BNL-97007-2012

Calculation of Kinetics Parameters for the NBSR

A.L. Hanson & D.J. Diamond

March 2012

Nuclear Science and Technology Department

Brookhaven National Laboratory

U.S. Department of Energy
National Nuclear Security Administration

DISCLAIMER

Calculation of Kinetics Parameters for the NBSR

Date: March 6, 2012

Prepared by: A.L. Hanson and D.J. Diamond

Nuclear Science and Technology Department
Brookhaven National Laboratory
Upton, NY 11973

Prepared for
National Nuclear Security Administration and
National Institute of Standards and Technology

ABSTRACT

The delayed neutron fraction and prompt neutron lifetime have been calculated at different times in the fuel cycle for the NBSR when fueled with both high-enriched uranium (HEU) and low-enriched uranium (LEU) fuel. The best-estimate values for both the delayed neutron fraction and the prompt neutron lifetime are the result of calculations using MCNP5-1.60 with the most recent ENDFB-VII evaluations. The best-estimate values for the total delayed neutron fraction from fission products are 0.00665 and 0.00661 for the HEU fueled core at startup and end-of-cycle, respectively. For the LEU fuel the best estimate values are 0.00650 and 0.00648 at startup and end-of-cycle, respectively. The present recommendations for the delayed neutron fractions from fission products are smaller than the value reported previously of 0.00726 for the HEU fuel. The best-estimate values for the contribution from photoneutrons will remain as 0.000316, independent of the fuel or time in the cycle.

The values of the prompt neutron lifetime as calculated with MCNP5-1.60 are compared to values calculated with two other independent methods and the results are in reasonable agreement with each other. The recommended, conservative values of the neutron lifetime for the HEU fuel are 650 μs and 750 μs for the startup and end-of-cycle conditions, respectively. For LEU fuel the recommended, conservative values are 600 μs and 700 μs for the startup and end-of-cycle conditions, respectively. In all three calculations, the prompt neutron lifetime was determined to be longer for the end-of-cycle equilibrium condition when compared to the startup condition. The results of the three analyses were in agreement that the LEU fuel will exhibit a shorter prompt neutron lifetime when compared to the HEU fuel.

ACKNOWLEDGEMENTS

The authors benefitted greatly by the close support of the staff at the NIST Center for Neutron Research, which runs the NBSR. Thank you Sean O'Kelly, Robert Williams, and Michael Rowe for your close collaboration. The authors appreciate the financial support of the National Nuclear Security Administration.The authors would like to thank Kirrah Jones who as a winter intern from M.I.T. performed much of the data analysis for the calculation of the prompt neutron lifetime for the LEU fuel. Thanks also to Lynda Fitz who ably performed all the administrative work for this report.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. INTRODUCTION

The parameters that enter into the point kinetics model used to assess transient/accident behavior of a reactor include the delayed neutron fraction and the prompt neutron lifetime. Of the total number of neutrons emitted during the fission process , a small, but important, contribution to the total number of neutrons are emitted sometime after fission has occurred. This number is referred to as the delayed neutron fraction, β. There are two components to the delayed neutron fraction. The first contribution is from the decay of fission products that emit a neutron and the second is from (γ,n) reactions (photoneutrons) where the gamma rays originate from the decaying fission products and activated materials. The contribution to the overall delayed neutrons from photoneutrons is more important in a heavy water cooled and moderated system, such as the NBSR, than it is in a system cooled and moderated by light water because of the $^{2}H(\gamma,n)^{1}H$ nuclear-reaction. The delayed neutron fraction from a specific fissionable isotope is usually measured, independent of any photoneutron contribution. Taking into account the fissionable species present in a reactor, those data are integrated over the reactor using the appropriate weighting. This is discussed in Section 3.

The prompt neutron lifetime is the time from the birth of a neutron, when its energy is typically in the MeV range, through thermalization to when it is absorbed. Therefore this parameter is important in determining how fast the reactor control system needs to respond to perturbations in order to maintain the reactor at the desired power. The prompt neutron lifetime was calculated for the NBSR using three independent methods. The first method is the 1/v-absorber method, where a 1/v absorber, such as ^{10}B is placed uniformly throughout a nuclear reactor and the change in reactivity is calculated. This prompt neutron lifetime is then extracted from the changes in the reactivity as ^{10}B concentration approaches zero. The second method calculates the decay characteristics of a neutron pulse in a sub-critical nuclear reactor. The decay constant of the neutron pulse is a function of the prompt neutron lifetime. The third method uses the fundamental definition of the neutron lifetime with adjoint weighting that has recently been included in MCNP. The results of the three calculations are compared in Section 4.

2. DETERMINATION OF DELAYED NEUTRON FRACTION β

A. Delayed neutrons from the decay of fission products

Most of the delayed neutrons are the result of the decay of fission products and each fissionable isotope has its own distribution of fission products. Therefore the value of β for the reactor will depend on the mix of fissionable isotopes that contribute to the fission process. However, the distribution of fission products also exhibits a dependence on the neutron spectra that the fissionable materials experience. Hence, the value of β will exhibit some degree of position and time dependence. For the purpose of calculating the response of the NBSR to a perturbation, it is assumed that the time dependence is small enough that values calculated for the startup (SU) and end-of-cycle (EOC) conditions of the core are sufficiently representative. The SU condition is not at equilibrium (^{135}Xe is not yet present) and exhibits the largest power peaking of any time during the cycle. The beginning-of-cycle (BOC) equilibrium core occurs approximately 1.5 days into each cycle though calculations with MCNP5-1.60 have shown that the kinetics parameters at BOC are similar to the values for the SU condition.

