# 6 Radiation Protection and Health Physics

## 6.1 Definitions and Equations

### 6.1.1 Dose concepts

**Equivalent dose**

Equivalent dose is defined as

$$H_{\mathrm{T,R}} = w_{\mathrm{R}} D_{\mathrm{T,R}} \tag{6.1.1}$$

where $w_{\mathrm{R}}$ is the radiation weighting factor and $D_{\mathrm{T,R}}$ is the mean absorbed dose from radiation $R$ in tissue or organ $T$. Numerical values of $w_{\mathrm{R}}$ are given in Table 6.1 (ICRP, 2007).

The total equivalent dose, $H_{\mathrm{T}}$, is the sum of $H_{\mathrm{T,R}}$ over all radiation types,

$$H_{\mathrm{T}} = \sum H_{\mathrm{T,R}} \text{ Unit : sievert, } 1\,\mathrm{Sv} = 1\,\mathrm{J\,kg^{-1}} \tag{6.1.2}$$

**Table 6.1:** ICRP recommended radiation weighting factors (ICRP, 2007).

| Radiation type | Radiation weighting factor, $w_{\mathrm{R}}$ |
|---|---|
| Photons | 1 |
| Electrons and muons | 1 |
| Protons and charged pions | 2 |
| Alpha particles, fission fragments, heavy ions | 20 |
| Neutrons | A continuous function of neutron energy |

**Effective dose**

Effective dose is defined as

$$E = \sum w_{\mathrm{T}} H_{\mathrm{T}} \text{ Unit : sievert, } 1\,\mathrm{Sv} = 1\,\mathrm{J\,kg^{-1}} \tag{6.1.3}$$

where $w_{\mathrm{T}}$ is the tissue weighting factor and $H_{\mathrm{T}}$ is the equivalent dose in a tissue or organ. Numerical values of $w_{\mathrm{T}}$ are given in Table 6.2

Summing all tissues in the body will give $\Sigma w_{\mathrm{T}} = 1$

**Committed equivalent dose**

Committed equivalent dose is defined as

$$H_{\mathrm{T}}(\tau) = \int_{t_0}^{t_0+\tau} \dot{H}_{\mathrm{T}}(t)\,\mathrm{d}t \tag{6.1.4}$$

**Table 6.2:** ICRP recommended tissue weighting factors (ICRP, 2007).

| Tissue | Tissue weighting factor, $w_T$ | $\sum w_T$ |
|---|---|---|
| Bone marrow (red), colon, lungs stomach, breast, remainder tissues | 0.12 | 0.72 |
| Gonads | 0.08 | 0.08 |
| Bladder, oesophagus, liver, thyroid | 0.04 | 0.16 |
| Bone surface, brain, salivary glands, skin | 0.01 | 0.04 |
| Total | | 1.00 |

Remainder tissues: Adrenals, extrathoracic region, gall bladder, heart, kidneys, lymphatic nodes, muscle, oral mucosa, pancreas, prostate, small intestine, spleen, thymus, uterus/cervix.

where $t_0$ is the time of intake, $\dot{H}_T(t)$ is the equivalent dose rate at time $t$ in organ or tissue $T$ and $\tau$ is the time elapsed after time of intake.

If $\tau$ is not specified it is taken as 50 y for adults and 70 y for children.

**Committed effective dose:**

Committed effective dose is defined in a similar way as

$$E(\tau) = \sum w_T H_T(\tau) \tag{6.1.5}$$

**Collective effective dose**

The collective effective dose, due to individual effective dose values between $E_1$ and $E_2$ from a specified source within a specified time period $\Delta T$, is defined as

$$S(E_1, E_2, \Delta T) = \int_{E_1}^{E_2} E[\frac{dN}{dE}]_{\Delta T}\, dE \tag{6.1.6}$$

It can be approximated as $S = \Sigma_i E_i N_i$ where $E_i$ is the average effective dose for a subgroup $i$ and $N_i$ is the number of individuals in this subgroup. The unit of the collective effective dose is Joule per kilogram and the special name is person-sievert.

**Annual limit on intake (ALI)**

ALI is the value of $I$ that satisfies the following inequality

$$I \sum w_T H_{50,T} \le 0.02\,\text{Sv} \tag{6.1.7}$$

$H_{50,T}$ is the committed equivalent dose for $\tau$=50 y. This means that if a person has an intake of activity corresponding to an ALI for 50 y, the equivalent dose year 50 will not be larger than 0.02 Sv, which fulfills the ICRP recommendation.

**Derived air concentration (DAC)**

DAC is defined as the concentration in air that would result in an activity inhalation of an ALI

$$DAC = \frac{ALI}{2000 \cdot 1.2}\ \mathrm{Bq\,m^{-3}} \tag{6.1.8}$$

where it is assumed that a person (Reference Man) works 2000 h per year and inhales 1.2 $m^3$ air per h.

### 6.1.2 Transport of radionuclides in the body

**I. Single uptake and exponential excretion.**
The activity in the body after a single intake and exponential excretion is given by

$$A(t) = A_0 e^{-\lambda_{\mathrm{eff}} t} \tag{6.1.9}$$

where $A_0$ is the initial uptake of activity, $\lambda_{\mathrm{eff}} = \lambda_{\mathrm{b}} + \lambda_{\mathrm{f}}$ (effective decay constant), $\lambda_{\mathrm{b}}$ is the biological decay constant and $\lambda_{\mathrm{f}}$ is the physical decay constant.

The relation can be rewritten including the half lives instead. Then

$$A(t) = A_0 e^{(-\ln 2t/T_{\mathrm{eff}})} \tag{6.1.10}$$

$$T_{\mathrm{eff}} = \frac{T_{\mathrm{b}} T_{\mathrm{f}}}{T_{\mathrm{b}} + T_{\mathrm{f}}} \tag{6.1.11}$$

**II. Single uptake and general excretion equation.**
The variation of activity in the body after a single intake and with a general excretion equation is given by

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = -E(t) - \lambda_{\mathrm{f}} A(t) \tag{6.1.12}$$

where $E$(t) is the excreted activity per time unit at time $t$ and $A(t)$ is the activity in the body after time, $t$.

The relative decrease in activity is then given by

$$\frac{\mathrm{d}R(t)}{\mathrm{d}t} = \frac{\mathrm{d}(\frac{A(t)}{A_0})}{\mathrm{d}t} = \frac{-E(t)}{A_0} - \lambda_{\mathrm{f}} \frac{A(t)}{A_0} = -Y(t) - \lambda_{\mathrm{f}} R(t) \tag{6.1.13}$$

where $R(t)$ is the fraction of initial activity, $A_0$, that is remaining in the tissue at time $t$ and $Y(t)$ is the fraction of the initial activity that is excreted per time unit at time $t$.

$R(t)$ and $Y(t)$ do not include the physical decay and can thus be used for different radioactive isotopes of the same element.

**III. Continuous contamination of the radionuclide.**
The activity in the tissue at time $t$ is given by the relation

$$A(t) = I \int_0^t R(t) e^{-\lambda_{\mathrm{f}} T\,\mathrm{d}T} \tag{6.1.14}$$

where $I$ is the intake/time unit.

**IV. Cumulated activity.**
By integrating the activity $A(t)$ over time, the cumulated activity $\tilde{A}$ is obtained.

$$\tilde{A} = \int_0^t A(\tau)\,\mathrm{d}\tau \tag{6.1.15}$$

If $A(t)$ is decreasing exponentially with time then

$$\tilde{A} = \frac{A_0}{\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}} t}) \tag{6.1.16}$$

If $t \to \infty$ then

$$\tilde{A} = \frac{A_0}{\lambda_{\text{eff}}} = \frac{A_0 T_{\text{eff}}}{\ln 2} \tag{6.1.17}$$

This approximation holds when $t$ is much larger than the effective half life.

The absorbed dose rate and absorbed dose is obtained by multiplying the activity or the cumulated activity with a factor $S$ Gy/(Bqs), giving the relation between absorbed dose rate and activity or absorbed dose and cumulated activity. The factor $S$ is depending on type and energy of radiation, and the size and shape of the organ. This factor can also be used to calculate the absorbed dose due to activity in one organ to other organs, and then also the distance between the organs will be of interest. MIRD (Medical Internal Radiation Dose Committee) has calculated and tabulated data for $S$ for different organs and human phantoms corresponding to different sizes, from babies to adults. These tables and other publications from MIRD can be downloaded from http://interactive.snm.org/.

### 6.1.3 Radiation shielding calculations

**Radiation sources**

Radiation sources are characterized by their strength and geometry. The simplest source is the point source and the other sources can be obtained by a summation of point sources. Normally one can divide the sources into the following categories:

Point source $S$ (Bq)
Line source $S_L$ (Bq m$^{-1}$)
Area source $S_A$ (Bq m$^{-2}$)
Volume source $S_V$ (Bq m$^{-3}$)

Instead of defining the source strength in Bq, it is often defined as the number of particles, in our cases mainly photons, emitted per second. In the solutions discussed in this material, the definitions using Bq are used. Then it is important to multiply with the number photons per decay to obtain the fluence rate. As often photons of different energies are emitted, separate calculations have to be performed for each energy. The energy fluence rate is obtained by multiplying the fluence rate for each energy with the respective energy. This means that all calculations are a sum of calculations for the different energies.

**Primary photons**

**Plane parallel beams**. The attenuation of the fluence rate of a plane parallel beam passing through an absorber with thickness $x$ and the linear attenuation coefficient $\mu$ is obtained through the relation

$$\dot{\Phi}(x) = \dot{\Phi}(0)e^{-\mu x} \tag{6.1.18}$$

This holds if only primary photons are included in the calculations. This can be assumed to hold for a narrow well collimated beam, both in front of and behind the absorber (see Fig. 6.1). This situation is sometimes called narrow beam or good geometry.

