Final report: Tech-X Corporation work for the SciDAC Center for Simulation of RF Wave Interactions with Magnetohydrodynamics (SWIM)

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Abstract

Work carried out by Tech-X Corporation for the DoE SciDAC Center for Simulation of RF Wave Interactions with Magnetohydrodynamics (SWIM; U.S. DoE Office of Science Award Number DE-FC02-06ER54899) is summarized and is shown to fulfil the project objectives. The Tech-X portion of the SWIM work focused on the development of analytic and computational approaches to study neoclassical tearing modes and their interaction with injected electron cyclotron current drive. Using formalism developed by Hegna, Callen, and Ramos [*Phys. Plasmas* **16**, 112501 (2009); *Phys. Plasmas* **17**, 082502 (2010); *Phys. Plasmas* **18**, 102506 (2011)], analytic approximations for the RF interaction were derived and the numerical methods needed to implement these interactions in the NIMROD extended MHD code were developed. Using the SWIM IPS framework, NIMROD has successfully coupled to GENRAY, an RF ray tracing code; additionally, a numerical control system to trigger the RF injection, adjustment, and shutdown in response to tearing mode activity has been developed. We discuss these accomplishments, as well as prospects for ongoing future research that this work has enabled (which continue in a limited fashion under the SciDAC Center for Extended Magnetohydrodynamic Modeling (CEMM) project and under a baseline theory grant). Associated conference presentations, published articles, and publications in progress are also listed.

1 Overview of the SWIM project

The Center for Simulation of Wave Interactions with Magnetohydrodynamics (SWIM) had two scientific objectives:

- To improve understanding of interactions that both RF wave and particle sources have on extended MHD phenomena, and substantially improve capability for predicting and optimizing the performance of burning plasmas,
- To develop an integrated computational system for treating multi-physics phenomena with the required flexibility and extensibility to serve as a prototype for the Fusion Simulation Project, address the mathematics issues related to the multi-scale, coupled physics of RF waves and extended MHD, and optimize the integrated system on high performance computers.

To reach these goals, the project efforts were directed along three major lines of research:

- **IPS development campaign (ORNL)** — Development of the Integrated Plasma Simulator, a computational platform that allows efficient coupling of a broad range of fusion codes and is flexible enough to allow exploration of various physics models and solution algorithms.
- **Fast MHD physics campaign (PPPL)** — Research addressing long timescale discharge evolution in the presence of sporadic fast MHD events (e.g. sawtooth modes).
- **Slow MHD physics campaign (Tech-X/Wisconsin)** — Research modeling the direct interaction of RF and extended MHD for slowly growing modes (e.g. neoclassical tearing modes).

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This work reports on the numerical aspects of the Slow MHD physics campaign, which is complementary to the analytic efforts carried out by Dr. Jesus Ramos (MIT). The initial work reported here was performed at the University of Wisconsin-Madison by Dr. T. Jenkins, under the supervision of Dr. Dalton Schnack. In early 2010, Dr. Jenkins left UW-Madison and began working for Tech-X Corporation; work relevant to UW-Madison's portion of the Slow MHD campaign was subsequently subcontracted to Tech-X beginning in March 2010. Because Tech-X was already receiving separate SWIM funding at that time, the two contracts were consolidated in 2011. For clarity of presentation, this report will summarize work carried out by Tech-X personnel from the time Dr. Jenkins joined Tech-X until the end of the SWIM funding period (January 2013).

2 Tech-X project contributions

In this section we discuss the major accomplishments of Tech-X personnel in furtherance of SWIM project goals.

2.1 Derivation of RF–induced quasilinear terms

Tech-X researchers have made significant expansions of a formulation for including the effects of externally applied RF sources in plasma fluid models. The formulation was originally developed by Hegna and Callen [1] and extended by Ramos [2, 3]; the latter works build on the basic principles outlined by Hegna and Callen to rigorously determine appropriate orderings and closures in low-collisionality, fusion-relevant regimes wherein neoclassical tearing modes grow. The Hegna/Callen formulation is valid when RF-induced perturbations to the distribution function are small; in such cases, the lowest-order distribution function can be written as a local Maxwellian (an approximation valid for electron cyclotron heating and current drive). The formulation also relies on a multiple timescale ordering, wherein RF processes are assumed to occur much more rapidly than MHD-like processes. RF sources are shown to introduce additional terms in the fluid equations (brought about by the transfer of momentum and energy from the RF fields to the plasma) and to affect the closure calculations (necessary to formally solve the system of fluid equations with full self-consistency). These additional terms, and their effect on the closures, are not calculated analytically by Hegna and Callen in Ref. [1]; instead, they are represented formally in terms of linear operators (and their inverses) acting on the fields and distribution functions of the RF-scale and MHD-scale processes. Likewise, in Ramos' works, the additional physics imparted by the RF appears only in formal expressions.

Tech-X researchers have extended the Hegna/Callen/Ramos formalism to obtain *analytic* expressions for the additional source terms appearing in the extended MHD equations and closures when injected RF power is resonant near the electron cyclotron frequency. The ensuing expressions are compatible with the hybrid fluid/drift-kinetic closure scheme proposed by Ramos, and thus completely specify the equations which are necessary to model ECCD stabilization of NTMs in the extended MHD formalism. We summarize the salient points of the ensuing publication [4] here.