The delayed neutrons emitted from the decay of fission products are not all emitted with the same decay constants so it is common to combine the emission of delayed neutrons into six precursor groups (denoted with i), each with its own representative decay constant and delayed neutron fraction, β_{ij} where j represents the fissionable isotope. The total value of β_j for each fissionable isotope is the sum of the contributions from each group:

$$\beta_j = \sum_i \beta_{ij}.$$

The value β for the core is then the sum of the β_j's for each fissionable isotope in the core weighted by the fraction of fissions contributed by each fissioning isotope w_j:

$$\beta = \sum_j w_j \beta_j.$$

The values of β_{ij} from each fissionable isotope include the contributions from the decay of k fission products, so each i group will have a representative decay constant, λ_{ij}. If the fraction of delayed neutrons from each isotope within the i^{th} group is h_{ijk}, the representative decay constant for each group is:

$$\lambda_{ij} = \sum_k h_{ijk} \lambda_{ijk}$$

where λ_{ijk} is the decay constant for isotope k in group i for the fissionable isotope j.

There have been several tabulations of β_{ij} over the past 50+ years. The published tabulation which includes the most fissionable isotopes was a review article by R.J. Tuttle [1]. The common values in this tabulation are in reasonable agreement with other more limited tabulations attributed to Keepin, Wimett, and Ziegler [2] and the DOE Fundamentals Handbook [3]. One must be aware that some tabulations report the total number of delayed neutrons per fission ($\beta\nu$), such as the Tuttle tabulation, and other tabulations show the fraction per emitted neutron (β). Here ν is the average number of neutrons emitted during the fission process.

The values for β and β_i that were reported in the NBSR SAR [4] were for ^{235}U and were taken from an earlier version of the NBSR SAR and were reported as the recommended values in Tuttle [1]. The recommended values in Tuttle were tabulated as delayed neutrons per fission ($\beta\nu$). In order to reproduce the values in the NBSR SAR [4], one needs to divide the recommended values in Tuttle by ν = 2.34. However, Lamarsh [5] and the present evaluations in the NNDC ENDFB-VII libraries [6] show values of ν = 2.43 and 2.44, respectively, for thermal neutron fission of ^{235}U.

Extracting values of β and β_i is complicated by the fact that ν has a dependence on the energy of the incident neutron as is shown in Figure 1 (from the ENDFB-VII libraries [6]) for the fissioning of ^{235}U and ^{239}Pu. A plot of the neutron flux energy distribution (flux per energy bin) averaged over the NBSR fuel is shown in Figure 2. The data in Figure 2 were calculated with MCNP [7] for the NBSR with both the HEU and the LEU fuel at startup (SU). In the fueled region of the NBSR there is only a few percent difference in the neutron flux between the HEU and LEU fuel. The same statement is true for the neutron flux energy distribution for the core at EOC.

The most recent ENDFB-VII [6] libraries contain evaluations of the delayed neutrons grouped into six precursor groupings and it is interesting to compare them with the earlier evaluations by Tuttle [1]. Since the values provided by Tuttle were quoted to be for thermal neutrons, the values in ENDFB-VII were calculated for the energy range of 0.0252 to 0.0254 eV using the NJOY [8] computer program[1]. Table 1 shows the direct comparison of ($\beta_i\,\nu$) and ($\beta\,\nu$) for the six delayed neutron groups as presented by Tuttle and by the ENDFB-VII tabulations for the major fissionable materials (j) that are expected to be in the NBSR when it is fueled with LEU fuel. The total values are similar, with the largest discrepancies in the decay constant for the shortest lived precursor group (Group 6).

[1] Note that the NJOY computer program calculates values within groups that have defined energy bounds.

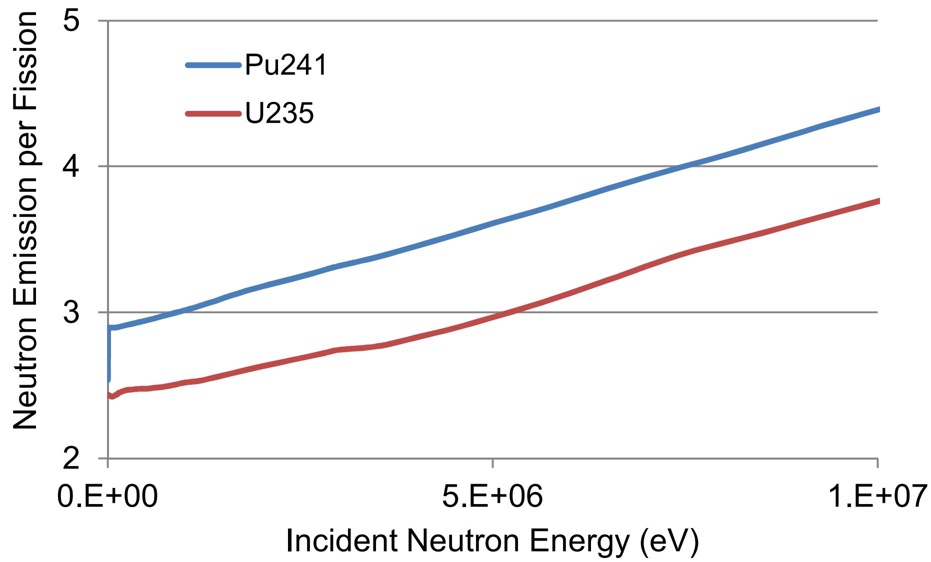

Figure 1. The neutron emission per fission (ν) as a function of incident neutron energy.

