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Momentum Compaction and Phase Slip Factor

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Abstract

Section 2.3.11 of the Handbook of Accelerator Physics and Engineering on Landau damping is updated. The slip factor and its higher orders are given in terms of the various orders of the momentum compaction. With the aid of a simplified FODO lattice, formulas are given for the alteration of the lower orders of the momentum compaction by various higher multipole magnets. The transition to isochronicity is next demonstrated. Formulas are given for the extraction of the first three orders of the slip factor from the measurement of the synchrotron tune while changing the rf frequency. Finally bunch-length compression experiments in semi-isochronous rings are reported .

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2.3.11 Momentum Compaction and Phase Slip Factor

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The phase slip factor η is the relative slip in revolution period T for a particle with fractional off-momentum $\delta=\Delta p/p_0$, i.e. $\Delta T/T_0=\eta\delta$, where the subscript zero stands for on-momentum. The various orders of momentum-compaction factor α_i give the relative increase in closed-orbit length C for an off-momentum particle, or $\Delta C/C_0=\sum_{i=0}^{\infty}\alpha_i\delta^{i+1}$. With $\eta=\sum_{i=0}^{\infty}\eta_i\delta^i$, we have [1]

$$\eta_i=\alpha_i-\frac{\eta_{i-1}}{\gamma_0^2}+\frac{3\beta_0^2\eta_{i-2}}{2\gamma_0^2}+\frac{(1-5\beta_0^2)\beta_0^2\eta_{i-3}}{2\gamma_0^2}-$$
$$-\frac{5(3-7\beta_0^2)\beta_0^4\eta_{i-4}}{8\gamma_0^2}+\cdots,\;\; \eta_i=\begin{cases}1, & i=-1,\\ 0, & i<-1,\end{cases} \quad (1)$$

where β_0 and γ_0 are the on-momentum Lorentz factors. The transition gamma is defined as $\gamma_t=\sqrt{1/\alpha_0}$. To lowest order, all off-momentum particles have the same transition gamma when $\alpha_1/\alpha_0\approx-\frac{1}{2}$, and cross transition at the same time when $\alpha_1/\alpha_0\approx-\frac{3}{2}$.

For a FODO lattice with *thin* quadruples of length ℓ and strength $B'\ell/(B\rho)=\pm S/L$, where L is the half cell length with dipole bending angle θ, we have [2, 3] (see also Eq.(8), Sec.2.2.3)

$$\alpha_0\approx1-\frac{S(\hat{D}_0-\check{D}_0)}{L\theta},\quad \alpha_1\approx-\frac{S(\hat{D}_1-\check{D}_1)}{L\theta},$$
$$\alpha_2\approx-\frac{S(\hat{D}_2-\check{D}_2)}{L\theta}-\frac{S^3(\hat{D}_0^3-\check{D}_0^3)}{6L^3\theta}, \quad (2)$$

where the dispersions at the F- and D-quadrupoles have been power expanded, respectively, as $\hat{D}=\sum_{i=0}^{\infty}\hat{D}_i\delta^i$ and $\check{D}=\sum_{i=0}^{\infty}\check{D}_i\delta^i$. When $S\ll12$, which is usually true because $S=2\sin\frac{\mu}{2}$ and μ is the phase advance per cell, $\alpha_1/\alpha_0\to+\frac{3}{2}$ and reduces to $+\frac{1}{2}$ after chromaticities are corrected by sextupoles.

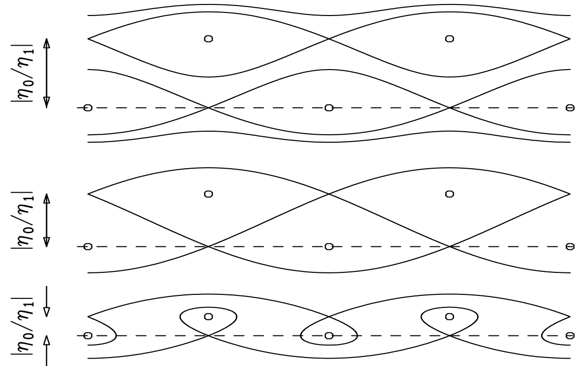
For an isochronous or quasi-isochronous ring, we must require the spread in η for off-momentum particles to be small also. Therefore, α_1 and α_2 need to be controlled in addition to α_0. In fact, first-order effect of sextupoles alters α_1, that of octupoles alters α_2, etc. For example, let $S_n=B^{(n)}\ell/(B\rho)$ be the strength of a *thin* quadrupole ($n=1$), *thin* sextupole ($n=2$), or *thin* octupole ($n=3$) of length ℓ at a location where the horizontal and vertical dispersions are, respectively D_x and D_y. Their first-order effects are [4] $\Delta\alpha_0=-S_1(D_x^2-D_y^2)/C_0$, $\Delta\alpha_1=-S_2(D_x^3-3D_xD_y^2)/C_0$, $\Delta\alpha_2=-S_3(D_x^4-6D_x^2D_y^2+D_y^4)/C_0$.