2.1.1 Fluid equations

When RF is injected near ECCD resonant frequencies, the momentum and energy equations of extended MHD can be written in the form

$$m_\alpha n_{\alpha s}\left(\frac{\partial \mathbf{V}_{\alpha s}}{\partial t}+(\mathbf{V}_{\alpha s}\cdot\nabla)\mathbf{V}_{\alpha s}\right)=-\nabla(n_{\alpha s}T_{\alpha s})-\nabla\cdot\mathbf{\Pi}_\alpha+q_\alpha n_{\alpha s}[\mathbf{E}_{0s}+\mathbf{V}_{\alpha s}\times\mathbf{B}_{0s}]+\mathbf{R}_\alpha+\mathbf{k}_{rs}H_\alpha\delta_{\alpha,e}\ ; \quad (1)$$

$$\frac{3}{2}n_{\alpha s}\left(\frac{\partial T_{\alpha s}}{\partial t}+(\mathbf{V}_{\alpha s}\cdot\nabla)T_{\alpha s}\right)+n_{\alpha s}T_{\alpha s}\nabla\cdot\mathbf{V}_{\alpha s}=-\nabla\cdot\mathbf{q}_\alpha-\mathbf{\Pi}_\alpha:\nabla\mathbf{V}_{\alpha s}+Q_\alpha+\omega H_\alpha\delta_{\alpha,e}\ , \quad (2)$$

where all but the final terms in the equations are standard and have their conventional meaning. In these final terms (which are added only for electrons), ω and $\mathbf{k}_{rs}$ are the frequency and real wavevector of the injected RF wave. The quantity H_α is well approximated by

$$H_\alpha = \frac{\epsilon_0}{4}\sum_{n=-\infty}^{\infty}\frac{\omega_{p\alpha s}^2}{\omega^2}e^{-\lambda_{\alpha s}}\xi_{0s}\sqrt{\pi}e^{-\xi_{nrs}^2}\times$$
$$\Bigg[[I_n(\lambda_{\alpha s})-I_{n+1}(\lambda_{\alpha s})]\left(2\lambda_{\alpha s}|E_{ys}|^2+n\left|E_{xs}-iE_{ys}+\frac{\sqrt{\lambda_{\alpha s}}(\omega-n\Omega_{\alpha s})E_{zs}}{nk_{\parallel s}v_{t\alpha s}}\right|^2\right)$$
$$+[I_n(\lambda_{\alpha s})-I_{n-1}(\lambda_{\alpha s})]\left(2\lambda_{\alpha s}|E_{ys}|^2-n\left|E_{xs}+iE_{ys}+\frac{\sqrt{\lambda_{\alpha s}}(\omega-n\Omega_{\alpha s})E_{zs}}{nk_{\parallel s}v_{t\alpha s}}\right|^2\right)\Bigg]\ . \qquad (3)$$

wherein $\omega_{p\alpha s}$ is the species plasma frequency, $\lambda_{\alpha s}=k_{\perp s}^2T_s/m_\alpha\Omega_{\alpha s}^2$ is the argument of the modified Bessel function $I_n(\lambda_{\alpha s})$, $\xi_{ns}=(\omega-k_{\parallel s}V_{\parallel\alpha s}-n\Omega_{\alpha s})/\sqrt{2}k_{\parallel s}v_{t\alpha s}$ is the (complex) conventional argument of the plasma dispersion function, and $\Omega_{\alpha s}$ is the cyclotron frequency. Quantities with the s subscript vary on the macroscopic (MHD) scales. Because of the Gaussian dependence of H_α on $\xi_{rns}=\mathrm{Re}(\xi_{ns})$, all but one of the terms in the infinite sum are negligible, and that term is likewise negligible except when $\xi_{rns}\sim(\omega-k_{\parallel s}V_{\parallel\alpha s}-n\Omega_{\alpha s})\approx 0$ – that is, when the Doppler-shifted frequency of the injected RF matches up with that particular resonant cyclotron harmonic.

From the RF point of view, the waves obey a dispersion relation

$$\left[\mathbf{N}_s\mathbf{N}_s+\mathbb{I}(1-N_s^2)+\sum_\alpha\chi_\alpha\right]\cdot\mathbf{E}_s\equiv\mathbb{D}_s\cdot\mathbf{E}_s=0 \qquad (4)$$

whose real part (associated with the Hermitian component of the susceptibility χ_α) is associated with the trajectory of RF power through the plasma, and whose imaginary part (associated with the anti-Hermitian component of the susceptibility) reflects the transfer of power from the RF to the plasma. One can construct a Poynting theorem

$$\nabla\cdot\mathbf{S}=-\omega\sum_\alpha H_\alpha \qquad (5)$$

and express

$$H_\alpha\equiv\frac{\epsilon_0}{2}\mathbf{E}_s^*\cdot\chi_\alpha^a(\mathbf{k},\omega)\cdot\mathbf{E}_s\ , \qquad (6)$$

demonstrating that the loss of energy from the RF wave [right hand side of Eq. (5), for which only electrons contribute] is exactly matched by the gain of energy by the plasma [last term of right-hand side of Eq. (2), for electrons].

