# Final Report
# DOE SC0002218 (DOE Award ER25942)

Award to the University of Maryland, College Park, MD:
"Interior-Point Algorithms for Optimization Problems with Many Constraints"
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Covering the period September 2009 through August 2013

## Progress/Accomplishments

In this work, we have built upon our results from previous DOE funding (DEFG 0204ER25655), where we developed new and more efficient methods for solving certain optimization problems with many inequality constraints. This past work resulted in efficient algorithms (and analysis of their convergence) for linear programming, convex quadratic programming, and the training of support vector machines; see, for example, [5]. The algorithms are based on using *constraint reduction* in interior point methods: at each iteration we consider only a smaller subset of the inequality constraints, focusing on the constraints that are close enough to be relevant. Surprisingly, we have been able to show theoretically that such algorithms are globally convergent and to demonstrate experimentally that they are much more efficient than standard interior point methods.

**Extension of Constraint Reduction to Linear/Quadratic Programming with Infeasible Start.** A significant advance was made here in that a provably convergent scheme, based on an exact penalty function, was devised and analyzed that caters to dual-infeasible initial points. This idea involves a conditional increase of the penalty parameter at each iteration, and it is proved that such increase will occur at most finitely many times, producing an appropriate value, which will guarantee convergence to the solution of the original problem.

Our work on allowing *dual-infeasible* starting points with constraint reduction for linear programming is complete [10]. The approach involves an "exact" penalty function scheme; an appropriate value for the penalty parameter is determined adaptively, as the iteration proceeds. The scheme can be applied to an entire class of interior-point methods, and its convergence was proved under weak assumptions. Remarkably strong numerical performance was observed. In fact, surprisingly, when used in conjunction with the acclaimed Mehrotra predictor-corrector algorithm, the penalty scheme leads to significant improvement, sometimes even in the absence of constraint reduction! We observed this on randomly generated problems and more strikingly on a model predictive control problem for the control of a rotorcraft [4].

This idea was generalized for the case of convex quadratic programming [14]; the analysis is underway.

This work was carried out by graduate student Meiyun He, under the supervision of André Tits, in consultation with Dianne O'Leary. It has been the object of several conference presentations [2,3,9,15], and a paper accepted for journal publication [13].

**Semidefinite Programming.**

We focused considerable attention on a more difficult optimization problem, semidefinite programming [6]:

| The Primal Problem | The Dual Problem |
|---|---|
| $\min_{X \in \mathcal{S}^n} C \bullet X$ | $\max_{\substack{y \in \mathcal{R}^m \\ S \in \mathcal{S}^n}} b^T y$ |
| $A_k \bullet X = b_k, \quad k = 1, \ldots, m,$ | $\sum_{k=1}^m y_k A_k + S = C,$ |
| $X \succeq 0,$ | $S \succeq 0.$ |

Here, $\mathcal{S}^n$ is the set of $n \times n$ symmetric matrices, $C \bullet X$ denotes the trace of the product of $C$ and $X$, and $X \succeq 0$ means that $X$ is positive semidefinite.

Our first insight is that our reduction technique for linear and quadratic programming has three interpretations: adaptively reducing

- far-away (dual) inequality constraints,
- (primal) variables that are near zero,
- small terms in the matrix defining the Newton direction.

In semidefinite programming, we can reduce dual constraints if the matrices have block structure.

Graduate student Sungwoo Park investigated this idea with thesis advisor Dianne O'Leary and consultation from André Tits. Sungwoo adapted an algorithm of Florian Potra and Rongqin Sheng to include constraint reduction. He also proved that the resulting algorithm was globally convergent and had polynomial complexity [7,16]. *This is the first constraint-reduced predictor-corrector interior point method to be proven to have this complexity*; even for linear and quadratic programming we had not previously been able to establish polynomial complexity.

**Determining Protein Shape.** A major application of semidefinite programming is in determining protein configurations. Practical measurement techniques (e.g., nuclear magnetic resonance) can give estimates of the distances between atoms that are close to each other in a protein. To determine the configuration of a protein, we must determine the missing distances, by solving the so-called *distance matrix completion problem.* Semidefinite programming is a currently popular way to solve this problem, which is one of the reasons we decided to focus on it. Ironically, in collaboration with Haw-ren Fang (Argonne, now at University of Minnesota), we developed an algorithm that is much faster than semidefinite programming for this class of problems [1,8,11,12].

**Quadratic Programming.**

We provided software using constraint reduction to solve quadratic programming problems to support the research of Cory Hauck (Oak Ridge National Lab).

## Impact

- This grant provided support to PhD student Meiyun He (PhD 2012), PhD student Ming Tse Paul Laiu (work in progress), and PhD student Sungwoo Park (PhD 2012).
- We developed an algorithm for solving the distance matrix completion problem and therefore can determine protein configurations from partial distance measurements. Our algorithm is much faster than previous algorithms. This enables the determination of the structure of proteins with a larger number of atoms than was previously possible, although sensitivity to error in the measurements needs to be studied.
- We developed practical schemes for dealing with the lack of an initial feasible guess in linear programming, overcoming an important limitation of our previous algorithms.
- Our work on semidefinite programming has many important applications: for example,
  - design of detectors and decoders,
  - camera calibration,
  - solution of certain combinatorial problems,
  - minimum-trace factor analysis.
  - solution of linear programming, quadratic programming, quadratic programming with quadratic constraints, and second-order cone programming problems.
- Our new SDP algorithm also does not require a primal- or dual-feasible initial guess and has polynomial complexity and global convergence.
- Sungoo Park completed the requirements for his PhD degree and has taken a position with Knight Investments.
- Meiyun He completed the requirements for her PhD degree and has taken a position with Amazon.
- Sungwoo Park was one of winners of the SIAM Student Paper Prize for 2011. This prestigious award earned him $1000 plus travel expenses to present his work at a SIAM National Meeting.