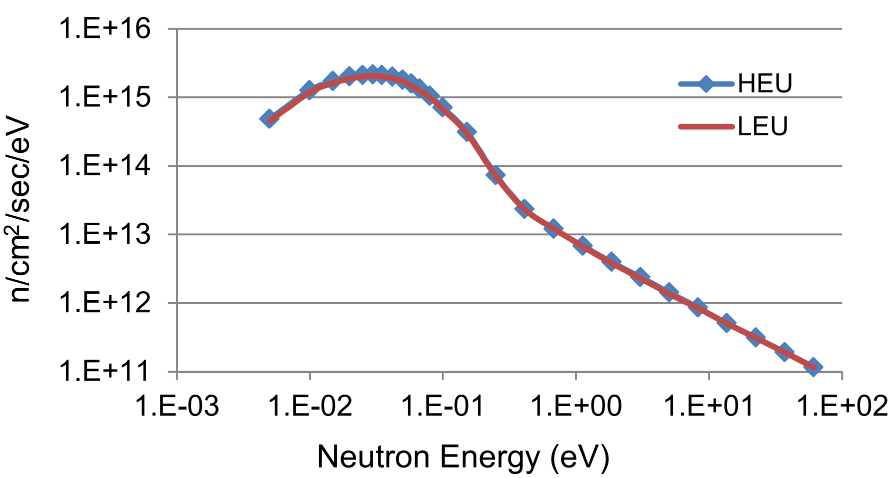

Figure 2. Truncated neutron flux energy distribution averaged over the fueled region.

Table 1. A comparison of delayed neutron parameters from Tuttle and those from the ENDFB-VII libraries.

^{235}U	Tuttle		ENDFB-VII	
Group	λ_i (1/s)	$\beta_i\ \nu$	λ_i (1/s)	$\beta_i\ \nu$
1	1.27E-02	0.000645	1.25E-02	0.000507
2	3.17E-02	0.003615	3.18E-02	0.002637
3	1.15E-01	0.003190	1.09E-01	0.002557
4	3.11E-01	0.006907	3.17E-01	0.007285
5	1.40E+00	0.002172	1.35E+00	0.002116
6	3.87E+00	0.000441	8.64E+00	0.000748
$\beta\ \nu = \Sigma\ \beta_i\ \nu$		0.01697		0.01585
^{238}U	Tuttle		ENDFB-VII	
Group	λ_i (1/s)	$\beta_i\ \nu$	λ_i (1/s)	$\beta_i\ \nu$
1	1.32E-02	0.000586	1.25E-02	0.000455
2	3.21E-02	0.006176	3.03E-02	0.005052
3	1.39E-01	0.007303	1.16E-01	0.005624
4	3.58E-01	0.017491	3.41E-01	0.019881
5	1.41E+00	0.010143	1.32E+00	0.010274
6	4.02E+00	0.003381	9.98E+00	0.002714
$\beta\ \nu = \Sigma\ \beta_i\ \nu$		0.04508		0.04400
^{239}Pu	Tuttle		ENDFB-VII	
Group	λ_i (1/s)	$\beta_i\ \nu$	λ_i (1/s)	$\beta_i\ \nu$
1	1.29E-02	0.000249	1.25E-02	0.000213
2	3.11E-02	0.001834	2.99E-02	0.001729
3	1.34E-01	0.001415	1.07E-01	0.001207
4	3.31E-01	0.002148	3.18E-01	0.002520
5	1.26E+00	0.000675	1.35E+00	0.000670
6	3.21E+00	0.000229	1.07E+01	0.000110
$\beta\ \nu = \Sigma\ \beta_i\ \nu$		0.00655		0.00645
^{241}Pu	Tuttle		ENDFB-VII	
Group	λ_i (1/s)	$\beta_i\ \nu$	λ_i (1/s)	$\beta_i\ \nu$
1	1.28E-02	0.000160	1.36E-02	0.000292
2	2.99E-02	0.003664	3.00E-02	0.003634
3	1.24E-01	0.002768	1.17E-01	0.002310
4	3.52E-01	0.006240	3.07E-01	0.005658
5	1.61E+00	0.002912	8.70E-01	0.003201
6	3.47E+00	0.000256	3.00E+00	0.001105
$\beta\ \nu = \Sigma\ \beta_i\ \nu$		0.01600		0.01620

The most recent version of MCNP, MCNP5-1.60 [7] has incorporated the capability of calculating the values of the delayed neutron fraction from each precursor group. For each fissionable isotope *j*, β_i is averaged over the core. The calculation is performed with the equation:

$$\beta_i = < \psi^\dagger, B_i\, \psi> / < \psi^\dagger, F\, \psi>$$

where $\psi^\dagger$ is the adjoint neutron flux several (usually 10) calculation cycles after the creation of the neutron flux, ψ; B_i is the delayed neutron operator for each precursor group, *i*, and *F* is the fission term operator. The bracket, <>, represents the integration of the of the adjoint flux*operator*flux over all energies, angles, and positions. As before, the delayed neutron precursors are separated into six groups each with the *i* designation.

Table 2 shows the contributions to the fission process from each major actinide (≥0.01% contribution) as calculated by MCNPX [9] when the inventories for the HEU and LEU fuels were being generated. The values in Table 2 are for the HEU and LEU fuels at both the startup (SU) and end-of-cycle (EOC) conditions of the core. With those inventories, MCNP5-1.60 was used to calculate β_i and β. For these analyses the MCNP5-1.60 code was run in the multiprocessor mode for 3000 kcode cycles with 125,000 particles per cycle. The calculated values of β_i and β are shown in Table 3. Also shown are the decay constants for each for each group along with the statistical standard deviation for the delayed neutron fraction as calculated by MCNP5-1.60. The values in Table 3 are recommended for use in the point kinetics models of the NBSR.