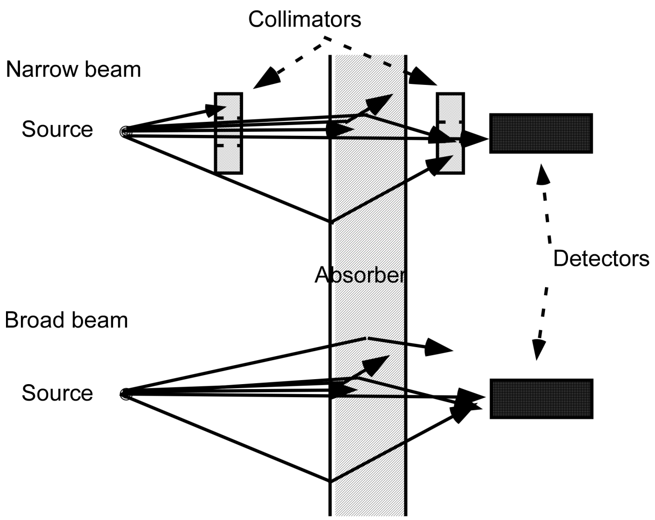


**Figure 6.1:** Transmission of photons trough a shielding material in “narrow” and “ broad” beam conditions.

*6.1.3.0.1 Point source.*
With a point source there will, besides the attenuation, be a decrease of the fluence rate due to the divergence of the beam. Sometimes the quantity

$$\frac{e^{-\mu r}}{4\pi r^2} \tag{6.1.19}$$

is called the *point kernel*, as it describes the variation of the fluence rate with the distance from the source. As all sources can be assumed to consist of an infinite number of point sources, the fluence rate from any source can be calculated using the point kernels.

The fluence rate of the primary photons at a distance $r$ from a point source with the source strength $S$ (Bq), and $f$ mono-energetic photons per decay, in an absorbing medium with the attenuation coefficient $\mu$, is obtained through the relation

$$\dot{\Phi}(r) = \frac{Sf}{4\pi r^2} e^{-\mu r} \tag{6.1.20}$$

**Secondary photons**
**Build-up factor**. Normally secondary photons produced by interaction of the primary photons have to be included in the calculations (see Fig. 6.1). These secondary photons are mainly Compton photons, but to a smaller extent also annihilation photons and fluorescence x rays. Calculation of the secondary photon fluence rate for a certain geometry is complex and often simplifications are made. Often the primary fluence is multiplied with a build up factor, $B$, that is defined as

$$B = 1 + \frac{\text{contribution from secondary photons}}{\text{contribution from primary photons}} \tag{6.1.21}$$

This can for an isotropic, monoenergetic point source in a medium be written as

$$B_k(r, E_0) = \frac{\int_0^{E_0} f_k(E)\Phi(r, E)\,dE}{f_k(E_0)\Phi_0(r, E_0)} \tag{6.1.22}$$

The factor $f_k$ defines which effect that the build-up factor is defined for. If $f_k$ is equal to unity, then the buildup factor is related to the fluence. If it is equal to $E$, then it is related to the energy fluence and if it is equal to $\mu_{en}E$ it is related to the absorbed dose and so on. $\Phi_0$ and $\Phi$ are the fluences of the primary and the total number of particles respectively.

The build up factor $B$ is a function of several parameters:

a) Photon energy. $B$ increases generally with decreasing photon energy in elements with low atomic numbers, where the Compton effect dominates down to low energies. For elements with a high atomic number, $B$ will instead decrease at low

**Table 6.3:** Ratio between the build-up factor in a finite and an infinite medium. Data from Berger and Doggett (Berger and Doggett, 1956).

| | | $[B(\mu x, \mu x])$-1]/[B($\mu x, \infty$])-1] | | |
|---|---|---|---|---|
| Material | Photon energy (MeV) | $1\mu x$ | $4\mu x$ | $16\mu x$ |
| Water | 0.66 | 0.66 | 0.78 | 0.78 |
| | 1.0 | 0.72 | 0.82 | 0.83 |
| | 4.0 | 0.89 | 0.92 | 0.93 |
| Lead | 0.66 | 0.95 | 0.98 | 0.98 |
| | 1.0 | 0.98 | 0.99 | 0.99 |
| | 4.0 | 0.99 | 0.99 | 0.99 |

photon energies as the photo-electric effect, where most of the photon energy is transferred to the electron, will start to dominate.

b) Atomic number of the absorber. $B$ decreases with increasing atomic number, as the probability for both the photo-electric effect and the pair production increases with increasing atomic number, giving rise to less secondary photons.

c) Penetration depth. $B$ increases continuously with increasing value of the penetration depth, $\mu x$.

d) Geometry. Normally two geometries are discussed, a plane parallel infinite beam or a point source in an infinite medium. Most tables and equations thus give data for $B$ in an infinite extension of the absorber. This generally does not correspond to the real geometry and $B$ overestimates the contribution from secondary photons, mainly due to the backscattered photons, which should not be included in calculations of the fluence rate behind an absorber. The difference between the correct and the calculated fluence rate using these values of $B$ is largest for low energies and low atomic numbers. The difference can be as large as up to 30%. Table 6.3 gives relations between the build up factor in a finite medium, $B(\mu x, \mu x)$ and an infinite medium $B(\mu x, \infty)$.

e) Quantity studied. The build-up factor is different depending on which quantity that is studied, fluence, energy fluence, absorbed dose and so on. The fluence build-up factor is larger than the buildup factor for the energy fluence as the energy of the secondary photons is lower than for the primary photons. It is thus important to use buildup factors for the quantity that is of interest.

Data for build-up factors can be found tabulated for some simple geometries in many textbooks and compilations on radiation shielding.

There are also some empirical equations where the parameters are fit to values calculated by analytical methods, that may be used for obtaining the buildup factor.

One example is the Taylor expression.

$$B(E_0, \mu r) = Ae^{-\alpha_1 \mu r} + (1 - A)e^{-\alpha_2 \mu r} \tag{6.1.23}$$

where A, $\alpha_1$ and $\alpha_2$ are parameters that vary with energy and medium.

Another commonly used expression is the Berger expression (Berger, 1956; Chilton, 1968).

$$B(E_0, \mu r) = 1 + a\mu r e^{b\mu r} \tag{6.1.24}$$

where the parameters $a$ and $b$ vary more slowly with energy than the parameters for the Taylor expression. Another advantage with this expression is that the contribution from secondary particles is separate.

Data for the parameters for the Berger expression are e.g. found in Tables A4:9 and A4:10 in Chilton et al (Chilton et al, 1984).

In some situations it is more practical to use experimental obtained transmission data. This is often the case when determining the shielding in diagnostic radiology and in therapeutic treatment rooms, where the approximation of infinite beams is inappropriate.

When more accurate calculations are needed the best method is to use the Monte Carlo method, where the real irradiation geometry can be simulated. These calculations may however be time consuming and in radiation protection a high accuracy is not always needed.

#### *6.1.3.0.2 Shielding including secondary photons.*

When including the secondary photons, the equation to calculate the fluence rate, is for a plane parallel beam given by

$$\dot{\Phi}(x) = \dot{\Phi}(0)B(\mu x)e^{-\mu x} \tag{6.1.25}$$

The corresponding equation for a point source is given by

$$\dot{\Phi}(r) = \frac{SfB(\mu r)}{4\pi r^2}e^{-\mu r} \tag{6.1.26}$$

Thus the total fluence is simply obtained by multiplying the relation for the primary fluence with the buildup factor $B$. Note that the buildup factor takes into consideration the different energies of the secondary and primary photons if needed. It is, as already mentioned, important to use the correct factors depending on the quantity one is interested in.

#### *6.1.3.0.3 Reflexion coefficient of radiation, Albedo.*

In many situations the backscattered radiation is of interest. If a medium has the shape of an infinite slab and radiation is incident on one of its sides, the reflexion coefficient or the Albedo factor is defined as the ratio of the amount of radiation reflected from the slab to the amount of radiation incident on the slab. The definition assumes that the reflected radiation is emitted from the same point as where it entered the slab (See Fig. 6.2), even if the reflexion is not a surface effect, but the interactions are at different depths in the slab.

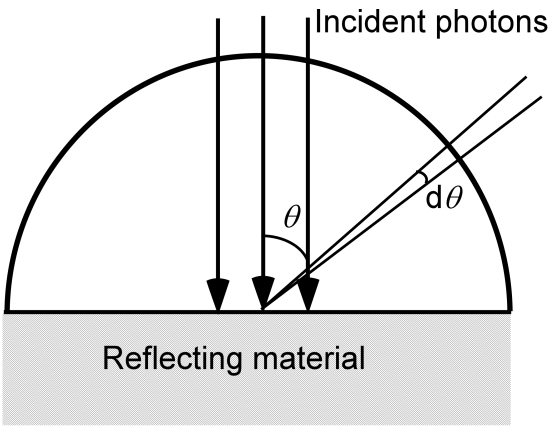


**Figure 6.2:** Reflexion of photons from a surface.

The reflexion coefficient for the number of particles may be defined as

$$R_N = 2\pi \int_0^{\pi/2} \int_0^{E_0} N(\theta, E) \sin\theta \, \mathrm{d}\theta \, \mathrm{d}E \tag{6.1.27}$$

where $(N(\theta, E))$ is the relative number of particles reflected differential in angle and energy.

**Extended sources**

**Fluence rate from line sources.** The fluence rate for primary photons from a line source at a point $P$ (See Fig. 6.3) and a total attenuation thickness of $\Sigma\mu_i x_i$ is given by

$$\dot{\Phi}_{\mathrm{P,L}} = \frac{S_{\mathrm{L}} f}{4\pi h} [F(\theta_2, \Sigma\mu_i x_i) + F(\theta_1, \Sigma\mu_i x_i)] \tag{6.1.28}$$

where $F$ is the Sievert integral defined as

$$F(\Theta, \mu x) = \int_0^{\Theta} e^{-\mu x/\cos\theta} \, \mathrm{d}\theta \tag{6.1.29}$$

Table 6.4 gives data for $F(\theta, \Sigma\mu_i x_i)$.