2.1.2 Converting to GENRAY variables

Knowing the form of H_α, one must now extract the appropriate data from GENRAY in order to construct it. This is generally straightforward, though a number of unit conversions need to made (GENRAY uses cgs and NIMROD uses SI). As well, we must examine normalization conventions. GENRAY uses a normalized Stix representation for the electric field, which we will designate as

$$e_{xs}\equiv\frac{E_{xs}}{|\mathbf{E}_{0s}|}\ ;\ e_{ys}\equiv\frac{E_{ys}}{|\mathbf{E}_{0s}|}\ ;\ e_{zs}\equiv\frac{E_{zs}}{|\mathbf{E}_{0s}|}\ ;\ |\mathbf{E}_{0s}|=\sqrt{|E_{xs}|^2+|E_{ys}|^2+|E_{zs}|^2} \qquad (7)$$

with the normalization yielding

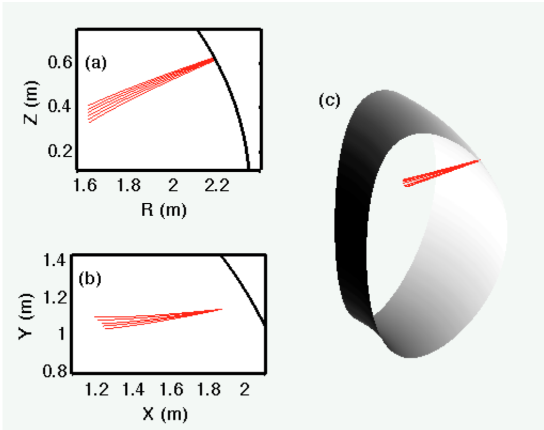

Figure 1: Typical trajectories of a few RF rays in a bundle are shown (a) as projected into the poloidal $(R-Z)$ plane of the torus; (b) as projected into the midplane $(Z = 0$, i.e. the view from the top) of the torus; and (c) in a three-dimensional cutaway section of the torus. For frequencies near electron cyclotron resonance, trajectories are typically nearly straight except near the constant-R resonant surface of the tokamak where power is deposited.

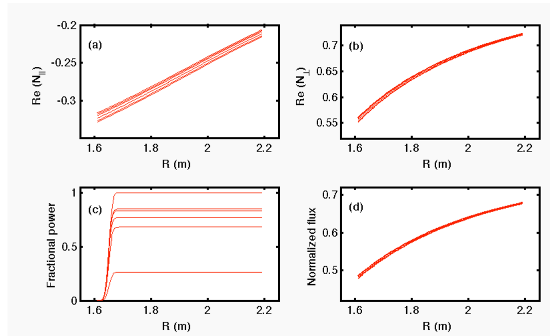

Figure 2: Variation with respect to major radius, for a subset of rays in the ray bundle, of (a) the real part of the parallel index of refraction $N_{\|}(= k_{\|rs}c/\omega)$; (b) the real part of the perpendicular index of refraction $N_{\perp}(= k_{\perp rs}c/\omega)$; (c) the fractional power content P/P_{max}, where P_{max} is the power of the central ray in the bundle as it exits the RF launcher, and (d) the normalized electromagnetic flux. Rays propagate from larger to smaller R values; the deposition of power in the plasma is indicated by the sharp drop-offs in (c) as the rays intersect the resonant surface.

$$\sqrt{|e_{xs}|^2 + |e_{ys}|^2 + |e_{zs}|^2} = 1 \, . \tag{8}$$

The $\{e_{xs}, e_{ys}, e_{zs}\}$ variables are complex quantities, and writing H_α in terms of these variables leaves a multiplicative factor of $|\mathbf{E}_{0s}|^2$ which is not known. However, GENRAY does calculate the power content P of a given ray as a function of distance along the trajectory, as well as the local power flux Γ along the ray. (Typical ray trajectories and data along these trajectories which GENRAY generates are shown in Figures 1 – 2). It can be shown (and is extensively discussed in our forthcoming article *"Coupling extended magnetohydrodynamic fluid codes with radiofrequency ray tracing codes for fusion modeling,"*; see section A for details) that the power satisfies the relation

$$P = \epsilon_0 c |\mathbf{E}_{0s}|^2 A \Gamma \tag{9}$$

where A is the cross-sectional area which corresponds to a given ray at a given point along its trajectory. This factor arises as a consequence of the the eikonal approximation providing solutions to the dispersion relation only along characteristic trajectories of constant phase, rather than global solutions for the wave fields. Consequently, one must use an adequately large *set* of such trajectories to construct the global solution, and we will discuss this further in the next section.

2.2 Development of computational methods: area elements

Although the ray trajectories and the associated fields obtained from ray tracing codes represent only local solutions to the underlying differential equations along these characteristic trajectories, a sufficiently dense number of such solutions can be used to construct the global solution. When rays pass through a finite volume, that volume can be subdivided into subvolumes whose points are nearer to one particular ray than to any other ray. The evolving cross-sectional area of the ensuing subvolume, as the trajectory is traversed, relates the local and global solutions as long as the spacing between neighboring rays is smaller than the spatial scales on which the global solution varies. The number and spacing of rays needed to achieve numerical convergence is determined by this condition. Fortunately, the needed resolution can be determined with reasonable accuracy before the NIMROD run begins.

Tech-X researchers have applied methods from computational geometry to calculate the value of these area elements. The basic method is indicated in Figure 3. Rays passing through a designated plane show up as points in the plane (Figure 3a); knowing these points, one can construct the associated Voronoi tessellation (i.e. computing the boundaries designating what parts of the plane "belong" to which points) shown by the solid black lines in the figure. Because the ray bundle is bounded, some Voronoi regions extend to infinity; each point in the plane, no matter how far from the intersection points, is still nearer to one particular intersection point than any of the others. Proper construction of the ray bundle implies that the physics of quasilinear diffusion at these outer points will be nearly negligible; however, one must still assign a finite cross-sectional area to these outer points. Thus, one also constructs the Delaunay triangulation of the points (Figure 3b), forming a mesh which is dual to the Voronoi tessellation. A subset of the triangles which comprise this mesh form the convex hull (effectively, the outer boundary of the original points or the solid black line in Figure 3c), and these triangles can then be reflected across the convex hull to form ghost points (Figure 3d). With the ghost points included, a new Voronoi tessellation can be constructed; within this tessellation, the regions associated with our original points all have finite area.