It should be noted that the evaluations for the delayed neutron fractions, β_i and β, in ENDFB-VII [6] are dependent on the energy of the incident neutron. However the relative fraction of each precursor group along with each group's decay constant is independent of incident neutron energy, that is although β_i and β are dependent on the incident neutron energy, the values of β_i / β are always constant meaning the mix of fission products in each precursor group is assumed to be constant.

Table 2. Fraction of fissions, f_j, from the major actinides, as calculated by MCNPX.

	Fraction of fissions, f_j, %			
	HEU-SU	HEU-EOC	LEU-SU	LEU-EOC
^{235}U	99.73	99.67	96.35	95.71
^{236}U	0.02	0.02	0.02	0.02
^{238}U	0.01	0.01	0.49	0.49
^{239}Pu	0.23	0.27	2.99	3.54
^{241}Pu	0.02	0.02	0.16	0.24

Table 3. Delayed neutron parameters for the HEU and LEU cores as calculated by MCNP5-1.60.

HEU SU				
	Group	λ_i (1/s)	β_i	σ
	1	0.01249	0.00022	0.00001
	2	0.03182	0.00111	0.00002
	3	0.10938	0.00107	0.00002
	4	0.31700	0.00301	0.00003
	5	1.35386	0.00092	0.00002
	6	8.63611	0.00032	0.00001
	$\beta = \Sigma \beta_i$		0.00665	0.00005
HEU EOC				
	Group	λ_i (1/s)	β_i	σ
	1	0.01249	0.00021	0.00001
	2	0.03182	0.00112	0.00002
	3	0.10938	0.00110	0.00002
	4	0.31700	0.00302	0.00003
	5	1.35374	0.00087	0.00002
	6	8.63558	0.00030	0.00001
	$\beta = \Sigma \beta_i$		0.00661	0.00004
LEU SU				
	Group	λ_i (1/s)	β_i	σ
	1	0.01249	0.00020	0.00001
	2	0.03177	0.00108	0.00002
	3	0.10942	0.00105	0.00002
	4	0.31731	0.00301	0.00003
	5	1.35205	0.00085	0.00002
	6	8.65543	0.00030	0.00001
	$\beta = \Sigma \beta_i$		0.00650	0.00005
LEU EOC				
	Group	λ_i (1/s)	β_i	σ
	1	0.01249	0.00020	0.00001
	2	0.03176	0.00109	0.00002
	3	0.10942	0.00102	0.00002
	4	0.3173	0.00301	0.00003
	5	1.35118	0.00087	0.00002
	6	8.65038	0.00030	0.00001
	$\beta = \Sigma \beta_i$		0.00648	0.00004

B. Photoneutron Contribution to the Delayed Neutrons

In a system such as the NBSR, where the reactor is cooled and moderated with D_2O, the delayed neutron fraction also has a non-negligible component from photoneutrons through the $^{2}H(\gamma,n)^{1}H$ nuclear reaction. The sources of the gamma rays for the delayed portion of those photoneutrons are primarily from the decay of fission products. As such, each isotope that emits a γ-ray with energy greater than the binding energy of the neutron in a deuteron (2.22 MeV [10]) can produce a photoneutron with that isotope's decay constant. Due to the large number of radioisotopes in a nuclear reactor, and since the fraction of delayed neutrons from photoneutrons are usually a measured quantity, the delayed neutrons from the (γ,n) reactions are also lumped into representative groupings. The NBSR SAR [4] used the values measured by Johns and Sargent [11] for the contribution of the delayed neutrons that result from (γ,n) reactions with the deuterium in the D_2O. Those values are shown in Table 4 with the groups numbered from 7 to 14. The contribution of the delayed neutrons from photoneutron interactions is on the order of 4.5% to 4.6% of the total. Since this is a small contribution, a more rigorous or recent analysis is not warranted and it is recommended to continue using the values of Johns and Sargent.

Table 4. Delayed neutrons from the $^{2}H(\gamma,n)^{1}H$ nuclear reaction.

Group	λ_i (1/s)	β_i
7	0.278	0.000203
8	0.0169	0.000065
9	0.0049	0.0000223
10	0.00152	0.0000107
11	4.27x10-4	0.0000066
12	1.16x10-4	0.0000074
13	4.41x10-5	0.000001
14	3.65x10-6	0.00000033
$\beta = \Sigma \beta_i$		0.000316

3. DETERMINATION OF THE PROMPT NEUTRON LIFETIME

A. The 1/v absorber insertion method

A commonly used method to calculate the prompt neutron lifetime is the 1/v insertion method [12] where a small amount (~1E-8 atoms/b-cm) of 1/v absorber is uniformly distributed throughout a reactor system. This absorber results in a negative reactivity insertion, ρ. The negative reactivity insertion is determined by calculating the value of k_{eff} after the 1/v absorber is introduced and is:

$$\rho = \Delta k/k = (k_{eff} - 1)/\, k_{eff}$$

Because models of operating nuclear reactors are not perfect, it is not unusual for the calculated value of k_{eff} to exhibit a bias [13]; that is, a consistent deviation from the expected value of k_{eff}. The calculations performed for this work used MCNP5 [7] with the ENDFB-VII libraries [6]. For a reactor such as the NBSR the bias from the model is determined by knowing the constituents of the fresh, unirradiated HEU fuel elements, and calculating equilibrium inventories for the NBSR at the end-of-cycle. The EOC occurs after 38.5 days of full 20 MW operation and at that point in time there is usually not enough excess reactivity to maintain operation and the reactor shuts itself down. The bias is then determined by calculating the value of k_{eff} at EOC, which is an equilibrium core, noting that the true value should be unity. Therefore any calculation for the reactivity is then compared to the value of k_{eff} with the built-in bias, and it is assumed that all calculations will contain a bias that is constant. Hence ρ is now:

$$\rho = \Delta k/k = 1/k_{eff,u} - 1/\, k_{eff,p}$$

where the subscripts *u* and *p* represent the unperturbed (no 1/v absorber) and the perturbed (with the 1/v absorber) values of k_{eff}. It is assumed that the bias is identical for each calculation of k_{eff}. For the prompt neutron lifetime calculation, the value of $k_{eff,p}$ is calculated for varying amounts of 1/v absorber added to the system. According to Bretscher [12], the reactivity insertion from the addition of a small amount of 1/v absorber is:

$$-\Delta k/k = N\ \sigma_o\, v_o\, l'_p$$

where:

N is the atomic density of the 1/v absorber (in atoms/b-cm)

σ_o is the thermal neutron absorption cross section in barns (b) of the absorber = 3837 b.

v_o is the speed of thermal neutrons = 2.2E5 cm/s: for ^{10}B, $\sigma_o v_o$ = 8.44E+8 b-cm/s

l'_p = neutron lifetime (s) when N atoms/b-cm of 1/v absorber is added to the system

The prompt neutron lifetime, l_p, is calculated as the amount of 1/v absorber approaches zero:

$$l_p\,(s) = \lim_{N\to 0} \; l'_p = \lim_{N\to 0} \; [-\Delta k/k\ /(N\ \sigma_o\ v_o)]$$

For this work, the 1/v absorber method was performed by including ^{10}B uniformly throughout all materials in the NBSR at concentrations between 4E-9 and 15E-9 a/b-cm. The calculations were performed at 1E-9 a/b-cm increments for a total of 12 calculations for the HEU and LEU fuels at startup (SU) and end-of-cycle (EOC). The value of k_{eff} was calculated for each value of ^{10}B loading and the reactivity change associated with the ^{10}B addition was calculated. Concentrations of ^{10}B below 4E-9 a/b-cm did not provide enough change in the reactivity to result in reliable values of ρ for inclusion in the neutron lifetime calculation. Although the statistical error in each calculation of k_{eff} is small, the calculation of Δk/k is the difference between two numbers that have similar values and the error in Δk/k becomes too large to be useful. A plot of the reactivity change as a function of ^{10}B loading is shown in Figure 3 for the two different fuels at the two different core conditions.

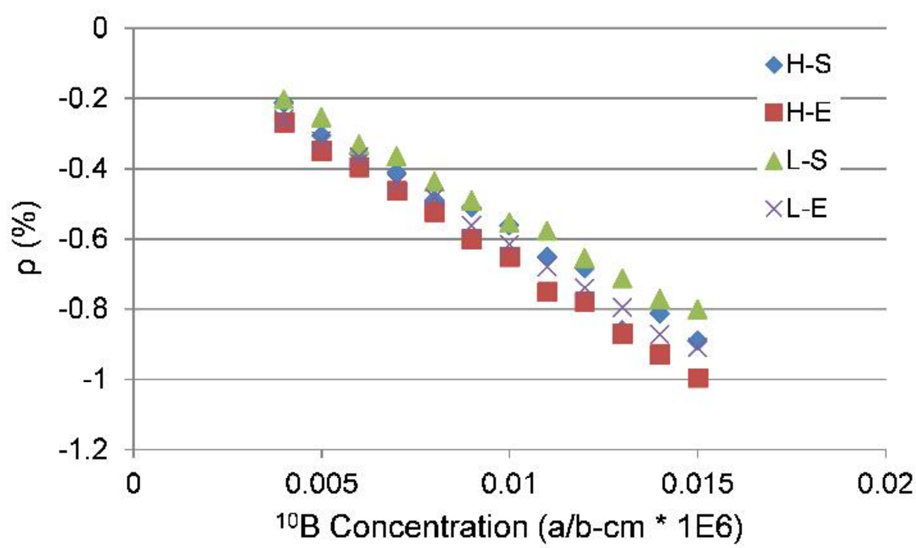

Figure 3. Plot of reactivity vs. ^{10}B where H-* represents HEU, L* LEU fuel, *-S startup and *-E end-of-cycle.

The value of [l'_p = -Δk/k /(N σ_o v_o)] is plotted as a function of N in Figure 4 for the HEU fuel at EOC with the value of N ranging from 4E-9 to 15E-9 a/b-cm. For the HEU fuel at EOC, the intercept, as is shown in Figure 4, is 801 μs. The error bars shown on this plot are from the statistical uncertainty in the calculation of ρ through the two calculated parameters, $k_{eff,u}$ and $k_{eff,p}$ The results are shown in Tables 5 and 6 (along with the other results discussed below) for the HEU and LEU fuels, respectively, at SU and EOC. These values are denoted as "1/v Insertion". The values of σ in these tables are calculated from variations of the values of l'_p from the curve fit (y=-1221.1x + 800.91).