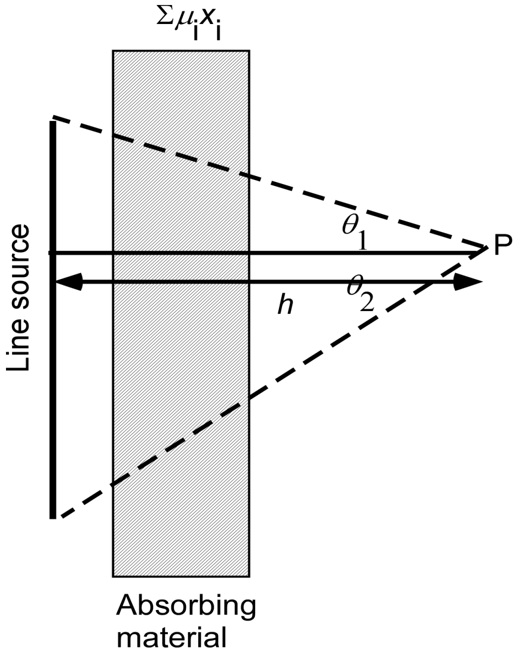


**Figure 6.3:** Line source with an absorbing material.

**Fluence rate from area sources.** The fluence rate of primary photons from a plane circular area to a point P (See Fig. 6.4), centrally positioned above the area at a height $h$ and with a total attenuation thickness of $\Sigma\mu_i x_i$ is given by

$$\dot{\Phi}_{\mathrm{P,A}} = \frac{S_A f}{2}[E_1(\Sigma\mu_i x_i) - E_1(\theta, \Sigma\mu_i x_i \sec\theta)] \tag{6.1.30}$$

When the radius of the area goes to infinity the second factor goes to zero and the relation becomes

$$\dot{\Phi}_{\mathrm{P,A}} = \frac{S_A f}{2} E_1(\Sigma\mu_i x_i) \tag{6.1.31}$$

$E_1(\Sigma\mu_i x_i)$ is the exponential integral of the first order obtained from the general relation

$$E_n(x) = x^{n-1} \int_x^\infty \frac{e^{-y}\,\mathrm{d}y}{y^n} \tag{6.1.32}$$

Observe that for an infinite area, the height over the surface is not included in the equation, and it is only the attenuation thickness that is of interest. For areas not circular but more or less symmetric it is possible to approximate the real area with a circular area of the same size. Table 6.5 gives values of the exponential integral functions $E_1(x)$ and $E_2(x)$.

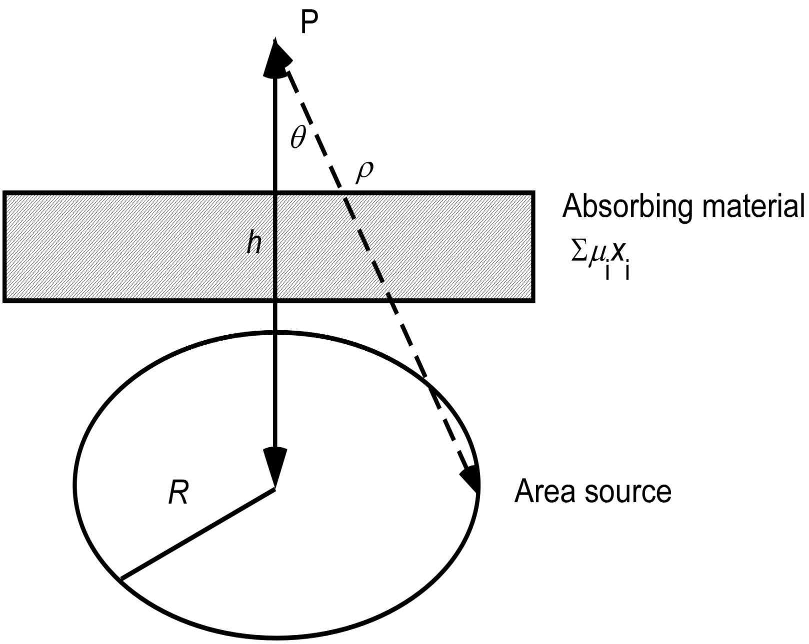


**Figure 6.4:** Area source with absorbing material.

**Fluence rate from volume sources.** The fluence rate at a point $P$ from a semi-infinite volume source with a thickness $d$ (See Fig. 6.5), attenuation coefficient $\mu_s$ and an absorbing material with an attenuation thickness $\Sigma\mu_i x_i$ is obtained by dividing the volume source into several thin slices and then integrate the fluence from each slice. The fluence rate of primary photons is then obtained as

$$\dot{\Phi}_{\mathrm{P,V}} = \frac{S_V f}{2\mu_{\mathrm{s}}}[E_2(\Sigma\mu_i x_i) - E_2(\Sigma\mu_i x_i + \mu_{\mathrm{s}} d)] \tag{6.1.33}$$

If the volume source can be assumed to have an infinite thickness then the equation is simplified to

$$\dot{\Phi}_{\mathrm{P,V}} = \frac{S_V f}{2\mu_{\mathrm{s}}} E_2(\Sigma\mu_i x_i) \tag{6.1.34}$$

If instead there is no absorbing material between the source and the calculation point P the equation becomes

$$\dot{\Phi}_{\mathrm{P,V}} = \frac{S_V f}{2\mu_{\mathrm{s}}}[1 - E_2(\Sigma\mu_{\mathrm{s}} d)] \tag{6.1.35}$$

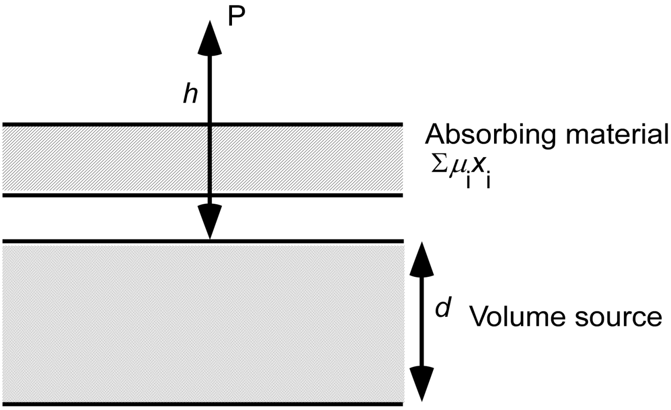


**Figure 6.5:** Volume source with absorbing material.

#### 6.1.3.1 Radiation shielding at radiotherapy installations

When installing a high energy radiation treatment facility, the protection of both patients and staff need to be considered. In this section only the determination of the shielding of the walls of the treatment room will be discussed. For further discussion see e.g. ICRP 33 (ICRP, 1982) and NCRP 151 (NCRP, 2005). Three main components have to be included in the discussion, primary radiation, leakage radiation and scattered radiation. Production of neutrons and induced activity will not be considered here. In the discussion it is important to consider the type of areas that shall be protected, as the acceptable permissible dose depends on the group of people to be protected (i.e. the general public or staff). The national recommendations should be followed here. Typical values could be a weekly dose equivalent of 0.02 mSv/week for uncontrolled areas and 0.1 mSv/week for controlled areas NCRP 151 (NCRP, 2005).

Most accelerator installations are isocentric. This will imply that two walls will not be hit by the primary beam, but only by leakage and scattered radiation. Thus the thickness of these walls may be reduced. Determination of the protective barriers for these three components will be discussed separately below.

**Primary radiation**. When determining the necessary shielding for walls which are directly hit by the beam the relation

$$B_{\mathrm{P}} = \frac{Pd^2}{WUT} \tag{6.1.36}$$

is sometimes proposed as a starting point (ICRP, 1982). $B$ is the transmission through the barrier to be determined, $P$ is the is the acceptable equivalent dose, $d$ is the distance from the source to the location of interest, $W$ is the work load (absorbed dose at 1 m), $U$ is the use factor (fraction of the treatments directed to the wall of interest), $T$ is the occupancy factor (taking into consideration of how much the space behind the barrier is used). The calculation time can be either per week or per year. It is important when deciding of the different factors, to realize that they may change in the future due to changes both in the use of the building and to changes in treatment methods. Thus it is important to use conservative assumptions and in practice the doses are often far below the regulations.

To determine the necessary wall thickness for a certain B, either half-value thickness (HVT), tenth-value thickness (TVT) or experimental transmission curves may be used (ICRP, 1982) as these beams have to be considered as broad beams.

**Scattered radiation**. In a similar way the relation for scattered radiation can be expressed as

$$B_S = \frac{Pd_s^2}{WTS} \tag{6.1.37}$$

$P$ and $T$ are the same and $W$ is the same unless the distance between source and scatterer is not 1 m. $d_s$ is the distance from scattering object (patient) to the location of interest. $S$ is the fraction of incident absorbed dose rate scattered to 1 m. When determining barrier thickness for secondary radiation it is important to remember that the scattered radiation has a lower energy than the primary one. Independent of the energy of the primary radiation, the energy of the radiation scattered more than 90° is lower than around 0.5 MeV. Thus the TVT and transmission curves for scattered radiation shall be used. In many installations the scattered radiation gives a small contribution to the total dose and can be neglected in barrier determinations.

**Leakage radiation**. For leakage radiation, transmission curves are not recommended to be used but only tenth-value thicknesses. The number of tenth-value thicknesses $N_{TVT}$ is given by

$$N_{TVT} = \log_{10} \frac{W_L T}{d^2 P} \tag{6.1.38}$$

$W_L$ is the leakage absorbed dose at 1 m from the source. For high energy x rays this value is at 1 m from the target often 0.1 % of the primary absorbed dose at 1 m (NCRP, 2005).

A wall is normally hit by both scattered radiation and leakage radiation. If the difference is larger than 1 TVT, which is the most common situation, the thicker shield thickness should be used. If the difference is smaller than 1 TVT, then 1 HVT should be added to the thicker shield.