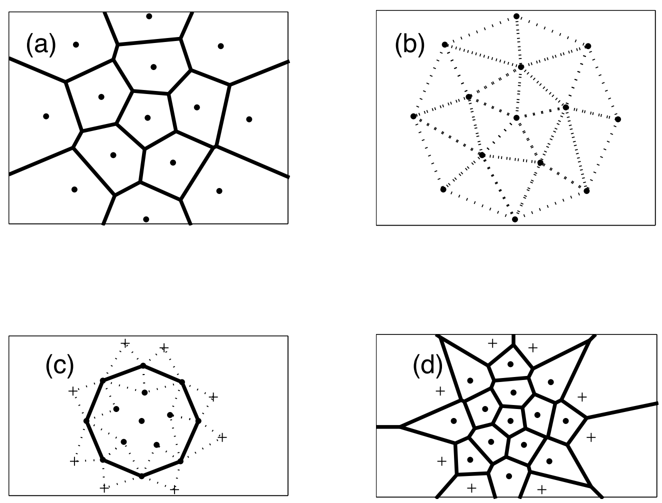

Figure 3: (a) A set of coplanar points representing the intersection of a ray bundle with a plane, together with the Voronoi tessellation (solid black lines) corresponding to these points. Note that the Voronoi regions corresponding to the outermost points extend to infinity. (b) Delaunay triangulation of the points in (a). (c) Convex hull (solid black lines) of the points in (a), together with the reflection across the hull of the Delaunay triangles whose edges lie on the convex hull. Ghost points (+ symbols) are thus created outside the convex hull. (d) Voronoi tessellation of the set of original and ghost points. Note that all original datapoints now correspond to Voronoi regions of finite area.

Using the QHULL software package [5], Tech-X personnel have applied these methods to the GENRAY data to determine approximate global solutions for the RF fields, and have developed mathematical methods to map these solutions onto NIMROD's mixed finite-element/spectral representation. These methods will be discussed in the next two sections.

2.3 Development of computational methods: resolving the Fourier space

NIMROD uses a mixed representation to model spatial quantities; in the toroidal direction, spatial variation is captured spectrally through the use of a dealiased finite Fourier series. Within a poloidal plane, high-order polynomial finite elements model the functional behavior. For many MHD problems, relatively few toroidal Fourier modes are needed since the characteristic scale lengths for toroidal variation are large. This is unfortunately not the case for the RF-induced quasilinear diffusion we investigate here; though the tearing modes we are attempting to suppress are generally low helicity and can be modeled with few toroidal modes, the resonant regions wherein electric current, momentum, and energy are induced by the RF are highly localized both toroidally and poloidally. Within the poloidal plane, achieving spatial resolution is usually not difficult; grid packing of finite element blocks, in tandem with the use of high-order polynomials within these blocks, generally captures even small-scale variation in the quasilinear diffusion coefficients (see Figure 4).

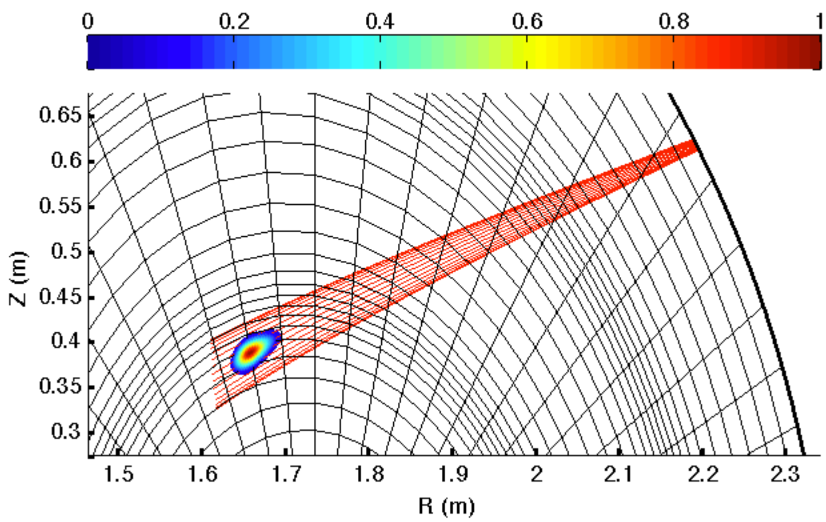

Figure 4: Projection into the poloidal plane, akin to plot (a) of Figure 1, of the normalized function H_e atop the ray trajectories. The grid boundaries corresponding to finite element blocks in a poloidal plane, within which high-order polynomials model physical quantities, are also displayed; grid packing has been employed and results in nonuniform block sizes. The grid resolution shown is adequate to capture variation in H_e in this representation.

In the toroidal direction, achieving adequate resolution is more complicated. Figure 5 demonstrates a case wherein planes of constant ϕ corresponding to moderately high toroidal resolution (relative to a conventional NIMROD run) are superposed atop a quasilinear diffusion coefficient computed from the ray trajectories. In this case, it is only by happenstance that any toroidal planes intersect the region of quasilinear diffusion at all; at least an eightfold increase in toroidal resolution is required to capture the variation shown in the figure.

Tech-X personnel have demonstrated that NIMROD is capable of running at these resolutions; simulations with $N = 512$ (171 dealiased Fourier modes) have been carried out at NERSC and have demonstrated weak scaling up to

$\sim$ 65,000 cores. Work on improvements to the scaling continues under a separate (baseline theory) grant, and is being carried out on the Titan machine at ORNL; the details of this work will be reported elsewhere.

