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Title: Detection Limits for Special Nuclear Material

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DETECTION LIMITS FOR SPECIAL NUCLEAR MATERIAL

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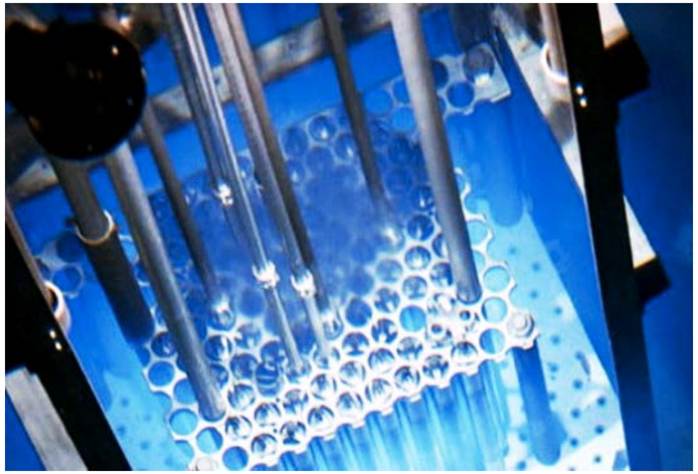
OVERVIEW

- Introduction
- What is special nuclear material (SNM)?
- Why detect SNM?
- How well can SNM be detected?
- How are detection limits calculated?
- Examples
- Summary

WHAT IS SPECIAL NUCLEAR MATERIAL (SNM)?

- According to Title I of the Atomic Energy Act of 1954, SNM consists of:
  - Plutonium
  - Uranium-233
  - Uranium enriched in the isotopes uranium-233 or uranium-235



- According to the International Atomic Energy Agency, SNM includes:
  - Plutonium
  - Uranium (depleted uranium (DU), low enriched uranium (LEU), high enriched uranium(HEU))
  - Thorium
  - Spent nuclear fuel
  - Neptunium-237 (E≥20%)

WHY DETECT SNM?

- To prevent the spread of nuclear material and weapons
  - Treaty verification
  - IAEA safeguards
- To track material in a material balance area
  - Material accountancy
  - Criticality safety
- To support national security
  - Port and border security
  - Terrorism

HOW WELL CAN SNM BE DETECTED?

- Detection limits
  - Can be calculated statistically
  - Depend on multiple factors
    - Type of SNM (e.g. metal, oxide, U vs. Pu)
    - Signatures used (e.g. gamma, neutron)
    - Detector properties (e.g. efficiency, resolution, size)
    - Background noise
    - Measurement geometry (e.g. distance between SNM and detector)
    - Presence of shielding
    - Measurement time
  - Impact ability to detect the presence of SNM and/or characterize its properties
  - Need measurement to be optimized
    - Minimize statistical uncertainty
    - Obtain best division of time for counting
    - Minimize background effects

HOW ARE DETECTION LIMITS CALCULATED?