## 6.2 Exercises in Radiation Protection

### 6.2.1 Radioecology

**Exercise 6.1.** A company is using high activities of $^{57}$Co. The waste activity is first released through waste water into a water basin and then, after passing through a filter, out into the sewage. The renewal of the water into the basin is 5.0 $m^3$ per day. The volume of the basin is 200 $m^3$. The contaminated water is assumed to immediately mix with the water in the basin. A check of the activity concentration in the water in the basin after 200 days shows that it is 2.50 kBq $m^{-3}$. This was regarded as too much and actions were made to reduce the release of the activity into the basin with a factor of 3.0. Calculate the activity concentration in the basin 100 days later.

**Exercise 6.2.** A laboratory can measure an activity of 0.20 Bq $^{89}$Sr in a urine sample. Assume that workers handling $^{89}$Sr, have a daily urine sample measured every 30 days. Calculate the lowest detectable intake in the "worst situation" i.e. when the worker was exposed to the contamination 30 days before the sample was taken. Assume that 30% of the activity is excreted through the urine and that the excretion equation is given by

$$Y(t) = 0.12 \cdot e^{-\ln 2 \cdot t/2.4} + 0.08 \cdot t^{-1.2} \text{ (t in days)}$$

**Exercise 6.3.** A person is working in an environment in which the concentration of activity of $^3$H is 0.10 DAC (derived air concentration). He is working in this environment for 180 days. Calculate the expected total effective dose he will obtain. $^3$H has a biological half life of 12.0 days.

**Exercise 6.4.** You are responsible for a laboratory using $^{210}$Po. There is a detector available that can measure the activity of $^{210}$Po in the urine. The detection limit is 15 Bq $^{210}$Po in the daily urine. You are interested in measuring a single uptake of 20.0 kBq $^{210}$Po. How often is it necessary to perform urine measurements to be able to detect such an uptake? The retention equation for $^{210}$Po in the kidneys is

$$R(t) = e^{-\ln 2 \cdot t/40} \text{ (t in days)}$$

10% of the activity is taken up by the kidneys and excreted through the urine.

**Exercise 6.5.** A person works for 60 days in an area with a high concentration of tritium in the drinking water. A continuous stay in this environment would result in an activity intake corresponding to one ALI, calculated for an effective dose of 20 mSv per year. Calculate the effective dose during the 60 days the person is staying in this environment. Calculate also the total effective dose integrated over infinite time. The biological half life is 10 days.

**Exercise 6.6.** A research institute is using $^3$H. By mistake the waste water became contaminated. This waste water is used by cattle drinking 70 $dm^3$ of the water every day. The activity concentration of the water is 390 kBq $dm^{-3}$. This is going on for 40 days until the contamination is discovered. The cattle will then drink uncontaminated water. Calculate the total absorbed dose to an animal with a mass of 400 kg (assuming infinite time). The biological half life is obtained by assuming that the water intake and excretion is the same and the water content is 260 kg. When calculating the mean absorbed dose assume that the activity is distributed uniformly in the body.

**Exercise 6.7.** The chemical toxicity for nickel carbonyl, $Ni(CO)_4$, gives the limit for the atmospheric concentration to $1.0 \cdot 10^{-10}$% per mass unit. In an experiment nickel carbonyl, corresponding to the chemical limit, tagged with 100% $^{63}$Ni is used. Is this in agreement with the radiological limit expressed in ALI for an effective dose of 20 mSv per year, i.e. the exposed persons are assumed to be radiological workers? The whole body is supposed to be the critical organ (70 kg). Assume that the breathing capacity is 10.0 $m^3\ d^{-1}$ and that 50% of the inhaled activity is absorbed. The biological half life is 2.0 years.

**Exercise 6.8.** When estimating risks from the food contaminated with radioactivity from fallout, it is necessary to consider how often a certain foodstuff is consumed. Compare the yearly emitted electron- and photon energy from the radioactivity in the body for the two following situations:
a) A single intake of 0.60 kg of lobster from Sellafield. The lobster contains 13000 Bq $kg^{-1}$ of $^{137}$Cs.
b) A “continuous” daily intake of 0.50 $dm^3$ milk, with the concentration 100 Bq $dm^{-3}$ of $^{137}$Cs. The excretion of Cs from the body can be approximated with a biological half life of 100 days.

**Exercise 6.9.** After the reactor power accident in Chernobyl the maximal concentration of $^{137}$Cs in food was in Sweden limited to 300 Bq $kg^{-1}$. The aim was that the population during their life shall not obtain an activity content higher than 30 000 Bq per person. Calculate the maximum mass intake of the contaminated food per day if this is to be fulfilled assuming constant activity concentration. The average length of life in Sweden is supposed to be 80 years. The retention function for $^{137}$Cs is given by

$$R(t) = 0.10 \cdot e^{-0.347t} + 0.90 \cdot e^{-0.00630t} \text{ (t in days)} \tag{6.2.1}$$

**Exercise 6.10.** A pond is by mistake contaminated with $^{137}$Cs. A test measurement shows that the concentration of the activity is 2.5 kBq $dm^{-3}$ water. The pond has an exchange of water of 1000 $dm^3$ per day and the volume of the pond is 100 000 $dm^3$ water. The mixture of the water is momentarily so the concentration of activity in the pond is uniform. The water in the pond is drunk by cows on a pasture surrounding the pond. Calculate the maximum activity in the cows if they drink 70 $dm^3$ water per day. The half life for Cs in the cow is assumed to be 100 days.

**Exercise 6.11.** One of the problems when using fusion reactors is the production of tritium. In a river the concentration of $^{3}$H in the water is 800 Bq dm$^{-3}$. Calculate the effective dose per year for a continuous intake of water, assuming that a person drinks 2.0 dm$^{3}$ water per day. Hydrogen is in this calculation approximated to be evenly distributed in the body with a mass of 70 kg. The biological half life for hydrogen is 10 days.

**Exercise 6.12.** You are working at a hospital, that is using $^{131}$I for treatment of diseases in the thyroid. There is an automatic system for distribution of the activity to the patient. At some occasion it is discovered that a tube in the system is broken and some activity is leaking out into a room with a volume of 20 m$^{3}$. The system is assumed to be continuously leaking $^{131}$I with a velocity of 0.20 MBq h$^{-1}$. The activity is mixed momentarily with the air and the concentration is approximated to be evenly distributed in the room. The ventilation exchange rate in the room is 0.50 h$^{-1}$. A laboratory assistant has been working in the laboratory the whole day, i.e. 8.0 h. Assume that the leakage started already in the morning, which means that there has been a leakage the whole day. Calculate the activity in the thyroid of the laboratory assistant at the end of the day, if 30% of the inhaled activity is taken up in the thyroid and excreted with a biological half life of 75 days. The inhaling rate is 0.020 m$^{3}$ min$^{-1}$.

### 6.2.2 Point Radioactive Sources

**Exercise 6.13.** A $^{60}$Co source is placed in a safety box in a storage room. In another room close to the storage, a person is sitting all the day, i.e. 40 h per week. Calculate the absorbed dose to the person during one week, using the following data. The activity of the source is 0.63 GBq. The wall of the safety box is made of 30 mm iron. The wall between the rooms is made of 15.0 cm concrete ($\rho = 2.35 \cdot 10^{3}$ kg m$^{-3}$). The absorbed dose is calculated to a small mass of water "free in air" at a distance of 60.0 cm from the source. The build-up factor due to the secondary photons is 3.0. Is it acceptable that the person is sitting there or is it necessary to improve the shielding?

**Exercise 6.14.** In a student's laboratory radioactive sources are stored in a safe. The dose rate (to a small mass of water) outside the safe should be less than 10 $\mu$Gy h$^{-1}$. Assume that the dominating radioactive source in the safe is $^{60}$Co. Which is the highest possible activity of this source in order to fulfill the radiation protection requirements? Assume the following geometry: The shortest distance from the radioactive source to the outside of the safe is 130 mm. The wall of the safe is made of 20 mm iron. For calculation of the build-up factor use the Berger expression with the parameters $a$=0.955 and $b$=0.024.

**Exercise 6.15** To decrease the absorbed dose when working with radioactive sources, it is possible to work behind a shield made of lead. Sometimes this however prolongs the working time and the absorbed dose can be higher than without the shield. Which of the following working situations gives the lowest absorbed dose?

Method 1: The radioactive source ($^{60}$Co) is positioned 50 cm from the person behind a 40 mm thick lead shield. The working moment takes 600 s.

Method 2: The radioactive source ($^{60}$Co) is positioned at a distance of 40 cm from the person without any lead shield. The working moment now takes 140 s.

For calculation of the build-up factor use the Berger expression with the parameters $a$=0.33 and $b$=-0.011. If the activity of the source is 200 GBq, calculate the kerma rate to air at the calculation point where the person is situated. The mean energy of the photons may be used.

**Exercise 6.16.** A young patient is to be treated with $^{60}$Co-$\gamma$-radiation and the father has then to be in the treatment room during the irradiation. The kerma rate to air is 21.8 mGy s$^{-1}$ at a distance of 1.00 m from the source. SSD is equal to 0.80 m and the treatment time is 150 s every time. The distance to the father is 4.00 m. He is only hit by 90$^o$ scattered radiation from the patient and leakage radiation from the treatment head. The thickness of the patient is 20 cm. The treatments are made 5 days per week and during 4 weeks.

The father is positioned behind a lead shield. How thick must this lead shield be if the father shall not get more than a total effective dose of 100 $\mu$Sv? The kerma rate of the scattered radiation 1.00 m from the scatterer (the patient) is 0.10% of the kerma rate of the primary beam at position of the center of the scatterer. Use transmission curves for broad beams to calculate the necessary shielding thickness. The leakage dose rate at 1.0 m is assumed to be 0.1% of the primary dose rate at the isocenter. Tenth value thickness in lead is 4.0 cm (ICRP, 1982). Conversion factor effective dose to air kerma = 1.09.