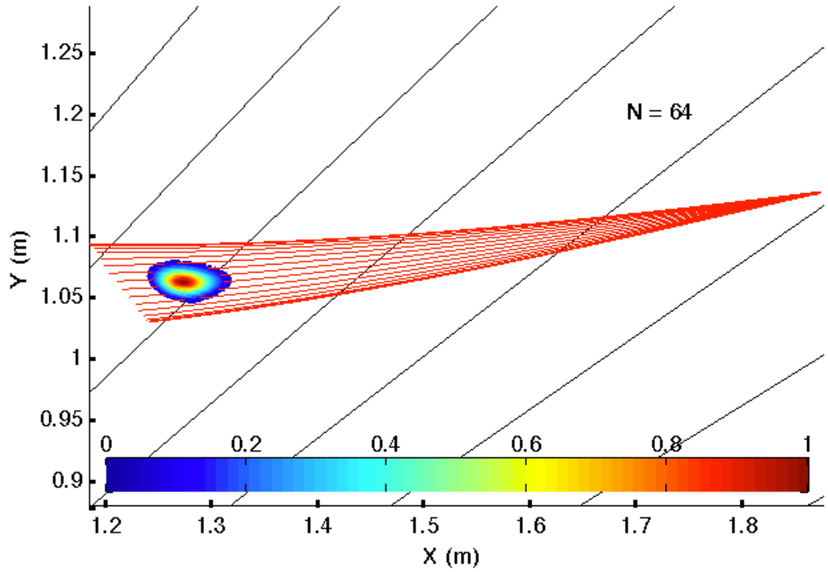

Figure 5: Midplane projection, akin to plot (b) of Figure 1, of the normalized function H_e atop the ray trajectories. Planes of constant ϕ corresponding to an $N = 64$ Fourier sampling are also displayed. The length scale of the H_e variation, on the order of 1 cm, demands at least an eightfold increase in the number of toroidal modes used to resolve the RF source.

Because the physics of tearing modes can be modeled in NIMROD using relatively few Fourier components in the toroidal direction (since the scale length for toroidal variation of these low-helicity modes is long), we have also experimented with various approximations for the toroidal form of the RF–induced quasilinear diffusion. Since the region where this diffusion occurs is highly localized, the above approach is a rather brute-force one; we effectively increase the toroidal resolution of conventional NIMROD runs by at least an order of magnitude to capture the diffusion at only a few points. But if the representation of the deposited RF, and its associated physics, can be broadened toroidally without substantial effects on the physics (in keeping with our observation that the scale length for tearing modes in this direction is also long, and that induced current and heat rapidly spreads over the relevant flux surfaces), then fewer Fourier modes will be needed to capture the variation. Tech-X work in this area is still ongoing, though we expect to publish initial results in our forthcoming journal article. For now, we turn to the discussion of how to construct smooth representations of functions within a plane which correctly model the data carried by rays which pass through that plane.

2.4 Development of computational methods: planar interpolation

Having addressed the transformation from characteristic local solutions to global solutions, and the issue of toroidal interpolation, we now discuss the interpolation of the RF data onto NIMROD's finite element basis within a poloidal plane. The data at this point takes the form of individual points at which the rays cross a particular plane, together with various measures of physical quantities associated with those points. In effect, we must carry out some kind of multivariate interpolation between these points to determine a smooth functional representation for them; this representation can then be projected onto the finite element basis via the orthogonality of the basis functions.

In collaboration with Dr. Eric Held at Utah State University, Tech-X personnel have implemented a modified Shepard algorithm in NIMROD to carry out these interpolations. A simple Shepard algorithm (inverse distance weighting) takes the form

$$f(x,y) = \sum_{i=1}^{N} \frac{f_i}{(x-x_i)^2 + (y-y_i)^2} \Bigg/ \sum_{i=1}^{N} \frac{1}{(x-x_i)^2 + (y-y_i)^2} \tag{10}$$

wherein $f(x,y)$ is the smooth approximation to the data f_i measured at the points (x_i, y_i). More complex algorithms, in which the data is functionally weighted to the distance from its observation point in different ways, have demonstrated better convergence properties. In NIMROD, cubic polynomial basis sets and cosine basis sets for the weighting were both implemented and compared; the latter method (described in detail Ref. [6]) was found to be more numerically reliable in constructing smooth representations of the RF data.

Another observation arising from the Shepard algorithm development and testing is that ray bundles at least twice as wide as the region of RF–induced quasilinear deposition are needed to make the algorithm stable. Numerical experiments with narrow ray bundles yielded sharp discontinuities at the edge of the diffusive region, and were found to be the cause of numerical instabilities within NIMROD in severe cases. The Shepard algorithm requires sufficiently smooth discrete data to work, and this requires that the discrete data goes to zero in places far from the resonant region. The easiest method is to broaden the ray cone such that it computes additional zero or nearly-zero quasilinear diffusion coefficients at the ray bundle's edge. This is shown in Figure 5, where the outermost rays in the bundle make no contributions to the quasilinear diffusion coefficient. Because the computational cost of calculating additional ray trajectories is negligibly small (ray tracing codes are embarrassingly parallelizable) the need for additional rays does not impose significant constraints.

2.5 Development of numerical Plasma Control System

Tech-X researchers have also developed a numerical control system to control the timing and location of the injected RF. The general concepts of the control system are shown in Figure 6. We have implemented synthetic diagnostics, Mirnov coils, in NIMROD to monitor tearing mode growth; when the mode amplitude exceeds a preset threshold, RF is introduced near the rational surface and allowed to dwell at that a fixed location for a time. Thereafter, its effect on the mode amplitude is assessed, and the RF can either be retained at that location, realigned slightly, or, turned off completely. NIMROD checks for the presence of a control system-generated status file to determine whether the RF is off or on; in the former case, it then calculates the quasilinear diffusion by reading other files generated by GENRAY and by the control system.

