

Superstrings and Quantum Superfields

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Final Year Project Report

In the last years of this eighteen-year grant project, the research efforts have focused mostly on the study of off-shell representations of supersymmetry, both on the worldline and on the world-sheet, *i.e.*, both in supersymmetric quantum mechanics and in supersymmetric field theory in 1+1-dimensional spacetime.

During this period of time, Mr. Gregory A. Katona and Mr. Shawn Eastmond have been recruited to work on the project, contributing to their PhD dissertation work.

Since the last report and within the budget period (06/01/2011–05/31/2013) covered by this report, the research under this grant included research work on the following topics: (1) off-shell supermultiplets of worldline N -supersymmetry [1,2,3], and (2) off-shell supermultiplets in higher-dimensional spacetimes [4], and (3) algebraic structure of common theoretical models [5].

In particular:

Ref. [1]: In collaboration with C.F. Doran, M.G. Faux, S.J. Gates, Jr., K.M. Iga, G.D. Landweber and R.L. Miller, we have completed the classification of the so-called Adinkraic off-shell representations of worldline N -supersymmetry without central charges. Surprisingly, this classification hinges on the classification of binary doubly even linear block error-detecting and error-correcting codes.

Ref. [2]: In collaboration with S.J. Gates, Jr, J. Hallett and K. Stiffler, we showed that a long-known but rarely used complex linear supermultiplet is not adinkraic, cannot be decomposed locally, but may be reduced by means of a Wess-Zumino type gauge and so described as a connected network of Adinkras. This then indicates that the already unexpectedly vast number of adinkraic off-shell supersymmetry representations is but the proverbial tip of the iceberg.

Ref. [3]: In collaboration with G.A. Katona, we study gauge-quotients of direct sums of Adinkras by a supersymmetric image of another Adinkra and thus solve a puzzle from Ref. [6]: The so-defined supermultiplets do not produce Adinkras but more general types of supermultiplets, each depicted as a connected network of Adinkras. Iterating this gauge-quotient construction then yields an indefinite sequence of ever larger supermultiplets, reminiscent of Weyl's construction that is known to produce all finite-dimensional unitary representations in Lie algebras.

Ref. [4]: I show herein that in all supersymmetric models with spacetime of four or more dimensions, this type of twisting, if nontrivial, necessarily maps regular (non-ghost) supermultiplets to ghost supermultiplets. This feature may well be indispensable in fully off-shell models with both supersymmetry and a BV-BRST treatment of constraints.

Ref. [5]: I show that the Hilbert space of the standard linear harmonic oscillator is a complete orbit of the $\mathfrak{osp}(2, 1; 2)$ spectrum-generating superalgebra, and that this is the smallest such algebraic structure. The ubiquitous appearance of the linear harmonic oscillator in virtually all domains of theoretical physics guarantees a corresponding ubiquity of appropriate generalizations of this spectrum-generating superalgebra.

These works rely on previous publications under this grant [7,8,9,10,6,11,12,13,14,15,16,17,18,19, 20,21,22,23,24].

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