**Exercise 6.17.** A 6 MV linear accelerator will be installed in a room previously used for a $^{60}$Co radiotherapy unit. The wall thickness will then need to be increased. The treatment room is outlined in Fig. 6.6 where the lines indicate the outer contour of the walls. At point A there is a control room and point B is situated outside the building, where it is unlikely that anybody will be there for a long time. According to national regulations, considering typical values for work load, use and occupancy factors, this implies that the absorbed dose rate at A shall be <10 $\mu$Gy h$^{-1}$ and at B <100 $\mu$Gy h$^{-1}$.

The linear accelerator is mounted isotropically and the primary beam can hit A, but not B. The maximum absorbed dose rate at the isocenter (1.00 m) is 4.0 Gy min$^{-1}$. The leakage dose rate at 1.0 m is assumed to be 0.1% of the primary dose rate at the isocenter. The scattered radiation 1.0 m from the isocenter is 0.03% of the primary dose rate at that point. For the calculation of the wall thicknesses, data from ICRP 33 (ICRP 1982) or similar compilations can be used. Calculate the necessary wall thicknesses for the walls at A and B.

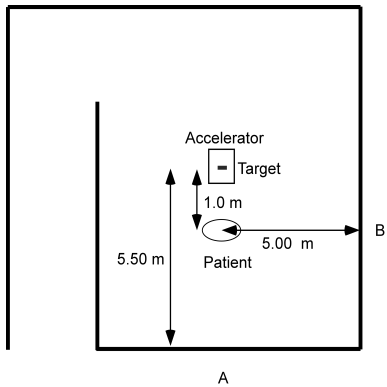


**Figure 6.6:** Outline of a radiotherapy treatment room with a linear accelerator.

**Exercise 6.18.** At a hospital, a room shall be used for brachytherapy patients, i.e. patients with intracavity radioactive sources. It is important that patients in a nearby room shall not receive absorbed doses which are too high. Calculate the necessary thickness for the wall between the rooms. See Fig. 6.7. Assume that there are patients with radioactive sources for 50 h per week. In the nearby room the same patient can stay for a week. The effective dose to the patient in the nearby room shall be lower than 0.020 mSv. The dose may be calculated as the kerma to a small mass of water "free in air" at the center of the patient, without including any absorption in the patient.

The brachytherapy sources are $^{137}$Cs with an activity of 2.60 GBq. The wall is made of concrete with a density of $2.35 \cdot 10^3$ kg m$^{-3}$. Each patient has only one source and only one patient is in the room. The self absorption in the brachytherapy patient is 30%. The air kerma constant for $^{137}$Cs=$20.3 \cdot 10^{-18}$ Gy s$^{-1}$ Bq$^{-1}$ m$^2$.

### 6.2.3 Extended Radioactive Sources

**Exercise 6.19.** In gynecological radiotherapy, radioactive sources which are inserted into the body, are used. Such a radiation source may consist of a line source inside a material which protects the source from the tissue, but also absorbs all electrons and low energetic photons that are emitted from the source. Such a radiation source has dimensions as in Fig. 6.8, where the cover made of platinum (Pt) has a thickness of

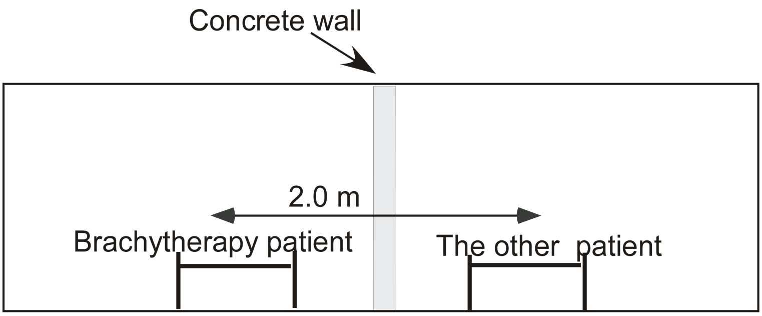


**Figure 6.7:** Brachytherapy patient in a room next to another patient.

0.65 mm. The source is placed in water. Which time is needed to obtain an absorbed dose at P of 2.0 Gy, if the activity of the source is 57.5 GBq? Only primary photons have to be included in the calculations.

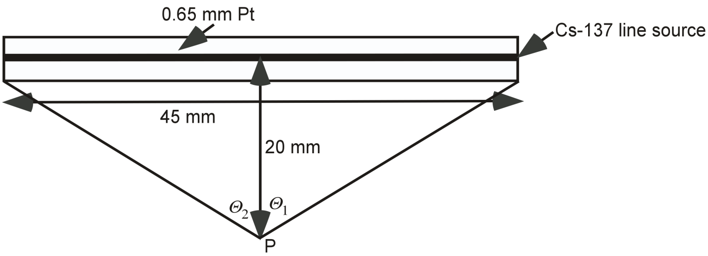


**Figure 6.8:** $^{137}$Cs brachytherapy source with a Pt-cover.

**Exercise 6.20.** A betatron (a circular electron accelerator), may be radioactively contaminated trough gamma-n and electron-n reactions when high accelerating energies are used. Assume that the betatron may be regarded as a circular radioactive source with the diameter 1.50 m. Calculate the kerma to a small mass of water at the center of the betatron during the first 30 min after the betatron has been switched off. The important radionuclide is $^{15}$O and the line source activity at the time of switch off is 1.30 GBq m$^{-1}$. Between the source and the calculation point is the wall of the betatron that is made of iron and has a thickness of 10.0 mm. Only primary photons have to be included in the calculations.

**Exercise 6.21.** In a room there is a tube in which a radioactive gas is passing (See Fig. 6.9). The closest distance from the tube to a work place is 2.00 m. The line source activity in the tube is 1.20 GBq m$^{-1}$ and consists of $^{18}$F. Calculate the kerma rate to air at the work place. The tube wall is made of iron and has a thickness of 2.0 mm. Assume that the positrons are annihilated directly at the inside of the tube and that the contribution from secondary photons can be neglected. The activity can be regarded as a line source and the contribution from the activity in the tube outside the room can be neglected. Is this work place acceptable regarding radiation protection recommendations?

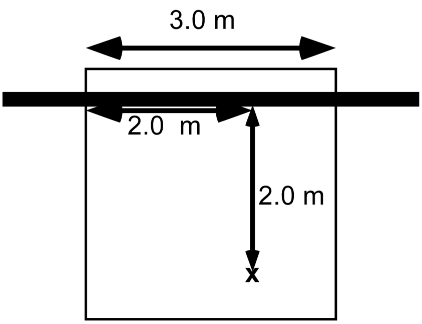


**Figure 6.9:** Room with a tube containing radioactive gas and a work place situated 2.0 m from the tube.

**Exercise 6.22.** In an experiment, cows on a pasture contaminated with $^{131}$I are irradiated. The concentration of activity is 450 MBq m$^{-2}$. The pasture is large so an infinite extension may be assumed in the calculations. Calculate the kerma to tissue at a point 1.50 m over the ground, if the cows are on the pasture for 14 days. The air pressure is 100 kPa and the mean air temperature 15 °C. Include only the most frequent photon energy in the calculations.

**Exercise 6.23.** After the Chernobyl accident, parts of Sweden were contaminated with radioactivity. The area around Gävle was heavily contaminated, and an area source activity up to 80 000 Bq m$^{-2}$ was measured. The main radionuclide was $^{137}$Cs. Calculate the maximal kerma rate to air 1.5 m above the ground for a circular area with the radius 200 m with this concentration. The air pressure is 101.3 kPa and the air temperature 20 °C. The contribution from the characteristic x rays and the production of secondary photons can be neglected.

**Exercise 6.24.** During winter Sweden is typically covered with snow. The snow will decrease the natural background radiation from the ground. Calculate the decrease in air kerma rate 1.50 m above the ground, if the ground can be approximated with an infinite plane surface source with the activity 10000 Bq m$^{-2}$. Assume that 0.80 photons with the energy 0.80 MeV are emitted per disintegration. The snow thickness is 0.50 m and the density is 0.25 kg dm$^{-3}$. The build-up factor is calculated using the formula according to Berger with the parameters $a$=1.74 and $b$=0.045. The same formula constants may be used for both air and snow (water). When calculating the kerma rate the same energy may be used for the primary and the secondary photons.

**Exercise 6.25.** Calculate the primary photon fluence rate at the water surface above a nuclear fuel rod, that is placed vertically, with its upper endpoint 2.00 m below the water surface.

The rod consists of uranium dioxide and is 4.00 m long, and may be regarded as a thin line source, but where the attenuation in the source should be considered along the length of the source. The activity in the source is 37·10$^{15}$ Bq and a photon with the energy 1.0 MeV is emitted per disintegration.

**Exercise 6.26.** A measuring room for low-activity measurements were built with a 0.40 m thick concrete wall, with the volume source activity 7.4 kB m$^{-3}$. The radioactive nuclides decay by emitting two photons per disintegration with the average energy of 1.0 MeV. To reduce the background contribution from the wall it was covered with 20 mm lead.

Calculate the fluence rate 1.0 m from the wall, with and without the lead. The wall may be assumed to be semi-infinite. Estimate what error this approximation introduces if the wall has the dimension 6.0x2.0 m$^2$. The build-up factor is calculated using the formula according to Berger with the parameters for concrete $a$=1.27 and $b$=0.032 and for lead $a$=0.30 and $b$=-0.015.

