

Final Report on DE-FG02-04ER46107: Glasses, Noise and Phase Transitions April 1, 2004 - December 31, 2011

Clare C. Yu
Department of Physics and Astronomy
University of California, Irvine
Irvine, CA 92697-4575

Abstract

We showed that noise has distinct signatures at phase transitions in spin systems. We also studied charge noise, critical current noise, and flux noise in superconducting qubits and Josephson junctions.

1 Introduction to Noise

Much of the work described in the proposal involves noise. So we begin with a brief introduction to noise. To an experimentalist, noise is a nuisance at best and a serious problem hindering measurements at worst. However noise comes from the fluctuations of microscopic entities and it can act as a probe of what is happening physically at the microscopic scale. Let us set up our notation and define what we mean by noise. Let $\delta x(t)$ be a fluctuation in some quantity x at time t . If the processes producing the fluctuations are stationary in time, i.e., translationally invariant in time, then the autocorrelation function of the fluctuations $\langle \delta x(t_2) \delta x(t_1) \rangle$ will be a function $\psi(t_2 - t_1)$ of the time difference. In this case the Wiener-Khintchine theorem can be used to relate the noise spectral density $S(\omega)$ to the Fourier transform $\psi(\omega)$ of the autocorrelation function [1]: $S(\omega) = 2\psi(\omega)$ where ω is the angular frequency.

The noise spectrum often goes as $1/f^\alpha$ where f is frequency and α is some power. If $\alpha = 0$, then the noise is white and independent of frequency. Thermal noise or Johnson noise is an example of white noise. $\alpha = 1$ is $1/f$ noise which is ubiquitous and dominates at low frequencies. It results from summing over a set of Lorentzian spectra [2, 3]:

$$S(\omega) \sim \int \frac{\tau}{1 + \omega^2 \tau^2} P(\tau) d\tau \quad (1)$$

Each Lorentzian spectrum comes from a fluctuation that decays exponentially with a characteristic relaxation time τ . $P(\tau)$ is the distribution of τ . If $P(\tau) \sim 1/\tau$ for $\tau_1 \leq \tau \leq \tau_2$, then one obtains $1/f$ noise, i.e., $S(\omega) \sim 1/\omega$ for $\tau_2^{-1} \ll \omega \ll \tau_1^{-1}$. This form of $P(\tau)$ arises, for example,

if the relaxation time is activated ($\tau = \tau_0 \exp(E/kT)$) and the distribution of activation energies is constant.

$\alpha = 2$ can be associated with a fluctuating two-state system [3, 4], or a random walk. α closer to 2 is characteristic of systems far from equilibrium [5].

Second Spectrum of the Noise

In addition to the first spectrum, there is something called the second spectrum of the noise. To understand the second spectrum, consider the following. Suppose we take a long time series, divide it into segments, and calculate the first spectrum of each segment, i.e., the noise spectra $S_i(f, t_2)$ where t_2 is the time in the middle of the i th segment. So now we have a set of noise spectra $S_i(f, t_2)$ taken at different times t_2 . The second spectrum is the power spectrum of the fluctuations of $S_i(f, t_2)$ with time t_2 , i.e., the Fourier transform of the autocorrelation function of the time series of $S_i(f, t_2)$ [6, 7, 8]. To calculate the second spectrum, we can divide each first spectrum into octaves. An octave is a range of frequencies from f_L to f_H where typically $f_H = 2f_L$. We can discretize the first spectrum by associating each octave with the total noise power in that octave. We do this for each data set. For each octave this gives us a set of numbers with one number from each data set labeled by t_2 . Now we can calculate the fluctuations in the noise power in a given octave labeled by frequency f_1 . Then we can calculate the autocorrelation function of these fluctuations, Fourier transform it and obtain the noise power $S_2(f_2, f_1)$ which is the second spectrum. Rather than doing a Fourier transform for the first spectrum $S(f_1)$, one can do a simple wavelet transform which is known as a Haar transform [9]; this is more computationally efficient. In addition the Haar transform allows us to access frequencies f_2 that are only slightly less than f_1 , while using only Fourier transforms restricts us to frequencies f_2 corresponding to the inverse of the duration of the data set used to produce each first spectrum.

