

FINAL TECHNICAL REPORT

Title: Nuclear Lattice Calculation

DOE Award #: DE-FG02-87ER40347

Period: April 1, 2006 – March 31, 2008

and no-cost one-year extension, April 1, 2008 – March 31, 2009

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A. ABSTRACT

This is the final progress report of DOE research grant DE-FG02-87ER40347, Nuclear Lattice Calculation for the period from April 1, 2006 to March 31, 2009 (including the no-cost one-year extension of April 1, 2008 – March 31, 2009.)

B. PROPOSED PROJECT

The **proposed objectives** were:

1. Reliable determination of the 1S_0 gap energy of low-density neutron matter as a function of the neutron density.
2. Determination of phase diagram of the neutron matter at those densities.
3. Clarification of crossover properties of the neutron matter of these densities, which are caused by Bose-Einstein condensate.

The NSAC Long Range Plan for nuclear science identified the research area central to the mission of nuclear physics, as five agenda. This project has addressed the second agenda: *What is the structure of nucleonic matter?*

The basic approach used was:

Determine the nucleon-nucleon potential on a simple cubic lattice by applying the method of effective field theory, and using the potential on the same lattice, carry out quantum Monte Carlo calculations of thermal properties of neutron matter. The project was an application of the effective field theory on a many nucleon system.

The personnel of the project was:

1) Ryoichi Seki (P.I.)

2) Takashi Abe (Graduate student until March, 2007; Post-docs at Tokyo Institute of Technology, April-October, 2007 and at University of Tokyo, since November, 2008.)

Note that T. Abe was not directly supported by this grant, but worked as a collaborator of the project.

C. SUMMARY OF WORK ACCOMPLISHED

The proposed objectives have been achieved, and two manuscripts were written in collaboration with T. Abe. The manuscripts include most of our results and represent the achievements of the project. They are:

- 1) "Lattice calculation of thermal properties of low-density neutron matter with pionless NN effective field theory," in Physical Review C, vol.79, 054002 (2009).
- 2) "From low-density neutron matter to the unitary limit," in Physical Review C, vol.79, 054003 (2009).

In the following, we summarize the contents of the manuscripts, so as to describe the work and accomplishment that have been made:

Lattice Calculation of Thermal Properties of Low-Density Neutron Matter with Pionless NN Effective Field Theory

Thermal properties of low-density neutron matter are investigated by determinantal quantum Monte Carlo lattice calculations on 3+1 dimensional cubic lattices. Nuclear effective field theory (EFT) is applied using the pionless single- and two-parameter neutron-neutron interactions, determined from the 1S_0 scattering length and effective range. The determination of the interactions and the calculations of neutron matter are carried out consistently by applying EFT power counting rules. The thermodynamic limit is taken by the method of finite-size scaling, and the continuum limit is examined in the vanishing lattice filling limit. The 1S_0 pairing gap at $T \approx 0$ is computed directly from the off-diagonal long-range order of the spin pair-pair correlation function, and is found to be approximately 30% smaller than BCS calculations with the conventional nucleon-nucleon potentials. The critical temperature T_c of the normal-to-superfluid phase transition and the pairing temperature scale T^* are determined, and the temperature-density phase diagram is constructed. The physics of low-density neutron matter is clearly identified as being a BCS-Bose-Einstein condensation crossover.

From Low-Density Neutron Matter to the Unitary Limit

Various quantities of an attractively interacting fermion system at the unitary limit are determined by extrapolating Monte Carlo results of low-density neutron matter. Smooth extrapolation in terms of $1/(k_F a_0)$ (k_F is the Fermi momentum, and a_0 S-wave scattering length) is found with the quantities examined: the ground-state energy, the pairing energy gap, and the critical temperature of the normal-to-superfluid phase transition. We emphasize proximity of the physics of low-density neutron matter to that at the unitary limit. The extrapolated quantities are in a reasonable agreement with those in the literature.

D. BUDGET

All allocated budget has been spent for carrying out the project. No dollar amount is left.

II. PROJECT DESCRIPTION

A. THE LONG-TERM GOAL OF THE PROJECT

Our long-term goal is to understand thermal properties of nuclear and neutron matter at relatively low density and temperature where the constituents are treated as hadrons. This region of the density and temperature is below the quark-gluon region currently investigated at RHIC. Our understanding of thermal properties of matter in the hadronic region, however, is still rather poor. Study of hadronic many-body systems has a long history, but its much success is mostly at or near zero temperature. We wish to obtain reliable **thermal properties** of nuclear and neutron matter by applying the method of **effective field theory** combined with the method of determinant **quantum Monte Carlo simulation**.