## 6.3 Solutions in Radiation Protection

### 6.3.1 Radioecology

**Solution exercise 6.1.**

The change of the concentration of activity in the basin is given by the equation

$$\frac{\mathrm{d}C}{\mathrm{d}t} = U - \lambda_{\text{eff}} C(t) \tag{6.3.1}$$

Solving the equation gives

$$C(t) = \frac{U}{\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}} t}) \tag{6.3.2}$$

$U$ is the input of activity to the basin per time and volume unit and $\lambda_{\text{eff}} = \lambda_f + \lambda_b$ (effective decay constant)

Data:
$C(200\,\mathrm{d}) = 2.5\,\mathrm{kBq\,m^{-3}}$ (activity concentration in the basin after 200 d)
$\lambda_f = \ln 2/270.9\,\mathrm{d^{-1}}$ (physical decay constant)
$\lambda_b = 5.0/200\,\mathrm{d^{-1}}$ (rate of exchange of water in the basin)
$\lambda_{eff} = 2.756 \cdot 10^{-2}\,\mathrm{d^{-1}}$

Data inserted in the equation gives

$$2.5 \cdot 10^3 = \frac{U}{0.02756}(1 - e^{-0.02756 \cdot 200})$$

$U = 68.62\,\mathrm{Bq\,m^{-3}\,d^{-1}}$

The concentration of activity 100 d after the decrease of the activity is given by the new activity rate released, which is 68.62/3 Bq ($\mathrm{m^{-3}\,d^{-1}}$) and the decrease of the activity in the basin after 200 d, when the change in activity was introduced.

$$C = \frac{68.62}{3 \cdot 0.02756}(1 - e^{-0.02756 \cdot 100}) + 2.5 \cdot 10^3 \cdot e^{-0.02756 \cdot 100} = 936\,\mathrm{Bq\,m^{-3}}$$

**Answer:** The concentration of activity 100 d after decrease of release of the activity is $0.94\,\mathrm{kBq\,m^{-3}}$.

**Solution exercise 6.2.**
Excretion of $^{89}Sr$ is given by the equation

$$Y(t) = 0.12 \cdot e^{-\ln 2 \cdot t/2.4} + 0.08 \cdot t^{-1.2} \tag{6.3.3}$$

The excreted activity per day at time $t$ is then given by

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t} = A_0 f Y(t) e^{-\ln 2 \cdot t/T_{1/2}} \tag{6.3.4}$$

where
$T_{1/2} = 50.5$ d (physical half life of $^{89}Sr$)
$\mathrm{d}A/\mathrm{d}t = 0.20\,\mathrm{Bq\,d^{-1}}$ (minimal measurable excreted activity in the urine per day)
$f = 0.30$ (fraction excreted through urine)
$t = 30$ d (time between two measurements)

Data inserted in Eq. 6.3.4 gives

$$0.20 = A_0 \cdot 0.30 \cdot (0.12 \cdot e^{-\ln 2 \cdot 30/2.4} + 0.08 \cdot 30^{-1.2}) \cdot e^{-\ln 2 \cdot 30/50.5}$$

This gives

$A_0$=734 Bq

**Answer:** The minimum detectable intake is 0.73 kBq $^{89}$Sr.

**Solution exercise 6.3.**
The activity in the body at time $t$ with continuous intake is given by

$$A(t) = \frac{I}{\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}} t}) \tag{6.3.5}$$

After a time corresponding to several half lives, equilibrium is obtained and the activity in the body is

$$A(\infty) = \frac{I}{\lambda_{\text{eff}}}$$

where $I$ is the intake/time unit and $\lambda_{\text{eff}}$ is the effective decay constant.

In this situation an air activity concentration corresponding to a DAC (derived air concentration) will give an effective dose, $E$, during a year corresponding to 20 mSv. This gives the relation

$$E = \frac{I}{\lambda_{\text{eff}}} ST \tag{6.3.6}$$

where $S$ is the factor giving the relation between activity and effective dose rate and $T$ is the time for dose calculation (one year).

Thus

$$I = \frac{E\lambda_{\text{eff}}}{ST} \tag{6.3.7}$$

This relation is inserted in Eq. (6.3.5) to obtain the activity at time $T_1$.

$$A_1(T_1) = \frac{E\lambda_{\text{eff}}}{ST\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}} T_1}) \tag{6.3.8}$$

The activity then decays and the variation of activity with time after $T_1$ is given by

$$A_2(t) = \frac{E}{ST}(1 - e^{-\lambda_{\text{eff}} \cdot T_1}) e^{-\lambda_{\text{eff}} \cdot t} \tag{6.3.9}$$

The effective dose can then be obtained through Eq. (6.3.8) and Eq. (6.3.9).

The first 180 d:

$$E_1 = S \int_0^{T_1} \frac{E}{ST}(1 - e^{-\lambda_{\text{eff}} t})\,\mathrm{d}t = \frac{E}{T}\left[T_1 + \frac{e^{-\lambda_{\text{eff}} T_1}}{\lambda_{\text{eff}}} + \frac{1}{\lambda_{\text{eff}}}\right] \tag{6.3.10}$$

Data:
$\lambda_{\text{eff}} = \lambda_b$ = ln2/12=0.05776 d$^{-1}$ as $\lambda_f$ can be neglected.
$E$=2.0 mSv

$T$=1.0 y=365 d
$T_1$=180 d

Data inserted gives

$$E_1 = \frac{2.0 \cdot 10^{-3}}{365}[180 + \frac{e^{-0.05776 \cdot 180}}{0.05776} + \frac{1}{0.05776}] = 0.891 \cdot 10^{-3}\ \text{Sv}$$

After 180 d

$$E_2 = \frac{2.0 \cdot 10^{-3} \cdot S}{S \cdot 365}(1 - e^{-0.05776 \cdot 180}) \int_0^\infty e^{-0.05776 \cdot t}\,\mathrm{d}t = \frac{2.0 \cdot 10^{-3}}{365}\frac{(1 - e^{-180 \cdot 0.05776})}{0.05776}$$

$E_2 = 0.095 \cdot 10^{-3}$ Sv.

The total effective dose is then $E$=0.891+0.095=0.988 mSv

**Answer**: The total effective dose is 0.99 mSv.

**Solution exercise 6.4.**
The variation of $^{210}$Po activity in the body is given by the retention function corrected with the physical half life.

$$A(t) = A_0 e^{-\lambda_{\text{eff}} t} \tag{6.3.11}$$

$\lambda_{\text{eff}} = \lambda_{\text{f}} + \lambda_{\text{b}}$

The excretion is given by differentiation of Eq. (6.3.11)

$$\frac{\mathrm{d}A}{\mathrm{d}t} = -A_0 \lambda_{\text{eff}} e^{-\lambda_{\text{eff}} t} \tag{6.3.12}$$

where
$A_0$=20.0 kBq (initial activity of $^{210}$Po)
$\lambda_\text{f}$=ln2/138.4 d$^{-1}$ (physical decay constant)
$\lambda_\text{b}$=ln2/40 d$^{-1}$ (biological decay constant)
$\lambda_{\text{eff}}$=2.234·10$^{-2}$ d$^{-1}$ (effective decay constant)

The minimum measured activity excreted in the daily urine should be 15 Bq. 10% of the activity is excreted through urine. Thus data inserted in Eq. (6.3.12) gives

$$15 = 0.1 \cdot 20 \cdot 10^3 \cdot 2.234 \cdot 10^{-2} \cdot e^{-2.234 \cdot 10^{-2} \cdot t}$$

Solving $t$ gives $t$=49 d

**Answer:** It is necessary to measure at least every 49th day.

**Solution exercise 6.5.**
ALI (annual limit of intake) shall be calculated over a time span of 50 years. However, with the short biological half life in this case ($T_b$=10 d) it is possible to integrate over infinity.

The effective dose is given by Eq. 6.3.13 if the disintegrations are integrated over infinite time

$$E = \frac{ALI}{\lambda_{\text{eff}}} S \tag{6.3.13}$$

Solving $S$ (Sv Bq$^{-1}$ s$^{-1}$) gives

$$S = \frac{E\lambda_{\text{eff}}}{ALI} \tag{6.3.14}$$

Assume continuous intake, then

$I = ALI/365$ Bq d$^{-1}$

The activity in the body after a time $t$ is then given by

$$A(t) = \frac{I}{\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}} t}) \tag{6.3.15}$$

Integration over a time $T$, during which there is an intake, gives the number of disintegrations during this time

$$N(T) = \int_0^T A(t)dt = \frac{I}{\lambda_{\text{eff}}}(T + \frac{e^{-\lambda_{\text{eff}} T}}{\lambda_{\text{eff}}} - \frac{1}{\lambda_{\text{eff}}}) \tag{6.3.16}$$

The number of disintegrations after the time $T$, where there is no intake of activity, is given by

$$N_\infty = \frac{A(T)}{\lambda_{\text{eff}}} = \frac{I}{(\lambda_{\text{eff}})^2}(1 - e^{-\lambda_{\text{eff}} T}) \tag{6.3.17}$$

Data:
$A(T)$=activity at time $T$
$E$=20 mSv (permissible effective dose per year for a person in radiological work)
$\lambda_f$=ln2/12.3 y$^{-1}$ (physical decay constant)
$\lambda_b$=ln2/10 d$^{-1}$ (biological decay constant)
$\lambda_{\text{eff}}=\lambda_f+\lambda_b$ (effective decay constant)
$\lambda_{\text{eff}}$=ln2/10 d$^{-1}$
$T$=60 d

The effective dose during the first 60 d is given by

$$E_{60} = SN(60) = \frac{20 \cdot 10^{-3} \cdot \lambda_{\text{eff}} \cdot ALI}{ALI \cdot \lambda_{\text{eff}} \cdot 365}(60 + \frac{e^{-(\ln 2/10)60}}{(\ln 2/10)} - \frac{10}{\ln 2}) = 2.51 \cdot 10^{-3} \text{ Sv}$$

Here $I_0$ is set equal to ALI/365.