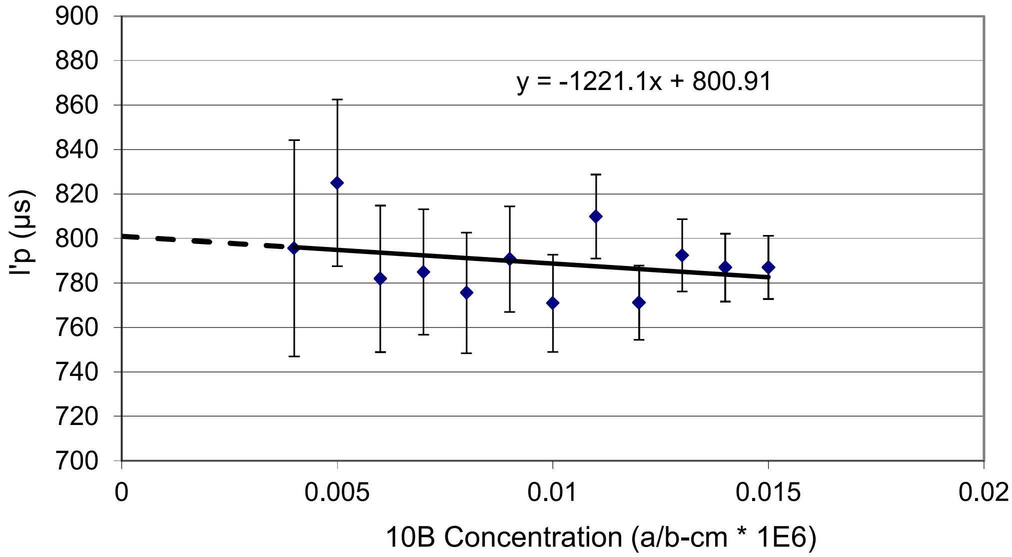

Figure 4 Plot of l'_p as a function of ^{10}B concentration for the HEU fuel at EOC. The solid line and the equation are the results of a linear fit of the data points.

B. The neutron pulse method

The second method determines the prompt neutron lifetime by using MCNP to calculate the decay of a pulse of neutrons in a subcritical nuclear reactor [14] at some point in the reactor. After the higher harmonics have died away, the fundamental mode exhibits an exponential decay:

$$N = N_o e^{\alpha t}$$

where

N_o = the neutron population of the fundamental mode at time zero
t = the decay time
$\alpha = (\rho - \beta) / l_p$
ρ = reactivity of the subcritical assembly ($1 - 1/k_{eff}$) relative to unity
β = delayed neutron fraction
l_p = prompt neutron lifetime (desired quantity)

In this situation, α is always negative and if the reactor was not subcritical, the pulse would not decay. The value of ρ is changed by positioning the shim arms and the decay of the pulse is related to the value of k_{eff} different from unity.

Plots of several decay curves as calculated by MCNP for the LEU fuel at EOC for several shim arm positions are shown in Figures 5 and 6 with Figure 6 being an expanded view of the decay regions that were used for the analyses. It is the slope of each curve in Figure 6 that provides the values of α, for each value of ρ in the equation above. In order to determine a representative value of α for each curve, ten subsets of calculated data points were chosen from each curve (usually including 50-90 points) and the value of α was determined for each subset of data. The value of α that is representative of the overall curve was then the average of the 10 different values of α. Each shim arm angle represents a negative insertion of reactivity, which is taken from the calculated shim arm worth curve [15] and the value of β used was from the analysis in Section 3 above. A value of the prompt neutron lifetime can be calculated for each shim arm position. Since the MCNP calculation does not include any delayed neutrons from the (γ, n) reactions on deuterium, they were not included in the value of β used for this calculation.

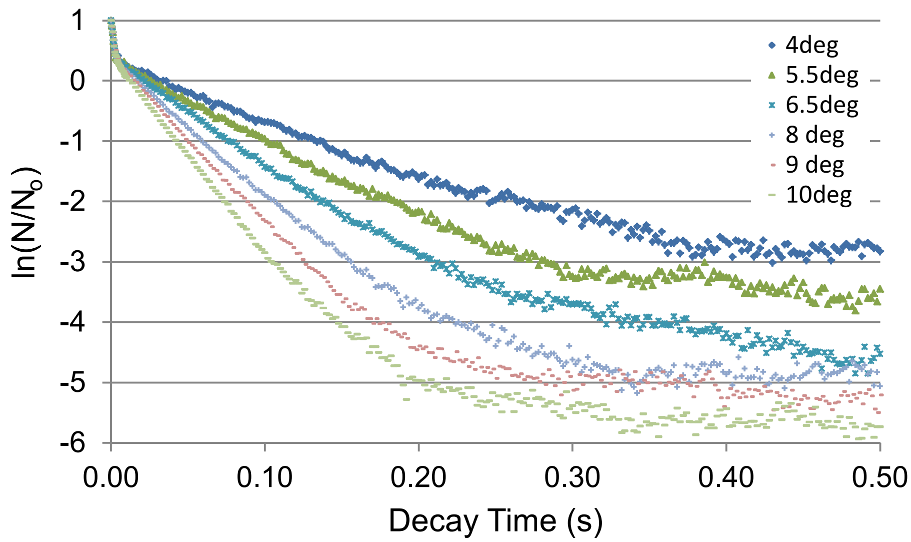

Figure 5. Decay curves for pulses of neutrons into an LEU core at EOC with the shim arms at several positions.