2 Work Done Under Grant

2.1 Noise spectra in the vicinity of ordered first and second order phase transitions show signatures of the transition.

Our goal was to see if noise can be used as a probe of microscopic fluctuations. There have been indications that the noise in the resistivity increases in the vicinity of the metal-insulator transition [5]. But what are the characteristics of the noise associated with well-understood first and second order phase transitions? It is well known that critical fluctuations are associated with second order phase transitions, but do these fluctuations lead to enhanced noise? On the other hand, phase transitions occur in the thermodynamic limit but the noise decreases with increasing system size. So how do we reconcile these two aspects? We addressed these questions using Monte Carlo simulations to study the noise in the 2D ferromagnetic Ising model which undergoes a second order phase transition, and in the 5-state Potts model which undergoes a first order phase transition. We monitored these systems as the temperature drops below the critical temperature. At each temperature, after equilibration was established, we obtained the time series of the energy and magnetization.

On a log-log plot of noise power versus frequency f , the noise power was white (flat) at low frequencies and then, as the frequency increased above a frequency that we call the knee

frequency, the noise decreased as a power law: $S(f) \sim f^{-\alpha}$. For both the magnetization per spin and the energy per spin, we found that for a finite size system, the total noise power (area under the $S(f)$ versus f curve) and the low frequency white noise below the knee frequency increased as the critical temperature is approached. (We normalized the total noise power to the variance.) In the thermodynamic limit the noise power below the knee frequency diverges but the knee frequency goes to zero and the total noise power vanishes. We showed that the inverse of the knee frequency is approximately the equilibration time [10].

At high frequencies above the knee frequency, the noise power decreases as a power law with increasing frequency. Using the fluctuation-dissipation theorem, we expressed the exponent α of this power law in terms of the critical exponents for the 2D Ising model which undergoes a second order phase transition. Our findings indicate that a maximum in the measurement noise can be used as a signature of a phase transition. So if one sees a maximum in the noise, this indicates that a phase transition has occurred. However, the absence of a maximum in the noise does not mean that there is no phase transition since disorder which can lead to an inhomogeneous transition, e.g., a transition occurring at slightly different temperatures in different parts of the sample. As a result, in these cases there may not be a clear signature of the transition in the noise. In addition some transitions are accompanied by a decrease in the noise, e.g., the fluctuations in the center of mass can decrease if a liquid freezes [11]. This work was published in Physical Review Letters [10].

2.2 Our new algorithm uses pulse parameters to determine the noise spectra of stochastic pulse sequences, e.g., by global spin flips in the Ising model.

Noise due to random pulses is ubiquitous. Examples include switching the rotational direction of the flagellar motor in *Escherichia coli* bacteria [12], two-state switching in electrical resistance [4], and the switching of the value of the magnetization of a system near a phase transition [10]. Yet another example is crackling noise in which slowly driven systems produce sudden discrete outbursts spanning a broad range of sizes [13]. Instances of crackling noise include the sound of paper crumpling, Barkhausen noise from domain movement in ferromagnets [14, 15], flux noise from vortices entering a superconductor as the external magnetic field is increased [16], and the seismic activity during earthquakes [17].

The question is how these pulses are reflected in the features of the power spectra commonly used to characterize noise. The answer could be used to estimate or predict the pulse noise spectrum as well as to separate the pulse contribution to the noise spectrum from other sources. For example, suppose one wants to determine the critical exponents of a second order phase transition from the noise spectra [18, 19, 10]. In a finite size system with a discrete broken symmetry, switching between degenerate ordered phases will also contribute to the noise spectra, and it is important to separate out this contribution before determining the critical exponents.