The effective dose for the time after 60 days is given by

$$E_\infty = SN_\infty = \frac{20 \cdot 10^{-3} \cdot ALI \cdot 10}{ALI \cdot 365 \cdot \ln 2}(1 - e^{-\frac{60 \cdot \ln 2}{10}}) = 0.78 \cdot 10^{-3}\ \mathrm{Sv}$$

**Answer**. The effective dose during the first two months is 2.5 mSv and the total absorbed dose is 3.3 mSv.

**Solution exercise 6.6.**
The change of activity in the animal is given by the equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = I - \lambda_{\mathrm{eff}} A \tag{6.3.18}$$

Solving the equation gives

$$A(t) = \frac{I}{\lambda_{\mathrm{eff}}}(1 - e^{-\lambda_{\mathrm{eff}} t}) \tag{6.3.19}$$

The total number of disintegrations in the animal is given by the sum of the number of disintegrations during the time $T$ when the cattle is drinking the activated water and the number of disintegrations from the day they start to drink uncontaminated water to infinity, as the effective half life is short.

$$\tilde{A} = \int_0^T A(t)\,\mathrm{d}t + \int_0^\infty A(T)e^{-\lambda_{\mathrm{eff}} t}\,\mathrm{d}t \tag{6.3.20}$$

or

$$\tilde{A} = \int_0^T \frac{I}{\lambda_{\mathrm{eff}}}(1 - e^{-\lambda_{\mathrm{eff}} t})\,\mathrm{d}t + \int_0^\infty \frac{I}{\lambda_{\mathrm{eff}}}(1 - e^{-\lambda_{\mathrm{eff}} T})e^{-\lambda_{\mathrm{eff}} t}\,\mathrm{d}t \tag{6.3.21}$$

Solving the integral gives

$$\tilde{A} = \frac{I}{\lambda_{\mathrm{eff}}}(T + \frac{e^{-\lambda_{\mathrm{eff}} T}}{\lambda_{\mathrm{eff}}} - \frac{1}{\lambda_{\mathrm{eff}}}) + \frac{I}{\lambda_{\mathrm{eff}}^2}(1 - e^{-\lambda_{\mathrm{eff}} T}) \tag{6.3.22}$$

This can be simplified to

$$\tilde{A} = (I \cdot T)/\lambda_{\mathrm{eff}} \tag{6.3.23}$$

Data:
$T$=40 d (time during which the animals are drinking contaminated water)
$\lambda_f = \ln 2/(12.3 \cdot 365.25) = 1.54 \cdot 10^{-4}\ \mathrm{d}^{-1}$ (physical half life of $^3H$)
$\lambda_b = 70/260\ \mathrm{d}^{-1} = 0.2693\ \mathrm{d}^{-1}$ (biological half life of hydrogen)

$\lambda_{\text{eff}}$=0.2694 d$^{-1}$ (effective half life of $^{3}$H)
$I$=70·390 kBq d$^{-1}$ (intake per day if physical decay is neglected)

Data inserted in Eq. 6.3.23 gives

$$\tilde{A} = \frac{70 \cdot 390 \cdot 10^{3} \cdot 40 \cdot 3600 \cdot 24}{0.2694} = 3.502 \cdot 10^{14}\ \text{Bqs}$$

$^{3}$H decays emitting low energetic $\beta$-radiation. All energy can then be assumed to be absorbed in the animal.

The absorbed dose is then given by

$$D = \frac{\tilde{A}_h \bar{E}}{m} \tag{6.3.24}$$

Data:
$\bar{E}$=0.00568 MeV (mean energy of the $\beta$-radiation)
m=400 kg (mass of the cattle)

Data inserted in Eq. (6.3.24) gives

$$D = \frac{3.502 \cdot 10^{14} \cdot 0.00568 \cdot 1.602 \cdot 10^{-13}}{400} = 7.97 \cdot 10^{-4}\ \text{Gy}$$

If the physical decay of $^{3}$H is not included, the absorbed dose will increase with less than 0.1 %.

**Answer**: The absorbed dose to the animal is 0.80 mGy.

**Solution exercise 6.7.**
The radiological limit for a radionuclide is given by ALI. From the definition of ALI the following relation is obtained

$$E = \int_0^{50} S \cdot ALIe^{-\lambda_{\text{eff}} t}\, \mathrm{d}t = \frac{S \cdot ALI}{\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}} \cdot 50}) \tag{6.3.25}$$

$S$ in the equation above is the relation effective dose to cumulated activity (Sv Bq$^{-1}$ s$^{-1}$), which in our case is the same as the absorbed dose per decay.

As $^{63}$Ni is emitting only $\beta$-radiation all energy may be assumed to be absorbed in the body and then $S$ is obtained as $S = \bar{E}_\beta / m$ if the radiation weighting factor $w_R$

and the tissue weighting factor $w_T$ are assumed to be equal to unity. $\bar{E}_\beta$ is the mean energy of the $\beta$-radiation, and $m$ is the mass of the reference man.

Thus the equation may be expressed as

$$E = \frac{\bar{E}_\beta ALI}{m\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}}\cdot 50}) \tag{6.3.26}$$

Data:
$\bar{E}_\beta$=17.1 keV (mean energy of the $\beta$-particles)
$m$=70 kg (mass of the reference man)
$\lambda_b = \ln 2/2.0\ \text{y}^{-1}$ (biological decay constant)
$\lambda_f = \ln 2/96\ \text{y}^{-1}$ (physical decay constant)
$\lambda_{\text{eff}} = \ln 2/2.0 + \ln 2/96 = 0.3538\ \text{y}^{-1}$ (effective decay constant)
$E$=20.0 mSv (permissible effective dose per year for radiological personal over several years)

Data inserted in Eq. (6.3.26) gives

$$20 \cdot 10^{-3} = \frac{17.1 \cdot 1.602 \cdot 10^{-16} \cdot ALI \cdot (3600 \cdot 24 \cdot 365.25)}{70 \cdot 0.3538}(1 - e^{-0.3538\cdot 50})$$

ALI=5.73·$10^6$ Bq

The limit for chemical uptake of air with a concentration of $C\,\text{kg}\,\text{m}^{-3}$ gives the yearly intake of nickel carbonyl

$$m_{\text{Ni}} = v\rho_{\text{air}}Cf \tag{6.3.27}$$

where
$v = 10.0 \cdot 10 \cdot 365\ \text{m}^3$ (inhaled air volume per year)
$\rho_{\text{air}} = 1.20 \cdot 10^{-3}\ \text{kg}\,\text{dm}^{-3}$ (t=20 $^o$C, p=101.3 kPa)
$C = 1.0 \cdot 10^{-12}$ (concentration of Ni in air per mass unit)
$f$=0.5 (fraction of Ni absorbed in the body)
This gives

$m_{\text{Ni}} = 1.0 \cdot 10^4 \cdot 365 \cdot 1.20 \cdot 10^{-3} \cdot 1.0 \cdot 10^{-12} \cdot 0.5 = 2.19 \cdot 10^{-9}$ kg (mass of nickel carbonyl per year)

The total molecule mass of the nickel carbonyl ($Ni(CO)_4$)) molecule is (63+4(12+16))=175

The number of nickel atoms is obtained by the relation using Avogadro's number.

$$N_{\text{Ni}} = \frac{m_{\text{Ni}}N_A}{m_a}$$

Data inserted gives

$$N_{\mathrm{Ni}} = \frac{2.19 \cdot 10^{-9} \cdot 6.023 \cdot 10^{26}}{175}$$

The activity is then obtained by using the relation $A = \lambda \cdot N$

$$A_{\mathrm{Ni}} = \frac{2.19 \cdot 10^{-9} \cdot 6.023 \cdot 10^{26}}{175} \frac{\ln 2}{96 \cdot 365.25 \cdot 24 \cdot 3600}$$

$A_{\mathrm{Ni}}$=1.72·$10^6$ Bq

This activity is smaller than ALI. Thus the chemical toxicity sets the limit.

**Answer:** ALI is 5.73 MBq and the chemical limit corresponds to an activity of 1.72 MBq, and thus the experiment is acceptable from radiological point of view.

**Solution exercise 6.8.**
a) Single intake

The total number $N_{\mathrm{a}}$ of decays during time $T$ after an intake $A_0$ is given by

$$N_{\mathrm{a}} = \int_0^T A_0 e^{-\lambda_{\mathrm{eff}} t}\,\mathrm{d}t = \frac{A_0}{\lambda_{\mathrm{eff}}}(1 - e^{-\lambda_{\mathrm{eff}} T}) \tag{6.3.28}$$

b) Continuous intake

The activity in the body at time $t$ after a continuous intake $I$ is given by

$$A = \frac{I}{\lambda_{\mathrm{eff}}}(1 - e^{-\lambda_{\mathrm{eff}} t}) \tag{6.3.29}$$

The total number of decays $N_{\mathrm{b}}$ during time $T$ is given by

$$N_{\mathrm{b}} = \int_0^T \frac{I}{\lambda_{\mathrm{eff}}}(1 - e^{-\lambda_{\mathrm{eff}} t})\,\mathrm{d}t \tag{6.3.30}$$

$$N_{\mathrm{b}} = \frac{I}{\lambda_{\mathrm{eff}}}(T + \frac{e^{-\lambda_{\mathrm{eff}} T}}{\lambda_{\mathrm{eff}}} - \frac{1}{\lambda_{\mathrm{eff}}}) \tag{6.3.31}$$

Data:
$A_0$=0.60·13000 Bq (0.60 kg with a concentration of 13000 Bq kg$^{-1}$)

$T$=365 d (integration time)
$\lambda_f = \frac{\ln 2}{100}\ \mathrm{d}^{-1}$ (physical decay constant)
$\lambda_b = \frac{\ln 2}{30 \cdot 365}\ \mathrm{d}^{-1}$ (biological decay constant)
$\lambda_{eff} = \lambda_b + \lambda_f = 6.995 \cdot 10^{-3}\ \mathrm{d}^{-1} = 8.096 \cdot 10^{-8}\ \mathrm{s}^{-1}$ (effective decay constant)
$I$=50 Bq d$^{-1}$ (continuous intake of activity; 0.5 dm$^3$ per day with a concentration of 100 Bq dm$^{-3}$)
$E$=0.946·0.1734+0.054·0.4346+0.662·0.946=0.813 MeV (energy emitted per decay)

Data inserted in Eq.(6.3.28) and (6.3.31) and multiplying with the energy per decay gives

a) Single intake

$$E_a = \frac{0.813 \cdot 1.602 \cdot 10^{-13} \cdot 0.60 \cdot 13000}{8.096 \cdot 10^{-8}}(1 - e^{-6.995 \cdot 10^{-3} \cdot 365}) = 1.16 \cdot 10^{-2}\ \mathrm{J}$$

b) Continuous intake

$$E_b = \frac{0.813 \cdot 1.602 \cdot 10^{-13} \cdot 50}{8.096 \cdot 10^{-8} \cdot 3600 \cdot 24}(365 \cdot 24 \cdot 3600 + \frac{e^{-6.995 \cdot 10^{-3} \cdot 365}}{8.096 \cdot 10^{-8}} - \frac{1}{8.096 \cdot 10^{-8}})$$
$E_b = 1.88 \cdot 10^{-2}$ J

**Answer:** The emitted energy the first year for the single intake is 12 mJ and for the continuous intake 19 mJ.

**Solution exercise 6.9.**
The retention equation

$$R(t) = 0.10 \cdot e^{-0.347t} + 0.90 \cdot e^{-0.00630t}\ (t \text{ in days}) \tag{6.3.32}$$

can be considered as if the activity of Cs is taken up into two compartments with the fractions $f_1$ and $f_2$, with the respective biological decay constants $\lambda_{b,1}$ and $\lambda_{b,2}$.