- Dianne O'Leary mentored high school student Urja Mittal on her science fair project, related to the optimization problems studied in this grant. Her project was "Evaluating and Improving the Efficiency of Mangasarian's Linear Programming Algorithm". She was given the Galileo Circle Award by the Washington Academy of Science and the Blair High School Dr. Vaccaro Award. She is currently an undergraduate student at the University of Pennsylvania.
- Dianne O'Leary was the 2011 Norbert Wiener Lecturer for Tufts University. On March 30 – April 1, she delivered the three lectures, one for a very broad audience, one at the level of a colloquium, and one more specialized. One of these talks was on work from this grant support: "Where am I? Position from Incomplete Distance Information." Past Wiener Lecturers have included Persi Diaconis, Nick Trefethen, Jim Yorke, Margaret Wright, Sigurdur Helgason and Jeff Weeks.

## Publications and Presentations

1. Haw-ren Fang and Dianne P. O'Leary, Euclidean Distance Matrix Completion Problems, *Optimization Methods and Software*, Available on-line: December 2011 DOI:10.1080/10556788.2011.643888
2. Meiyun Y. He, "An Infeasible Constraint-Reduced Interior-Point Method for Linear Programming," presented at the 6th Northeast Control Workshop, held at Johns Hopkins University, Baltimore, MD, April 2010. Abstract available at
   `http://www.ece.jhu.edu/NECW2010/NECW2010Abstracts.pdf`
3. Meiyun Y. He and André L. Tits, "Infeasible Constraint-Reduced Method for Linear Programming", poster presentation at the 2010 DOE Applied Mathematics Program Meeting, Berkeley, CA, May 2010.
   Abstract available at
   `https://outreach.scidac.gov/applmath10/`
4. Meiyun Y. He, Mary Kiemb, André L. Tits, Aaron Greenfield, and Vineet Sahasrabudhe, *Constraint-Reduced Interior-Point Optimization for Model Predictive Rotorcraft Control,* to appear in the Proceedings of the 2010 American Control Conference, Baltimore, MD, June–July 2010.
   `http://www.ece.umd.edu/%7eandre/DOE2010/HKTGA10.pdf`
5. Jin Hyuk Jung, Dianne P. O'Leary, and André L. Tits, "Adaptive Constraint Reduction for Convex Quadratic Programming," *Computational Optimization and Applications*, (March 2010) DOI:10.1007/s10589-010-9324-8 (from previous funding)
6. Dianne P. O'Leary, "Adaptive Reduction for Semidefinite Programming Problems," Presentation at the 2010 DOE Applied Mathematics Program Meeting, May 3-5, 2010, Berkeley, CA. `https://outreach.scidac.gov/applmath10/`

7. Sungwoo Park, "Matrix Reduction in Numerical Optimization," PhD dissertation, Computer Science Department, University of Maryland, 2011.

8. Dianne P. O'Leary, "Where am I? Position from Incomplete Distance Information," Tufts University seminar, March 2011.

9. Meiyun Y. He and André L. Tits, "Infeasible Constraint-Reduced Interior-Point for Linear Optimization," Special Session on Advances in Optimization (in honor of Florian Potra's 60th Birthday) 2011 Spring Southeastern Section Meeting Statesboro, GA, March 2011

10. Haw-ren Fang and Dianne P. O'Leary, "Euclidean Distance Matrix Completion Problems," Special Session on Advances in Optimization (in honor of Florian Potra's 60th Birthday) 2011 Spring Southeastern Section Meeting Statesboro, GA, March 2011

11. Dianne P. O'Leary "Where am I? Position from Incomplete Distance Information," Purdue University seminar, April 2011

12. Dianne P. O'Leary "Where am I? Position from Incomplete Distance Information," Householder Symposium, Lake Tahoe, CA, June 2011.

13. Meiyun Y. He and André L. Tits, "Infeasible Constraint-Reduced Interior-Point Methods for Linear Optimization", to appear in Optimization Methods and Software, 2011. `http://www.optimization-online.org/DB_HTML/2010/11/2822.html`

14. Meiyun Y. He and André L. Tits, "An Infeasible Constraint-Reduced Interior-Point Method for Quadratic Programming", working paper, 2011.

15. Meiyun Y. He and André L. Tits, "Infeasible Constraint-Reduced Interior-Point for Linear Optimization", 2011 SIAM Conference in Optimization, Darmstadt, Germany, May 2011. Abstract available from `http://www.siam.org/meetings/op11/op11abstracts.pdf`

16. Sungwoo Park and Dianne P. O'Leary, "A Polynomial Time Constraint Reduced Algorithm for Semidefinite Optimization Problems," Optimization Online preprint 2013-08-4011, `http://www.optimization-online.org/DB_HTML/2013/08/4011.html` submitted for journal publication August 2013.