As yet, we have not determined how best to optimize the parameters (dwell time, launch angle adjustment) which modify the position of the RF as the simulation runs, except on an *ad hoc* basis for specific cases. In this regard, these complex, coupled simulations begin to resemble experimental scenarios; one first needs to characterize the dominant physics of the scenario and then adjust control system parameters accordingly.

We have also developed an "exact" control system which uses Poincaré representations of NIMROD's magnetic fields to precisely calculate the position of magnetic islands and their O-points, and to inject RF at or near the O-point. From analytic theory, this configuration is the most optimal for RF stabilization and has guided our thinking in the development of other (more experimentally relevant) approaches to stabilization. Using this control system allows explorations of the optimal efficiency (given the more accurate geometry and deposition profiles) as well as sensitivity of the mode stabilization effects to RF misalignment. This approach is complicated by the presence of toroidal rotation, which effectively rotates the island within the poloidal plane while the position of the RF remains fixed.

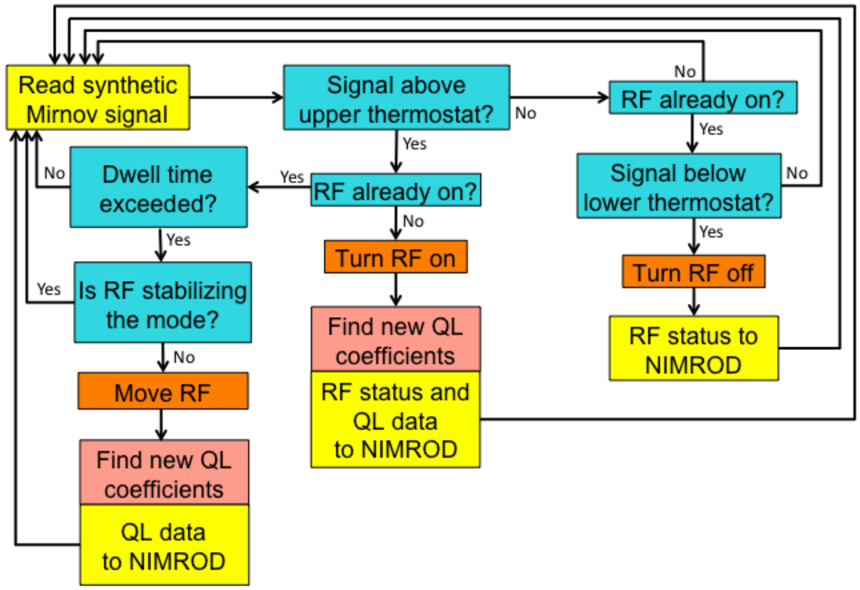

Figure 6: Schematic plasma control system. Data from synthetic Mirnov coils (implemented within NIMROD) is monitored by the control system and compared with upper and lower thermostats (the mode amplitude must exceed the threshold set by the upper thermostat before the RF attempts to suppress it, and must likewise fall below the lower thermostat threshold before the RF will turn off). Realignment of the RF is possible after it has remained in a fixed position long enough ("dwell time") to determine whether or not its effect on the mode is stabilizing. Thermostats, dwell times, and the increments in which the RF is realigned can all be varied for optimal RF performance.

2.6 Coupling of NIMROD to IPS

Having addressed all of the analytic and numerical issues with coupling, coupled RF/MHD simulations may now be run using the IPS framework developed by the SWIM project's Oak Ridge team. NIMROD, GENRAY, and an interface program QLCALC (which calculates quasilinear diffusion coefficients using data from the other two codes) have all been implemented as IPS components, together with their pre- and post-processors and a number of NIMROD's data analysis programs. A sample run that uses a version of the "exact" control system is shown in Figure 7. Here, a dominant $(2,1)$ resistive tearing mode at low β grows linearly; injection of RF just outside the island O-point arrests this growth and begins to shrink the magnetic island. However, when the mode amplitude drops beneath a certain threshold, the RF is switched off and the linear growth resumes. At the same time, rotational effects modify the position of the island O-point relative to the fixed RF position, and the RF is consequently unable to stabilize the mode as effectively when the amplitude rises again. Eventually, the mode achieves nonlinear saturation despite the efforts of the RF. Thus, this control algorithm delayed the saturation of the mode and considerably reduced its saturated amplitude. We anticipate that further developments to the control system will demonstrate more complete stabilization. Ultimately, we also wish to better mimic the experimental control system to provide more direct comparisons.