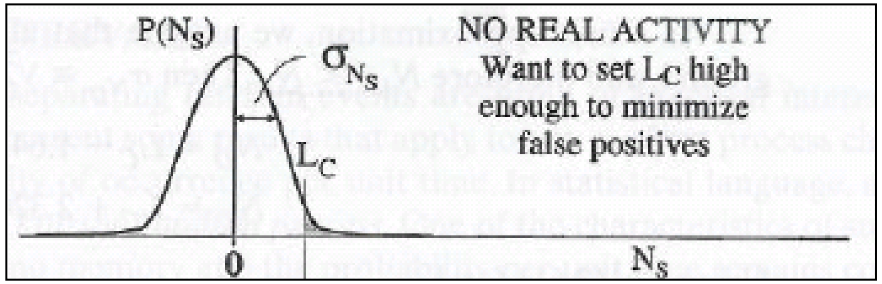
- First, a counting experiment should be optimized to ensure that the maximum number of counts is being obtained.
- The source count rate S is usually measured as $S = \frac{N_S+N_B}{T_{S+B}} - \frac{N_B}{T_B} = T - B$ where N_B is the number of background counts, T is the total count rate, B is the background count rate, and T_{S+B} and T_B are the respective measurement times
- The error associated with the above equation is $\sigma_S = \sqrt{\frac{S+B}{T_{S+B}} + \frac{B}{T_B}}$ and should be minimized
- The optimum ratio of measurement times (i.e. that minimizes error) is $\left.\frac{T_{S+B}}{T_B}\right|_{opt} = \sqrt{\frac{S+B}{B}}$
- The figure of merit is $\frac{1}{T} = \frac{\epsilon^2 S^2}{\left(\sqrt{S+B}+\sqrt{B}\right)^2}$, where $\epsilon = \frac{\sigma_S}{S}$
  - Used to compare experimental setups
  - Represents the fractional standard deviation of the source rate
  - Assumes optimal ratio of measurement times used
  - When S<<B, then $\frac{1}{T} = \frac{\epsilon^2 S^2}{4B}$ and the ratio $\frac{S^2}{B}$ needs to be maximized

HOW ARE DETECTION LIMITS CALCULATED?

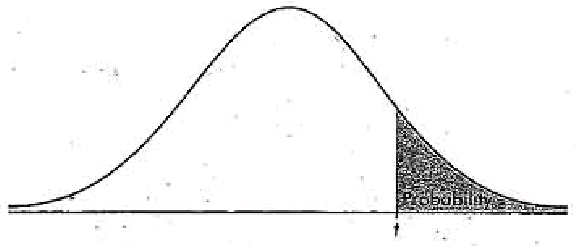
- With an experiment optimized, it is possible to determine the minimum detectable activity (MDA) which acts as the limit of detection
  - A critical source level L_C is set for the detection system in order to discriminate between a false-positive signal (e.g. a shipping container with bananas) and a true positive signal (e.g. a shipping container with SNM), while minimizing the likelihood of missing a true positive signal (a “false-negative”).
  - 5% false-negative rate and 95% true-positive rate usually selected
- Remember that the net counts for the source S can be found from $N_S = N_T - N_B$ and its standard deviation can be found from ${\sigma_{N_S}}^2 = {\sigma_{N_T}}^2 + {\sigma_{N_B}}^2$
- Two cases
  - Case 1: No source present (just background)
  - Case 2: Source present (source + background)

HOW ARE DETECTION LIMITS CALCULATED?

- Case 1: No source present (just background)
  - With no source present, any alarm will be a false-positive signal; thus L_C needs to be high enough to minimize this occurrence
  - When there is no source present, $N_S = 0 \rightarrow N_T = N_B \rightarrow \sigma_{N_T} = \sigma_{N_B}$
    - Plugging into ${\sigma_{N_S}}^2 = {\sigma_{N_T}}^2 + {\sigma_{N_B}}^2$ from the previous slide, $\sigma_{N_S} = \sqrt{2{\sigma_{N_B}}^2} = \sqrt{2}\sigma_{N_B}$
  - The Gaussian probability that there is a false-positive rate of 5% signifies that 95% of the counts lie below L_C (the mean plus 1.645σ)
    - This implies that 5% of the counts are in the right-hand tail
    - Therefore, using a t-table, $L_C = 1.645\sigma_{N_S} = 1.645\sqrt{2}\sigma_{N_B} = 2.326\sigma_{N_B}$