We developed a general algorithm that uses a distribution of pulse parameters to determine the noise spectrum of a sequence of stochastic pulses. These parameters include the height and duration of a pulse as well as the time between successive pulses. We applied the algorithm to the 2D ferromagnetic Ising model. From our Monte Carlo simulations, we noticed that just below the critical temperature T_C , rather than being flat, the low frequency noise spectra of

the magnetization has a second rise as the frequency decreases below the knee frequency. It finally flattens off at very low frequencies. We showed that this second rise was due to the magnetization flips of almost the entire (finite size) spin system by extracting a simplified signal from the magnetization time series. The simplified signal was a sequence of pulses due to magnetization jumps involving most of the spins of the system. Then we compared the noise spectrum of this simplified time series to the noise spectrum from the simulations and found that they agreed at low frequencies (below the knee frequency where the second rise is seen). We also deduced the distribution of pulse parameters and used our algorithm to predict the noise spectrum. The result was in excellent agreement with the spectrum of the simplified signal. This work has been published in *Physical Review B* [20].

2.3 Microscopic calculation shows fluctuating two level systems produce critical current noise in superconducting Josephson junctions.

The superconducting Josephson junction (JJ) qubit is a leading candidate in the design of a quantum computer, with several experiments recently demonstrating single qubit preparation, manipulation, and measurement [21, 22, 23, 24], as well as the coupling of qubits [25, 26, 27]. A significant advantage of this approach is scalability, as these qubits may be readily fabricated in large numbers using integrated circuit technology. A major obstacle to the realization of quantum computers with superconducting Josephson junction qubits is decoherence of the quantum mechanical wavefunction. The goal of this aspect of our research has been to elucidate the microscopic sources of this decoherence and to suggest ways to eliminate or reduce these culprits. We have been working closely with experimentalists who are leaders in the field, especially John Martinis (UC Santa Barbara) and Robert McDermott (Wisconsin).

Recent experimental evidence [28] indicates that the dominant source of decoherence is two level systems (TLS) in the insulating barrier of the junction as well as in the dielectric material (e.g., SiO₂) that is typically used as an insulator in the fabrication of integrated circuit chips. Two level systems have been used for years to describe the low energy excitations in amorphous materials at low temperatures (below 1 K) [29, 30, 31, 32]. The microscopic nature of two level systems is still unknown. However, one can think of a two level system as an atom or group of atoms that can sit in one of two positions. So think of a double well potential with an atom tunneling between the two positions.

Fluctuating two level systems produce low frequency $1/f$ critical current noise S_{I_c} [33, 34, 35, 36, 37]. However, a simple microscopic model showing this was lacking. In a common scenario for S_{I_c} [36], two level systems in the oxide tunnel barrier affect conduction through nanometer sized channels. But something is troubling about this scenario. Namely, how can tiny defects a few angstroms in width have a noticeable effect on a big superconducting wave function with a micron sized coherence length that is orders of magnitude larger than the perturbing defect? The answer is that the tunneling current is exponentially sensitive to perturbations of the tunnel barrier. We confirmed this with a microscopic calculation of S_{I_c} due to fluctuating TLS in the barrier and obtained good quantitative agreement with experiment. This work has been published in *Physical Review Letters* [38].

Previous theoretical work postulated that the qubit was coupled to fluctuating defects by