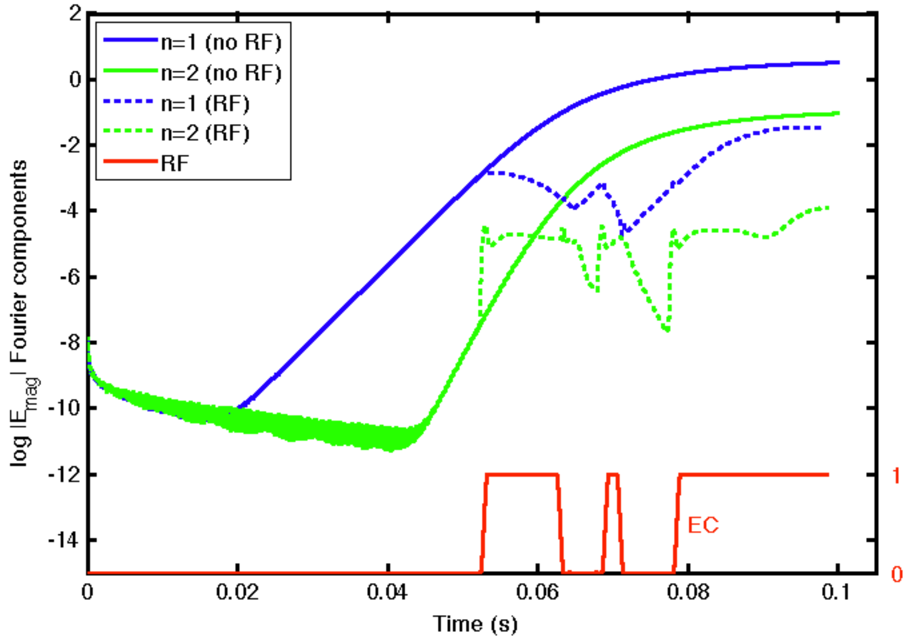

Figure 7: Behavior of the tearing mode in response to the injected RF of the plasma control system. The RF is held at a fixed position and is switched on or off according to the plasma response. In the absence of RF (solid lines), exponential growth of the mode amplitude as the magnetic island opens is eventually overcome by nonlinear saturation effects, which arrest the island growth. With RF injected slightly outside the island O-point, the initial growth is halted and reversed. However, mode growth resumes at the original rate as the control system is turned off, and since rotation of the plasma shifts the position of the O-point relative to the injected RF, eventually the injected RF cannot act to stabilize the mode.

3 Challenges

A number of challenges were encountered in this work which are likely to emerge again in future research efforts, and we briefly summarize these here for informational purposes.

- It is difficult to construct experimentally relevant high-β plasma equilibria which are near enough to marginal stability that one can actually produce neoclassical tearing modes. (While one can scarcely avoid NTMs experimentally, it's presently a challenge to get them into extended MHD simulations in the first place without using unphysically large mode seeding parameters.) Useful guidance from Dylan Brennan (U. Tulsa) and Val Izzo (GA) allowed us to make some progress in this regard, but much remains to be done in developing and refining equilibria near the marginal stability boundary (which is the most experimentally-relevant parameter regime).

- The development of neoclassical closures for NIMROD (intrinsically related to the previous issue) has been slower than was originally anticipated at the outset of SWIM. Eric Held (Utah State) has done a lot of good work on the problem and made considerable progress, but we haven't had a lot of time to incorporate that work into SWIM problems. In order to capture all of the relevant NTM physics and stabilization mechanisms at high β (Fisch-Boozer, Ohkawa, etc.) this will need to be carefully looked at.

- GENRAY, though adequate for the purposes at hand, is not a particularly modern code (fortran 77) and its build system, output data formats, and frequent use of unconventional coding practices have all presented difficulties at various points in the project.

4 Future prospects

Tech-X research for the SWIM project has opened up a number of other possible future research avenues. We received encouraging reviews (though no award) on an INCITE proposal submitted in 2012, this work centered (among other things) on further refinement of the numerical control system, further improvements to NIMROD scaling at high toroidal resolution, and detailed investigation of the physics basis for toroidal approximations to the quasilinear diffusion operator (noted in section 2.3). A number of these issues are currently under study through a Director's Discretionary account on Titan at ORNL (6 million hours), and we hope to reapply for INCITE in 2013.

Ongoing developments in the areas of neoclassical closures and high-β equilibrium refinement will enable the simulation of increasingly realistic experimental scenarios, including more accurate modeling of neoclassical tearing mode stability thresholds, growth, and stabilization via the work discussed here.

In addition, the formalism developed for coupled ECRH/MHD interactions enables study of the effect of applied ECRH on the plasma rotation profile. Such an effect has been noted in the KSTAR device [7], in which rotation profiles were altered by applied ECRH without substantive alterations to the plasma stored energy or ion temperature profile. This effect is not well understood, but exploration of the underlying physics could enable the use of targeted ECRH to optimize rotation profiles in existing and future devices.

5 Summary

Under the SWIM project, Tech-X researchers have carried out physics-based calculations of the quasilinear diffusion terms induced by RF in the fluid equations. We have demonstrated that the resultant equations conserve energy as it is transferred from the RF wave to the MHD plasma via electron cyclotron resonance, and have determined the relationship of GENRAY's ray tracing variables to NIMROD's extended MHD variables. Sophisticated computational geometry calculations have been carried out to convert the discrete, characteristic solutions of the RF ray tracing equations to continuous quasilinear diffusion coefficients within NIMROD; the interpolation of the ensuing wave fields onto Fourier and finite element bases has also been investigated and successfully demonstrated. A numerical control system which mediates the interaction between the RF and fluid codes has been developed and is being refined, and the entire simulation workflow has been integrated into the SWIM IPS framework and successfully executed on high-performance machines at NERSC and OLCF. Several publications summarizing key developments in this work also have been or are being prepared. In addition, we have identified a number of prospective research avenues which this work has made possible and which we hope to pursue in future collaborations.

A Publications and presentations

Here we provide a list summarizing talks, presentations, and publications by Tech-X personnel in connection with the SWIM project, from January 2010 to the present. The majority of the relevant slides/abstracts/manuscripts can be accessed via the links embedded in electronic versions of this document.