Study of excitations of finite nucleonic matter, based on our findings, would be a desirable extension of our project. But this requires much of novel techniques beyond what is described in this proposal, and is considered as a possible future direction of the long term goals of the project.

B. THE TIMELINE OF THE PROJECT

For clarity, we list our **timeline** of the proposed work for the three year period, together with **their brief explanation**.

Phase 1:

The current three years:

Pionless Hamiltonian fit to the scattering length for the neutron density up to $\rho = 0.006 \rho_0$ or $k_F = 0.3 \text{ fm}^{-1} = 60 \text{ MeV}/c$. Preparatory work of mean-field calculations and of the effective-range expansion on lattice.

Work on algorithm: Low temperature instability

Phase 2:

Year 1

Pionless Hamiltonian fit to the scattering length and the effective range

For the neutron density up to $\rho = 0.07 \rho_0$ or $k_F = 0.7 \text{ fm}^{-1} = 140 \text{ MeV}/c$.

Work on algorithm: Hybrid Monte Carlo

Year 2

Close examination of the BCS-BEC crossover phenomena in neutron matter.

Phase 3:

Years 2-3

Pion-full Hamiltonian fit to the phase shift over all momentum range up to

260 MeV/c (or possibly to 300 MeV/c) for the neutron density up to

$\rho = 0.46 \rho_0$ or $k_F = 1.3 \text{ fm}^{-1} = 260 \text{ MeV}/c$.

2. A brief explanation of the timeline:

We plan to carry out the project step-by-step, from the simplest and least time-consuming computation to more complicated. The degree of complication depends on that of the neutron-neutron potential used in the computation. The potential is constructed by applying effective field theory, following its power-counting rules. We increase the number of terms in the potential as we widen the range of application of our potential in the center-of-mass momentum p of the interacting neutrons.

Widening the range of p corresponds to widening the range of the neutron matter density that we calculate, as the density ρ is roughly represented its Fermi momentum k_F through the noninteracting Fermi gas formula,

$$= \rho [2/(2\pi)^3][4\pi/3] k_F^3 = k_F^3 / 3 \pi^2,$$

and the relevant p is roughly set to be $p \sim k_F$.

The coupling constants in the potential are determined to fit the observed neutron-neutron phase shifts with the suitably chosen regularization scale Λ by applying the method of effective field theory. In our lattice calculation, the inverse of the lattice spacing “ a ” play the role of Λ as

$$\Lambda \sim 1/a, \pi$$

and the same values of “ a ” are used for the Monte Carlo calculation of neutron matter of the density with

$$k_F \sim p < \pi / a$$

As will be discussed below, the $T=1$, 1S_0 nucleon-nucleon phase shift is described overall reasonably well in terms of the scattering length and the effective range without explicitly introducing the pion degree of freedom, up to near $p \sim m_\pi$, about 140 MeV/c. In this momentum range, we set the potential to have two terms by fitting to the scattering length and the effective range. For $p \geq 140$ MeV/c up to 260 MeV and beyond, say, about 300 MeV near the momentum for which the 1S_0 phase shift changes its sign, the pion interactions must be included explicitly, and we plan to include the pion field as an explicit degree of freedom in effective field theory.

Currently (PHASE I) we are carrying out a series of numerical calculations for $p \leq 60$ MeV/c, since a reasonably accurate description can be achieved in this momentum range, using the potential of a single term fit to the scattering length. This calculation will be completed in the current three-year period. The first numerical result is shown as a part of II.C.3.c First Numerical Result in PHASE I in p12.

Summarizing, the momentum range in our calculation is roughly divided to the following three overlapping regions, including the current calculation, as

Momentum range Potential

$p = 0 - 60 \text{ MeV/c}$ one term, fit to the scattering length

$0 - 140 \text{ MeV/c}$ two terms, fit to the scattering length and effective range

$0 - 260 \text{ MeV/c}$ (possibly up to 300 MeV/c)

two terms and pion, fit to the whole range of the phase shift.

The calculations in the three regions will be compared in the overlapping momentum ranges so as to ensure reliability of the results.