putting a coupling term into the Hamiltonian [39, 40, 41, 42, 43, 44], but no one had shown how this coupling arises microscopically. We calculated the $1/f$ critical current noise S_{I_c} due to thermally fluctuating TLS that have electric dipole moments. We assumed that the current I through the Josephson junction is given by $I = I_c \sin \delta$ where I_c is the critical current and δ is the phase difference between the superconductors. We started by calculating how a dipole modifies the junction's potential barrier $U(\mathbf{r})$. We then used a WKB formalism to compute the tunneling matrix element $\mathcal{T}_{LR} \sim \exp(-\sqrt{U})$ between the left (L) and right (R) electrodes. The critical current I_c is proportional to $\langle |\mathcal{T}_{LR}|^2 \rangle$ averaged over the junction [45]. We considered elastic electron tunneling where different orientations of the dipole corresponded to different values of \mathcal{T}_{LR} and hence, I_c . We could obtain S_{I_c} since each fluctuating dipole behaves as a random telegraph variable that has a Lorentzian noise spectrum. By averaging over the standard TLS distribution, and each dipole's orientation and position along the z -axis, we obtained S_{I_c} . At low frequencies we found $1/f$ behavior for S_{I_c} , and our values are in good agreement with the corresponding experimental values [46, 47, 37]. Our model predicts that the noise is very sensitive to the tunnel barrier thickness d and that $S_{I_c} \sim d^5$, implying that the noise can be greatly reduced by decreasing the tunnel barrier thickness.

2.4 Saturation of two level systems does not affect charge noise in Josephson junction qubits.

Fluctuating two level systems (TLS) with electric dipole moments in the substrate and in the tunnel junction produce charge noise in Josephson junction qubits. (The tunnel junctions are so small that most of the TLS are in the substrate.) This occurs because the two level fluctuators that have electric dipole moments can induce image charges in the nearby superconductor and hence produce charge noise $S_Q(\omega)$. [46, 48, 47, 49, 50, 41, 42, 43]

Microwaves used to manipulate the qubit also drive the TLS with electric dipole moments. Let us denote the electric field intensity of the microwaves by I . It was not widely appreciated in the qubit community that TLS can easily be strongly saturated if $I \gg I_c$, where I_c is the critical electric field intensity. Saturated TLS are in steady state with equal populations of their upper and lower levels. Previous theories of charge noise [41, 42, 43] neglected the important issue of the saturation of two level systems by electric fields used to manipulate the qubits. Dielectric (ultrasonic) experiments on insulating glasses at low temperatures found that when the electric (acoustic) field intensity I used to make the measurements exceeds the critical intensity I_c , the dielectric (ultrasonic) power absorption by the TLS is saturated, and the attenuation decreases as the field intensity increases [51, 52, 53, 54, 28]. (If $\mathcal{E} \cos(\Omega t)$ denotes the electric field, then we define the intensity $I = \mathcal{E}^2$.) Previous theories of charge noise in Josephson junctions assumed that the TLS were not saturated, i.e., that $I \ll I_c$. This seems sensible since charge noise experiments [55] are done in the limit where the qubit absorbs only one photon. However, simple estimates show that the stray electric fields associated with this photon could saturate two level systems in the dielectric substrate which supports the qubit.

So we explored the consequences of TLS saturation on charge and polarization noise. We considered both the random fluctuations of the TLS as well as the fact that the dipole moments of these TLS couple to the applied ac electric field of the microwaves that drive the system. To do the calculation, we expressed the noise spectral density in terms of density matrix elements.

To determine the dependence of the density matrix elements on the ratio I/I_c , we found the steady state solution for the density matrix using the Bloch-Redfield differential equations [56]. We then obtained an expression for the spectral density of charge fluctuations as a function of frequency f and the ratio I/I_c . We found $1/f$ charge noise at low frequencies, and white (constant) charge noise at high frequencies. Using a flat density of states, we found that TLS saturation has no effect on the charge noise at either high or low frequencies. This work has been published in *Physical Review B* [57].

2.5 Noise in Spin Glasses

Having studied noise in the vicinity of phase transitions in ordered systems, we turned our attention to disordered systems. In particular we used computer simulations to see if noise could be used as a probe of a spin glass transition. In a spin glass [58], the exchange constants J_{ij} between the i th and j th spins are random, resulting randomly oriented spins even at low temperatures.

Can noise be a probe of spin glass transitions?