Knoll, p. 95



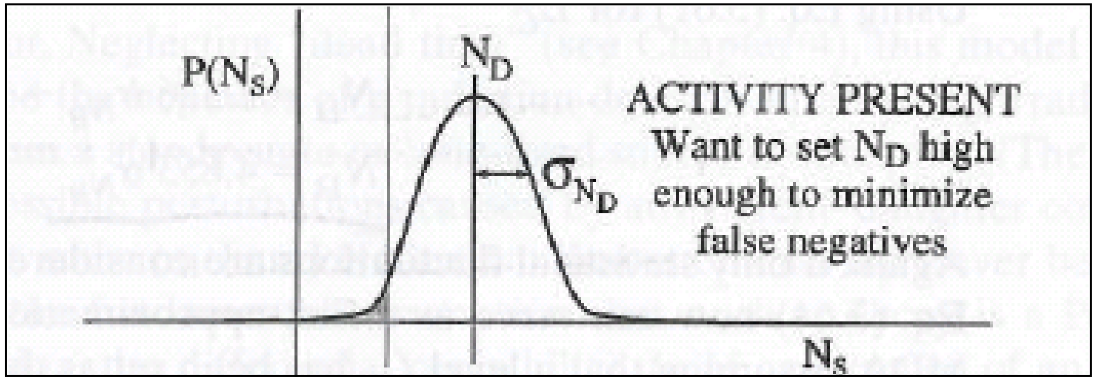
UF Dept. of Statistics

TABLE B: t-DISTRIBUTION CRITICAL VALUES

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	$\boxed{1.645}$	1.960	2.054	2.326	2.576	2.807	3.091	3.291

HOW ARE DETECTION LIMITS CALCULATED?

- Case 2: Source present (source + background)
  - With a source present, any negative will be a false-negative signal
  - Let N_D be the mean number of counts that yields a false-negative rate of 5%
    - This means that 95% of the N_D distribution lies above L_C (the mean minus 1.645σ)
    - $N_D = L_C + 1.645\sigma_{N_D}$
    - If we assume that $N_D \ll N_B$, then $\sigma_{N_D} = \sqrt{2}\sigma_{N_B}$
    - Substitute into the first equation to yield $N_D = 1.645\sqrt{2}\sigma_{N_B} + 1.645\sqrt{2}\sigma_{N_B}$ → $N_D = 2(2.326\sigma_{N_B})$ → $N_D = 4.652\sigma_{N_B}$



Knoll, p. 95

HOW ARE DETECTION LIMITS CALCULATED?

- Thus, for a 5% false-negative rate and 95% true-positive rate, the minimum detectable counts is $N_D = 4.652\sigma_{N_B} = 4.652\sqrt{N_B}$
  - Want this value to be as low as possible
- Applying this to a detection system implies that detector parameters must be included in the calculation
  - Minimum detectable activity (MDA)
    - $\alpha = \frac{N_D}{f\epsilon T} = \frac{4.652\sqrt{N_B}}{f\epsilon T_{S+B}}$, where f is the yield per disintegration, ϵ is the absolute detection efficiency, and T is the counting time for each sample
    - Want this value to be as low as possible

EXAMPLES

- Question 1: You are an inspector for the IAEA. You have just been alerted that a sample of HEU has gone missing, and you need to find it. You have been given a NaI(Tl) handheld detector (efficiency of 20%) and told to walk slowly around the site until you find the sample. Before you begin, you take a background count for 60 s, which yields a net peak count of 1,000 counts in the 186 keV U-235 gamma line (branching ratio of 57%).
  - Part A: If you need 10,000 counts to limit relative uncertainty to 1%, how long will you have to count to have a minimum error?
  - Part B: If it is a 1 Bq source, will you be able to find it?
  - Part C: How does the error compare if you use $T_{S+B} = 100\ s$?

- Question 2: Refer back to the equations for minimum detectable counts and activity.
  - Part A: Derive N_D for a 1% false-positive and a 1% false negative rate.
  - Part B: Re-answer the above questions. How do the values change?