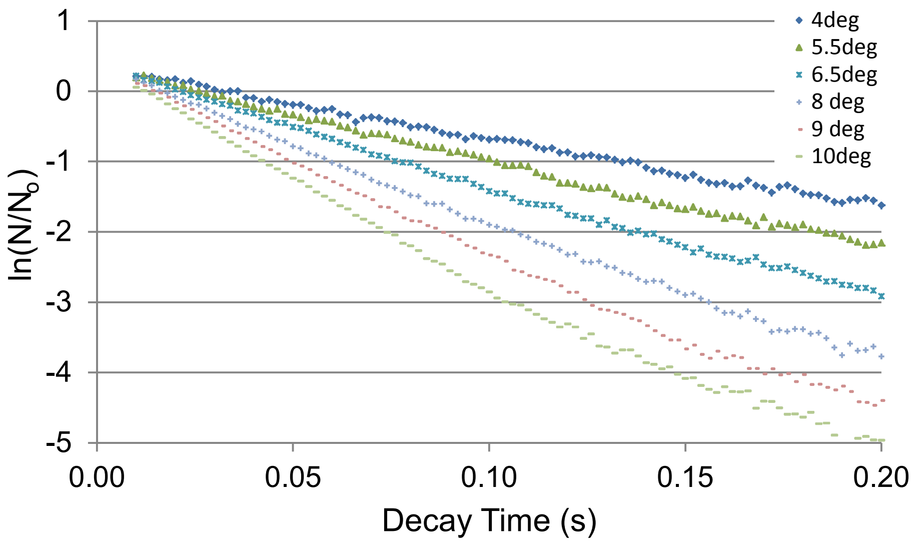

Figure 6. Figure 5 expanded over the region of analyses.

The calculations from the decay of pulsed neutrons were performed for a total of 10 different shim arm positions, and hence 10 different values of ρ. From those calculations, 10 neutron lifetime values were determined and averaged to determine the prompt neutron lifetime for the NBSR with LEU fuel at SU and EOC. Those calculated values of prompt neutron lifetime are shown in Figure 7 as a function of reactivity insertion. The average values are included in Tables 5 and 6 under the "Pulse" heading.

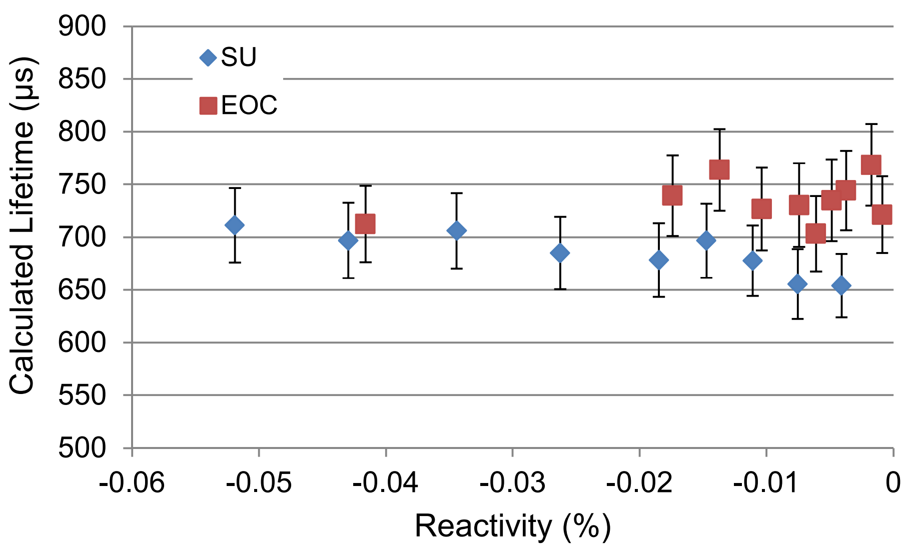

Figure 7. Calculation of neutron lifetime in LEU core as a function of reactivity insertion by the pulse method.

The values of the prompt neutron lifetime for the HEU fuel were originally presented in Reference [14] but have been recalculated with the values of β presented in the first section of this report for the HEU fuel. The uncertainties presented in these tables were calculated from the combination of (1) the spread in the 10 values of α for each decay curve, (2) the deviation between the calculated values of the shim arm worth curve and the fit of those values (a negligible contribution), and (3) the uncertainty in β, which is assumed to be 5%.

C. The adjoint flux weighting method

The third method of calculating the prompt neutron lifetime utilizes the definition of l_p obtained by deriving the point kinetics equations from the more detailed transport equation. It employs tallies (integrals over space, angle and energy) weighted by the adjoint flux. However in the MCNP calculation, the adjoint flux is calculated several (usually 10) cycles after the calculation of the flux [16,17]. This method is similar to the method of calculating the delayed neutron fraction discussed in Section 3. Like the calculation of delayed neutrons, this technique has been implemented in MCNP5-1.60 and is used by invoking the *kopts* card (new to MCNP5-1.60). In this method ψ is the neutron flux, $\psi^\dagger$ is the adjoint flux, *F* is the total fission operator and the operator in the numerator is now 1/*v*. The prompt neutron lifetime is calculated by integrating over all positions, directions and neutron energies:

$$l_p = \langle \psi^\dagger, 1/v\ \psi \rangle / \langle \psi^\dagger, F\ \psi \rangle$$

MCNP calculates the initial flux, ψ in one cycle of calculations. It processes several cycles before it determines the adjoint flux, $\psi^\dagger$, with the default being 10 cycles. For these analyses the MCNP code was run for 3000 kcode cycles with 125,000 particles per cycle. The results of the analyses for the HEU and LEU fuels at SU and EOC are presented in Tables 5 and 6 under the heading "MCNP". The uncertainties quoted in these tables are the statistical uncertainties as reported by MCNP. Changing the number of neutron generations for the adjoint flux to 15 resulted in no significant change in the value of the neutron lifetime.

Table 5. Comparison of the Calculations of the Prompt Neutron Lifetime for the HEU core.

	SU		EOC	
	l_p, μs	σ	l_p, μs	σ
1/v Insertion	712	35	801	14
Pulse	732	34	774	48
MCNP	698	1	802	1
Recommended	650		750	

Table 6. Comparison of the Calculations of the Prompt Neutron Lifetime for the LEU core.