In ordered systems we saw that the total noise power and the low frequency noise have a maximum at the critical temperature T_C for the second order phase transition in the 2D Ising model and for the first order phase transition in the 2D 5-state Potts model. We wanted to see if this was also true for a spin glass. To the best of my knowledge, no one had done simulations to look at the noise in spin glasses.

Noise in spin glasses could be related to flux noise in SQUIDS and Josephson junction qubits.

One of the reasons why noise in spin glasses is interesting and important is that it can be related to flux noise seen in superconducting quantum interference devices (SQUIDS) and in superconducting Josephson junction qubits [59]. Flux noise is one of the dominant sources of noise and decoherence in Josephson phase qubits [60] and flux qubits [61, 62]. It has a $1/f$ spectrum, and at $f = 1$ Hz, the magnitude is of order $1 \mu\Phi_0/\text{Hz}^{1/2}$, where $\Phi_0 = h/2e$ is the magnetic flux quantum. Despite its name, flux noise is not caused by fluctuating magnetic vortices. This has been shown experimentally by making SQUIDS with superconducting wires that are too narrow ($\sim 2 \mu\text{m}$) to trap or nucleate a vortex within them [63, 64, 65, 66]. Nevertheless, these narrow line SQUIDS still exhibit flux noise.

Recent work from Robert McDermott's group at Wisconsin [66] has shown that flux noise is produced by fluctuating magnetic defects of unknown origin. These magnetic defects appear in aluminum and niobium SQUIDS which are not supposed to have magnetic impurities. They are thought to be on the surface and their density is estimated to be about $5 \times 10^{17} \text{ m}^{-2}$ which corresponds to a typical spacing of about 1 nm. The Moler group at Stanford has recently seen paramagnetic spins on gold and AlO_x using scanning SQUID microscopy between 25 mK and 0.6 K [67]. The fact that they see magnetic spins on oxide-free gold implies that the oxide is not an essential ingredient in these paramagnetic impurities. The Stanford group found that the susceptibility versus temperature T approximately followed a Curie law $\chi \sim 1/T$. The Wisconsin group uses SQUIDS to measure the flux ϕ as a function of temperature. Their data can be fit to a Curie-Weiss law ($\phi \sim \chi \sim 1/(T - T_C)$) with a small Curie temperature T_C between 0 to 60 mK. The value of T_C is difficult to pinpoint since there is an arbitrary flux offset. However, they estimate an interaction energy scale between the spins of order 10 mK

[66]. At 55 mK in one of the flux versus temperature curves [66], they also observed a cusp that is reminiscent of the cusp seen in the susceptibility of a spin glass at the spin glass transition temperature. Assuming that the flux is due to the magnetization of the magnetic spins, these findings indicate that the spins either are noninteracting, or have a weak ferromagnetic interaction with each other, or are randomly interacting with each other as in a spin glass.

If the spins are interacting, what is the nature of the interaction? Dipole-dipole interactions with a magnetic dipole moment of a Bohr magneton at a distance of 1 nm are too weak to account for $T_C \sim 50$ mK. Direct exchange is also too weak. Faoro and Ioffe [68] have proposed that RKKY interactions are responsible. RKKY interactions between two localized spins a distance r apart are mediated by conduction electrons with spins that oscillate in sign as $\cos(2k_F r)/r^3$ where k_F is the Fermi wavevector. RKKY interactions between randomly placed spins are well known to produce a spin glass.

Ising spin glass noise is consistent with flux and inductance noise in SQUIDs

We did Monte Carlo simulations of 2D and 3D Ising spin glasses that produce magnetization noise S_M consistent with flux noise. At low frequencies S_M is a maximum at the critical temperature T_C in 3D, implying that flux noise should be a maximum at T_C . The second spectra of the magnetization noise and the noise in the susceptibility are consistent with experimentally measured SQUID inductance noise. This work was published in *Physical Review Letters* [69].

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