Invited talks

- T. G. Jenkins (with S. E. Kruger and E. D. Held), *ECCD-induced tearing mode stabilization via active control in coupled NIMROD/GENRAY HPC simulations*, to be presented at International Sherwood Fusion Theory Conference, Santa Fe, New Mexico, April 2013.
- T. G. Jenkins (with S. E. Kruger, E. D. Held, R. W. Harvey, D. D. Schnack, W. R. Elwasif, and the SWIM project team), *Coupled IPS/NIMROD/GENRAY simulations of ECCD-induced tearing mode stabilization* , CU-Boulder Center for Integrated Plasma Studies Seminar, Boulder, Colorado, April 2011.

Journal articles

- T. G. Jenkins and S. E. Kruger, *Fluid equations in the presence of electron cyclotron current drive* , Phys. Plasmas **19**, 122508 (2012).
- T. G. Jenkins, S. E. Kruger, C. C. Hegna, D. D. Schnack, and C. R. Sovinec, *Calculating electron cyclotron current drive stabilization of resistive tearing modes in a nonlinear MHD model* , Phys. Plasmas **17**, 012502 (2010).

Journal articles in progress

- T. G. Jenkins, S. E. Kruger, and E. D. Held, *Coupling extended magnetohydrodynamic fluid codes with radiofrequency ray tracing codes for fusion modeling*, to be submitted to J. Comp. Phys. or Comput. Phys. Commun., spring 2013.
- T. G. Jenkins and S. E. Kruger, *Numerical plasma control algorithms for tearing mode stabilization*, to be submitted to Nucl. Fusion.

Conference proceedings

- S. E. Kruger, T. G. Jenkins, E. D. Held, J. J. Ramos, J. King, D. D. Schnack, and R. W. Harvey, *Coupled Simulations of RF Effects on Tearing Modes* , in Proceedings of the 24th International Conference on Plasma Physics and Controlled Nuclear Fusion Research, San Diego, California, USA, 2012 (International Atomic Energy Agency, Vienna, Austria, 2012, Paper TH/P3-11).
- S. E. Kruger, T. G. Jenkins, E. D. Held, J. J. Ramos, R. W. Harvey, and D. D. Schnack, *Coupled Simulations of RF Feedback Stabilization of Tearing Modes* , in Proceedings of the 5th IAEA Technical Meeting on the Theory of Plasma Instabilities, Austin, TX, USA, 2011 (International Atomic Energy Agency, Vienna, Austria, 2011, Paper B1.3).

Conference presentations

- T. G. Jenkins, S. E. Kruger, E. D. Held, R. W. Harvey, and the SWIM project team, *ECCD-induced tearing mode stabilization via active control in coupled NIMROD/GENRAY HPC simulations* , Annual Meeting of the Division of Plasma Physics, American Physical Society, Providence, Rhode Island, November 2012. Abstract published in *Bulletin of the American Physical Society* **57**, 318 (2012).

- T. G. Jenkins, S. E. Kruger, E. D. Held, R. W. Harvey, D. D. Schnack, and the SWIM project team, *ECCD-induced tearing mode stabilization in coupled IPS/NIMROD/GENRAY HPC simulations* , Annual Meeting of the Division of Plasma Physics, American Physical Society, Salt Lake City, Utah, November 2011. Abstract published in *Bulletin of the American Physical Society* **56**, 213 (2011).
- T. G. Jenkins, S. E. Kruger, E. D. Held, R. W. Harvey, D. D. Schnack, W. R. Elwasif, and the SWIM project team, *ECCD-induced tearing mode stabilization in coupled IPS/NIMROD/GENRAY HPC simulations* , 22nd International Conference on Numerical Simulation of Plasmas, Long Branch, New Jersey, September 2011.
- T. G. Jenkins, S. E. Kruger, E. D. Held, R. W. Harvey, D. D. Schnack, W. R. Elwasif, and the SWIM project team, *Coupled IPS/NIMROD/GENRAY simulations of ECCD-induced tearing mode stabilization* , International Sherwood Fusion Theory Conference, Austin, Texas, May 2011.
- T. G. Jenkins, S. E. Kruger, E. D. Held, R. W. Harvey, D. D. Schnack, and the SWIM project team, *First results of coupled IPS/NIMROD/GENRAY simulations* , Annual Meeting of the Division of Plasma Physics, American Physical Society, Chicago, Illinois, November 2010. Abstract published in *Bulletin of the American Physical Society* **55**, 146 (2010).
- T. G. Jenkins, S. E. Kruger, E. D. Held, D. D. Schnack, R. W. Harvey, and the SWIM project team, *ECCD/MHD simulations with NIMROD, GENRAY, and the Integrated Plasma Simulator* , International Sherwood Fusion Theory Conference, Seattle, Washington, April 2010.

Other presentations

A number of other presentations on SWIM-related material, primarily presented at NIMROD, CEMM, and SWIM team meetings, are available at the respective organizational websites linked above or in summary form on Dr. Jenkins' website .

References

[1] C. C. Hegna and J. D. Callen, *Phys. Plasmas* **16**, 112501 (2009).

[2] J. J. Ramos, *Phys. Plasmas* **17**, 082502 (2010).

[3] J. J. Ramos, *Phys. Plasmas* **18**, 102506 (2011).

[4] T. G. Jenkins and S. E. Kruger, Phys. Plasmas **19**, 122508 (2012).

[5] C. B. Barber, D. P. Dobkin, and H. T. Huhdanpaa, ACM Trans. on Mathematical Software, **22**(4):469-483, December 1996, http://www.qhull.org.

[6] R. J. Renka and R. Brown, In Proceedings of ACM Trans. Math. Softw. 1999, 74-77.

[7] S. Ide et al, Proc. 21st IAEA Fusion Energy Conference, EX/P1-5 (2006).