	SU		EOC	
	l_p, μs	σ	l_p, μs	σ
1/v Insertion	610	16	766	12
Pulse	675	35	734	38
MCNP	651	1	730	1
Recommended	600		700	

4. DISCUSSION

The values for the delayed neutron fraction have been calculated using MCNP5-1.60 with the ENDFB-VII evaluations. These values are significantly lower than the values previously used in the NBSR SAR [4]. For the HEU fueled core the value of the total delayed neutron fraction, including the 0.000316 contribution from photoneutrons, based on the review paper by Tuttle [1], was 0.007574. However the values from the newer ENDFB-VII evaluations (adding in the contribution from photoneutrons) were 0.00697 at SU and 0.00693 at EOC. For the LEU fuel the total delayed fraction is lower, being 0.00682 and 0.00680 for SU and EOC, respectively. The lower value for the LEU core is presumably due to the increase in the fission contribution from ^{239}Pu, which has a delayed neutron fraction than that is approximately one third of that of ^{235}U.

The values of the prompt neutron lifetime calculations agree well between the three independent methods. The three methods all show that the prompt neutron lifetime for the SU condition is less than the prompt neutron lifetime for the EOC condition. The calculations also show that the prompt neutron lifetime for the LEU core is expected to be shorter than the prompt neutron lifetime for the HEU core. The value of the prompt neutron lifetime quoted in the NBSR SAR was ~800 μs which is approximately the value for the HEU at EOC, but longer than we expect for the HEU at SU. One reason that the present analysis would indicate a shorter lifetime is that the delayed neutron fraction in the SAR was assumed to be 0.007574 and the present values from the ENDFB-VII evaluations are 0.00665 at SU and 0.00661 at EOC.

The conservative value of the prompt neutron lifetime used for the safety analysis in the SAR was 650 μs. The smaller the lifetime, the faster a transient takes place and therefore, conservative values to be used in the future should not be greater than the smallest calculated value. Based on this approach, the values recommended for safety analysis in the future are 650 μs and 750 μs for the HEU core at SU and EOC, respectively, and 600 μs and 700 μs for the LEU core at SU and EOC, respectively, as shown in Tables 5 and 6.

5. REFERENCES

1. R.J. Tuttle, "Delayed-Neutron Data for Reactor-Physics Analysis", **Nuclear Science and Engineering, 56**, 37-71, 1975.
2. G.R. Keepin, T.F. Wimett, and R.K. Zeigler, "Delayed Neutrons from Fissionable Isotopes of Uranium, Plutonium, and Thorium**" Physical Review, 107,** 1044-1049, 1957.
3. DOE Fundamentals Handbook Nuclear Physics and Reactor Theory, Volumes 1 and 2, DOE-HDBK-1019/1-93, Department of Energy, January, 1993.
4. "Safety Analysis Report – NBSR 14", NISTR 7102, National Institute of Science and Technology, April 2004.
5. J.R. Lamarsh, **Nuclear Reactor Theory**, Addison-Wesley Publishing Company, Reading, MA, 1966.
6. M.B. Chadwick, P. Oblozinsky, M. Herman at al.,"ENDF/B-VII.0: Next Generation Evaluated Nuclear Data Library for Nuclear Science and Technology," Nuclear Data Sheets, 107, 2931-3060, 2006.
7. The general manual is "MCNP – A General Monte Carlo N-Particle Transport Code, Version 5", LA-UR-03-1987, Los Alamos National Laboratory, April 24, 2003; the updates to MCNP used for the delayed neutron fraction and prompt neutron lifetime calculations are in: B.C. Kiedrowski, T.E. Booth, F.B. Brown, J.S. Bull, J.A. Favorite, R.A. Forster, and R.L. Martz, "MCNP5-1.60 Feature Enhancements and Manual Clarifications," LA-UR-10-06217, Los Alamos National Laboratory.
8. R.E. MacFarlane and D.W. Muir, "The NJOY Nuclear Data Processing System", LA-12740-M Manual, UC-413, Los Alamos National Laboratory, October 1994.
9. D.B. Pelowitz, Ed., "MCNPX User's Manual Version 2.6.0," LA-CP-07-1473, Los Alamos National Laboratory, April 2008.
10. J.K. Tuli, Nuclear Wallet Cards, 7th edition, Brookhaven National Laboratory, April 2005, http://www.nndc.bnl.gov/wallet/.
11. M.W. Johns and B.W. Sargent, "Photoneutrons in a Heavy Water Pile," **Canadian J. Physics, 32**, 136-152, 1954.
12. M.M Bretscher, "Perturbation-Independent Methods for Calculating Research Reactor Kinetic Parameters," ANL/RERTR/TM-30, Argonne National Laboratory, December 1997.
13. J. Bess, "September, 2011 Status Update for the NRAD Reactor Benchmark Models," Presented at the TRTR Meeting, Idaho Falls, ID, September 13-15, 2011.
14. A.L. Hanson, H. Ludewig, D.J. Diamond, "Calculation of the Prompt Neutron Lifetime in the NBSR," **Nuclear Science and Engineering, 153**, 26-32, 2006.
15. A.L. Hanson and D.J. Diamond, "Calculation of Design Parameters for an Equilibrium LEU Core in the NBSR," BNL-96386-2011-IR, Brookhaven National Laboratory September 29, 2011.

16. B. Kiedrowski, F. Brown, and P. Wilson, "Calculating Kinetics Parameters and Reactivity Changes with Continuous-Energy Monte Carlo," PHYSOR-2010 – Advances in Reactor Physics to Power the Nuclear Renaissance, Pittsburgh, PA, May 9-14, 2010.
17. B. Kiedrowski, F. Brown, and P. Wilson, "Adjoint-Weighted Tallies for k-Eigenvalue Calculations with Continuous-Energy Monte Carlo," **Nuclear Science and Engineering, 168,** 226-241, 2011.