The Hamiltonian describing the longitudinal rf phase difference $\Delta\phi_{\rm rf}$ is (Sec.2.3.1) [5]

$$H=h\left(\tfrac{1}{2}\eta_0\delta^2+\tfrac{1}{3}\eta_1\delta^3+\tfrac{1}{4}\eta_2\delta^4+\cdots\right)+$$
$$+\frac{eV_{\rm rf}}{2\pi\beta_0^2E_0}\left[\cos(\phi_s+\Delta\phi_{\rm rf})+\Delta\phi\sin\phi_s\right], \quad (3)$$

where $V_{\rm rf}$ is the rf voltage with synchronous phase ϕ_s and harmonic h, while E_0 is the on-momentum energy. If only the η_0 and η_1 terms are considered, the two series of distorted pendulum-like buckets in the top figure ($\Delta\phi_{\rm rf}$ vs δ with $\phi_s=0$ or π, see Ref. [2] for nonzero or non-π ϕ_s) begin to merge to the middle figure when $|\eta_0/\eta_1|$ is lowered to

$$\left|\frac{\eta_0}{\eta_1}\right|=\sqrt{\left|\frac{6eV_{\rm rf}}{\pi\beta_0^2h\eta_0E_0}\left[\left(\frac{\pi}{2}-\phi_s\right)\sin\phi_s-\cos\phi_s\right]\right|}. \quad (4)$$



With further reduction of $|\eta_0/\eta_1|$, the buckets become α-like (lower figure), which shrink to zero when $|\eta_0/\eta_1|=0$. The total bucket height $|3\eta_0/(2\eta_1)|$ is small. It is asymmetric in momentum spread and is susceptible to longitudinal head-tail instability. If the η_1 term is eliminated, the Hamiltonian will be dominated by η_0 and η_2 and the bucket becomes pendulum-like again [3]. If the Hamiltonian is dominated by the η_2 term alone, the kinetic term is similar to a quartic potential providing maximal amount of synchrotron-frequency spread and therefore Landau damping.

The first three orders of the slip factor can be extracted by measuring the synchrotron tune ν_s while changing the rf frequency $f_{\rm rf}$: [2]

$$\nu_s^2\approx\frac{heV_{\rm rf}|\eta_0\cos\phi_s|}{2\pi\beta_0^2E_0}\left(1+\frac{s_1}{\eta_0}\left[\frac{\Delta f_{\rm rf}}{f_{\rm rf}}\right]+\frac{s_2}{\eta_0^2}\left[\frac{\Delta f_{\rm rf}}{f_{\rm rf}}\right]^2\right),$$
$$s_1=-\frac{2\eta_1-\eta_0^2}{\eta_0}+\frac{1}{\gamma_0^2},$$
$$s_2=\frac{3\eta_2\eta_0-2\eta_1^2}{\eta_0^2}-\frac{\eta_1}{\eta_0\gamma_0^2}+\frac{3\gamma_0^2\beta_0^2+2}{2\gamma_0^4}. \quad (5)$$

Notice that $\Delta f_{\rm rf}/f_{\rm rf}$ is typically $\mathcal{O}(\eta_0)$.

For the application of THz near-field imaging, THz spectroscopy, and others, bunch length compressed to the order of $\sigma_\tau \sim 1$ ps is desired. An obvious advantage is to store the bunch in the α-like buckets, where the bucket half width, $\Delta\phi_{\rm rf} \approx |\eta_0/\eta_1|\sqrt{2\pi\beta_0^2 E_0 h|\eta_0|/(3eV_{\rm rf}|\cos\phi_s|)}$, is intrinsically narrow. Low-alpha operation modes have been implemented in many light sources. Essentially, α_0 is reduced by making the dispersion outside the achromats of the Chasman-Green lattice negative by scaling the quadrupole strengths. At BESSY II, a reduction from $\alpha_0 = 7.3\times10^{-4}$ 100-fold or even more is possible. [6] At SPEAR III, α_0 has been reduced from 1.18×10^{-3} 240-fold. However, smaller α_0 implies shorter bucket height and therefore shorter beam lifetime. [7] To increase bucket height, sextupoles are used to minimize $|\alpha_1|$. For a more reliable operation of the machine, the low-alpha mode of BESSY II is compromised to $\alpha_0 = 3.5\times10^{-5}$ with zero-current rms bunch length reduced 5-fold to $\sigma_\tau = 3.5$ ps. For such an operation, no injection tuning of the optics is required and beam accumulates at a good rate up to a 200-bunch current of 5 mA with a 40-h lifetime. At SPEAR III, the $\alpha/21$-operation mode incorporates a 21-fold α_0-reduction at 100 mA in 280 bunches with a 30-h lifetime, and a measured bunch length $\sigma_\tau = 6.9$ ps. The shortest bunch length achieved has been 2.5 ps at the single bunch current $3.5\ \mu$A, when α_0 is reduced 240-fold. The beam lifetime is mostly limited by Touschek effect because of the short bucket height. When the bunch length is narrow enough, beam instability often occurs due to coherent synchrotron radiation.

References

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