We formulated the theory of the Lorentz force flowmeter, which connected the measured force to the unknown flow rate. The theory was applied to three specific cases, namely (i) pipe flow exposed to a longitudinal magnetic field, (ii) pipe flow under the influence of a transverse magnetic field and (iii) interaction of a localised distribution of magnetic material with a uniformly moving sheet of metal. These examples provided the key scaling laws of the method and illustrated how the force depends on the shape of the velocity profile and the presence of turbulent fluctuations in the flow. Moreover, we formulated the general kinematic theory valid for arbitrary distributions of magnetic material or electric currents and for any velocity distribution. The theory provided a rational framework for the prediction of the sensitivity of Lorentz force flowmeters in laboratory experiments and in industrial practice

II. Publications and presentations:

1. Publications in referred journals

Published or in press

1. Vorobev, A., Zikanov, O., Davidson, P., Knaepen, B. "Anisotropy of MHD turbulence of low magnetic Reynolds number," *Phys. Fluids,* **17**, N12, 125105, 2005

2. Thess, A., Votyakov, E., Knaepen, B., and Zikanov, O. "Theory of Lorentz force flowmeter," *New J. Phys.* **9** 299, 2007.

3. Thess, A., Krasnov, D., Boeck, T., Zienicke, E., Zikanov, O., Moresco, P., Alboussiere, T. "Transition to turbulence in the Hartmann boundary layer," *GAMM Mitt.* **30**, N1, 125-132, 2007.

4. Thess, A., Zikanov, O. "Transition from Two-Dimensional to Three-Dimensional MHD Turbulence," *J. Fluid Mech*. **579,** 383-412, 2007.

5. Vorobev, A., Zikanov, O. "Instability and transition to turbulence in a free shear layer affected by a parallel magnetic field," *J. Fluid Mech*. **574,** 131-154, 2007.

6. Ai, X., Li, B. Q., Zikanov, O. "Stability of electromagnetically-driven flows in induction channels," *Magnetohydrodynamics* **43,** No. 1, 3-32, 2007.

7. Vorobev, A., Zikanov, O. "Smagorinsky constant in LES modeling of anisotropic MHD turbulence," *Theor. Comp. Fluid Dyn.* **22** N2, 2008.

8. Krasnov, D., Rossi, M., Zikanov, O. and Boeck, T. "Optimal growth and transition to turbulence in channel flow with spanwise magnetic field," *J. Fluid Mech.*, **596,** 73-101, 2008,

Submitted

9. Krasnov, D., Zikanov, O. Schumacher, J., Boeck, T. "MHD turbulence in a channel with spanwise magnetic field," *Phys. Fluids*, 2008.

10. Boeck, T., Krasnov, D., Thess, A., Zikanov, O. "Large-scale intermittency of liquid-metal channel flow in a magnetic field," *Phys. Rev. Lett.*, 2008.

In preparation

11. Burattini, P., Zikanov, O., Knaepen, B. "Decay of MHD turbulence at low magnetic Reynolds number."

2. Selected publications in conference proceedings

1. Vorobev, A., Zikanov, O., “On stability of a free shear layer affected by parallel magnetic field,” *Proc. of 2005 MHD summer program.* ULB, Brussels, 79-90, 2005.

2. Zikanov, O., Vorobev, A., “LES modeling of MHD turbulence,” *Fundamental and Applied MHD. Proc. pf 15th Riga and 6th Pamir Conf.*, Riga, Latvia, ed. Alemany et al., 167-170, 2005.

3. Vorobev, A., Zikanov, O., “Anisotropic MHD turbulence at low magnetic Reynolds number,” *Turbulence and Shear Flow Phenomena, Proc. of TSFP-4,* Williamsburg, VA, USA, ed. Humphrey et al, v. 2, 519-524, 2005.

4. Thess, A., Votyakov, E., Zikanov, O., Knaepen, B. “Lorentz force flowmeter for a circular pipe,” *Proc. of 2006 CTR summer program.* 431-441, 2006.

5. Krasnov, D., Rossi, M., Boeck, T., Zikanov, O. “Transition to turbulence in channel flow with spanwise magnetic field,” *Proc. of 2006 CTR summer program.* 363-374, 2006.

6. Zikanov, O. and Vorobev, A. “LES Modeling of Anisotropic MHD Turbulence *Turbulence and Shear Flow Phenomena, Proc. of TSFP-5,* Garshing, Germany, ed. R. Friedrich et al, v.3, 1087-1092, 2007

7. Boeck, T. Krasnov, D. , Rossi, M. and Zikanov, O. “Transition in MHD Channel Flow with Spanwise Magnetic Field” *Turbulence and Shear Flow Phenomena, Proc. of TSFP-5,* Garshing, Germany, ed. R. Friedrich et al, v.3, 1081-1086, 2007

8. Vorobev, A., Zikanov, O., “Instability and transition to turbulence in a free shear layer affected by a parallel magnetic field,” *Advances in Turbulence XI, Proc. 11 Euromech Turb. Conf.*, Porto, Portugal, ed. J. M. L. M. Palma and A. Silva Lopes, 124-126, 2007.

9. Boeck, T., Krasnov, D., Rossi, M., Zikanov, O. “Transition to turbulence in plane channel flow with spanwise magnetic field,” *Advances in Turbulence XI, Proc. 11 Euromech Turb. Conf.*, Porto, Portugal, ed. J. M. L. M. Palma and A. Silva Lopes, 73-75, 2007.

III. Personnel involved in the project:

- Post-doctoral research associate, Dr. Anatoly Vorobev, 03/2004 – 09/2007.

- Graduate Student Research Assistant, Partha Sarathy, 05/2004 – 12/2006. Mr. Sarathy was a student at the Wayne State University involved in the project under a subcontract arrangements and co-supervised by Victor Berdichevski and Oleg Zikanov

- Dr. Victor Berdichevsky, professor, Wayne State University (at no charge to the federal grant)

- Dr. Oleg Zikanov, associate professor, PI, University of Michigan – Dearborn

IV. Changes in the project organization:

- Development of the project led to a conclusion that an extended student involvement was necessary. It was, therefore, decided to subcontract the corresponding portion of the project to the Wayne State University. Further advantages of the new arrangement included participation (at no cost for the federal grant) of the Professor Victor Berdichevsky, a known expert in statistical theory of turbulence. Prof. Berdichevsky helped to supervise the student and work on the research tasks of the project.

- Using additional support from the NSF, the PI was able to develop an active collaboration program with the MHD research center at the Ilmenau University of Technology in Germany. This helped to significantly broaden and extend the scope of the project.

- Another international collaboration was established with the MHD group at the Free University of Brusseles, Belgium.