QUESTION 1

- Part A: If you need 10,000 peak counts to limit relative uncertainty to 1%, how long will you have to count in the laboratory?

$$\left.\frac{T_{S+B}}{T_B}\right|_{opt} = \sqrt{\frac{S+B}{B}} = \sqrt{\frac{{N_{S+B}}/{T_{S+B}} + {N_B}/{T_B}}{{N_B}/{T_B}}} = \sqrt{\frac{{N_{S+B}}/{T_{S+B}}}{{N_B}/{T_B}} + 1}$$

$$\frac{T_{S+B}}{60\ s} = \sqrt{\frac{{10{,}000\ counts}/{T_{S+B}}}{{1000\ counts}/{60\ s}} + 1}$$

$$T_{S+B} = 138.5\ s$$

- Part B: If it is a 1 Bq source, will you be able to find it?

$$\alpha = \frac{N_D}{f\epsilon T} = \frac{4.652\sqrt{N_B}}{f\epsilon T_{S+B}}$$

$$\alpha = \frac{4.652\sqrt{1000\ counts}}{(0.57 * 0.9\ wt\%\ U - 235)(0.2)(138.5\ s)}$$

$\alpha = 10.4\ Bq > 1.0\ Bq$ → No!

QUESTION 1

- Part C: How does the error compare if you use $T_{S+B} = 100\ s$?

$$\sigma_S = \sqrt{\frac{S+B}{T_{S+B}} + \frac{B}{T_B}} = \sqrt{\frac{N_{S+B}}{(T_{S+B})^2} + \frac{N_B}{(T_B)^2}}$$

  - For an optimized T_{S+B}=138.5 s:

$$\sigma_S = \sqrt{\frac{10{,}000\ counts}{(138.5\ s)^2} + \frac{1000\ counts}{(60\ s)^2}} = 0.9\ cps$$

  - For an unoptimized T_{S+B}=100 s:

$$\sigma_S = \sqrt{\frac{10{,}000\ counts}{(100\ s)^2} + \frac{1000\ counts}{(60\ s)^2}} = 1.2\ cps$$

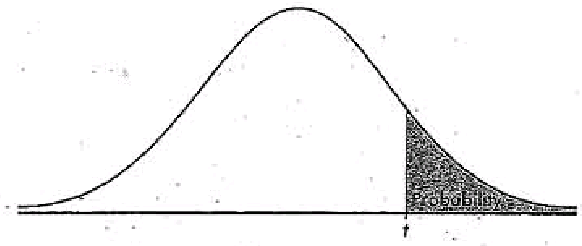
QUESTION 2

- Part A
  - Case 1: No source present (just background)
    - $L_C = 2.326\sigma_{N_S} = 2.326\sqrt{2}\sigma_{N_B} = 3.289\sigma_{N_B}$
  - Case 2: Source present (source + background)
    - $N_D = L_C + 2.326\sigma_{N_D} = 2.326\sqrt{2}\sigma_{N_B} + 2.326\sqrt{2}\sigma_{N_B} \rightarrow N_D = 2(3.289\sigma_{N_B}) \rightarrow N_D = 6.579\sigma_{N_B}$
  - Minimum detectable activity (MDA)
    - $\alpha = \frac{N_D}{f\epsilon T} = \frac{6.579\sqrt{N_B}}{f\epsilon T_{S+B}}$
- Part B
  - $T_{S+B} = 138.5\ s$ (this is unaffected)
  - MDA increases as accuracy increases:

$$\alpha = \frac{N_D}{f\epsilon T} = \frac{4.652\sqrt{N_B}}{f\epsilon T_{S+B}}$$

$$\alpha = \frac{6.579\sqrt{1000\ counts}}{(0.57 * 0.9\ wt\%\ U-235)(0.2)(138.5\ s)}$$

$\alpha = 14.6\ Bq > 1.0\ Bq \rightarrow$ No!



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SUMMARY

- SNM is regulated nationally and internationally, and its detection is important for a variety of reasons
- Many parameters affect detection limits, including the type of SNM, signatures used, detector properties, background noise, measurement geometry, presence of shielding, and measurement time
- Optimizing an experiment produces lower error and ensures maximum counts are being seen by the detector
- Limits of detectability and the minimum detectable activity should be determined for a detector system if unknown sources are being characterized with the system (e.g. search and recover a lost source, port security, treaty verification)
  - Background has the biggest effect on this calculation, followed detector and material parameters

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ANY
QUESTIONS
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