For each compartment the activity at time $t$ is then obtained from

$$\frac{\mathrm{d}A}{\mathrm{d}t} = fI - \lambda_{eff} A \tag{6.3.33}$$

$I$ is the activity intake per time unit.

Solving the equation gives

$$A = \frac{fI}{\lambda_{eff}}(1 - e^{\lambda_{eff} t}) \tag{6.3.34}$$

The activity after 80 years should be less than 30 000 Bq. As the biological half life is short (<100 d), equilibrium can be assumed and the total activity is obtained from

$$A_{\text{tot}} = \frac{f_1 I}{\lambda_{\text{eff},1}} + \frac{f_2 I}{\lambda_{\text{eff},2}} \tag{6.3.35}$$

Data:
$f_1$=0.10, $\lambda_{b,1}$=0.347 d$^{-1}$
$f_2$=0.90, $\lambda_{b,2}$=0.00630 d$^{-1}$
$\lambda_f = \ln 2/(365.25 \cdot 30)\,\text{d}^{-1}$
$\lambda_{\text{eff},1}$=0.347+ln2/(365.25·30)=0.347 d$^{-1}$
$\lambda_{\text{eff},2}$=0.00630+ln2/(365.25·30)=0.00636 d$^{-1}$
$A_{\text{tot}}$=30 000 Bq

Data inserted in Eq. 6.3.35 gives

$$30000 = \frac{0.1 \cdot I}{0.347} + \frac{0.9 \cdot I}{0.00636} \tag{6.3.36}$$

$I$=211.6 Bq d$^{-1}$

The concentration of the activity in the food is 300 Bq kq$^{-1}$. This gives a maximum intake of food per day of m=211.6/300=0.71 kg.

**Answer:** The maximum intake of the contaminated food should be 0.71 kg d$^{-1}$.

**Solution exercise 6.10.**
The activity in the water at time $t$ is

$$A_{\text{w}} = A_{\text{w},0} e^{-\lambda_{\text{eff,w}} t} \tag{6.3.37}$$

The activity in the cow at time $t$ is obtained from

$$\frac{\mathrm{d}A_{\text{c}}}{\mathrm{d}t} = I - \lambda_{\text{eff,c}} A_{\text{c}} \tag{6.3.38}$$

$I$ is the intake of activity by the cow per day. $I$ is varying with time as the water activity decreases. Thus

$$I = I_0 e^{-\lambda_{\text{eff,w}} t} \tag{6.3.39}$$

Solution of Eq. (6.3.38) and (6.3.39) gives the variation with time of the activity in the cow.

$$A_{\text{c}} = \frac{I_0}{\lambda_{\text{eff,c}} - \lambda_{\text{eff,w}}} \left(e^{-\lambda_{\text{eff,w}} t} - e^{-\lambda_{\text{eff,c}} t}\right) \tag{6.3.40}$$

Maximal activity is obtained by derivation of Eq. (6.3.40)

$$\frac{dA_c}{dt} = \frac{I_0}{\lambda_{eff,c} - \lambda_{eff,w}}(-\lambda_{eff,w}e^{-\lambda_{eff,w}t} + \lambda_{eff,c}e^{-\lambda_{eff,c}t}) \tag{6.3.41}$$

Maximum is obtained when $\frac{dA_c}{dt} = 0$. Thus

$$\lambda_{eff,w}e^{-\lambda_{eff,w}t} = \lambda_{eff,c}e^{-\lambda_{eff,c}t} \tag{6.3.42}$$

and

$$t_{max} = \frac{\ln \frac{\lambda_{eff,c}}{\lambda_{eff,w}}}{\lambda_{eff,c} - \lambda_{eff,w}} \tag{6.3.43}$$

Thus $A_{c,max}$ is obtained from

$$A_{c,max} = \frac{I_0}{\lambda_{eff,c} - \lambda_{eff,w}}(e^{-\lambda_{eff,w}t_{max}} - e^{-\lambda_{eff,c}t_{max}}) \tag{6.3.44}$$

Data:
$I_0$=70·$2.5 \cdot 10^3$ Bq d$^{-1}$ (initial activity intake in cow per day)

$\lambda_{eff,w} = \frac{1000}{100000} + \frac{\ln 2}{30 \cdot 365.25}$=0.01006 d$^{-1}$ (effective decay constant in water)

$\lambda_{eff,c} = \frac{\ln 2}{100} + \frac{\ln 2}{30 \cdot 365.25}$=6.995·10$^{-3}$ d$^{-1}$ (effective decay constant in cow)

This inserted in Eq. 6.3.43 gives

$$t_{max} = \frac{\ln \frac{6.995 \cdot 10^{-3}}{0.01006}}{6.995 \cdot 10^{-3} - 0.01006} = 118.56\,\mathrm{d} \tag{6.3.45}$$

Data inserted in Eq. (6.3.44) for $A_{c,max}$ gives

$$A_{c,max} = \frac{70 \cdot 2.5 \cdot 10^3}{6.995 \cdot 10^{-3} - 0.01006}(e^{-0.01006 \cdot 118.56} - e^{-6.995 \cdot 10^{-3} \cdot 118.56})$$

$A_{c,max} = 7.59 \cdot 10^6$ Bq

**Answer:** The maximal activity in the cow is 7.6 MBq.

**Solution exercise 6.11.**
The activity in the body at time $t$ is given by

$$A = \frac{I}{\lambda_{eff}}(1 - e^{-\lambda_{eff}t}) \tag{6.3.46}$$

where $I$ is the intake per time unit and $\lambda_{eff}$ is the effective decay constant.

After some months, equilibrium is obtained and then the equation is reduced to

$$A = \frac{I}{\lambda_{\text{eff}}} \tag{6.3.47}$$

$^3$H disintegrates by emitting low energetic $\beta$-radiation. Thus all energy may be assumed to be absorbed in the body.

The absorbed dose during a time $t$ is then obtained from the relation

$$D = \frac{A\bar{E}_\beta}{m}t \tag{6.3.48}$$

Data:
$I$=2·800 Bq d$^{-1}$ (intake in the body per day)
$\lambda_{\text{eff}}$=ln2/10 d$^{-1}$ (physical half life may be neglected (12.3 y)).
$\bar{E}_\beta$=5.68·10$^{-3}$ · 1.602 · 10$^{-13}$ J (mean energy per decay)
$m$=70 kg (body mass)

Data inserted in Eq. (6.3.48) gives the dose during a year when equilibrium has been obtained

$$D = \frac{2 \cdot 800 \cdot 10 \cdot 5.68 \cdot 10^{-3} \cdot 1.602 \cdot 10^{-13} \cdot 365 \cdot 24 \cdot 3600}{\ln 2 \cdot 70} = 9.46 \cdot 10^{-6}\,\text{Gy}$$

During the first year a lower dose is obtained as it takes some time to obtain the equilibrium activity in the body and the cumulated activity is then obtained according to the relation

$$\tilde{A} = \int_0^{365} \frac{I_0}{\lambda_{\text{eff}}}(1 - e^{-\lambda_{\text{eff}}t})dt = \frac{I_0}{\lambda_{\text{eff}}}[t + \frac{e^{-\lambda_{\text{eff}}t}}{\lambda_{\text{eff}}}]_0^{365} \tag{6.3.49}$$

Data inserted gives

$$\tilde{A} = \frac{2 \cdot 800 \cdot 10}{\ln 2}[365 \cdot 24 \cdot 3600 + \frac{e^{-365 \cdot \ln 2/10}}{\ln 2/(10 \cdot 24 \cdot 3600)} - \frac{1}{\ln 2/(10 \cdot 24 \cdot 3600)}]$$

$\tilde{A} = 6.992 \cdot 10^{11}$ Bq s

The absorbed dose is obtained from the relation

$$D = \frac{\tilde{A}\bar{E}_\beta}{m} \tag{6.3.50}$$

Inserted data gives

$D$=9.09·10$^{-6}$ Gy

The effective dose is obtained by multiplying with the radiation quality factor $w_R$ and the tissue weighting factor $w_T$. As $^3H$ is decaying by emitting $\beta$-radiation and the dose distribution is homogeneous, both factors are equal to unity and the effective dose is equal to absorbed dose.

**Answer:** The effective dose during the first year is 9.1 µSv and in the future 9.5 µSv per year.

**Solution exercise 6.12.**

Assuming continuous release of activity, the activity concentration in the room is given by

$$C(t) = \frac{I}{V\lambda_{\mathrm{room}}}(1 - e^{-\lambda_{\mathrm{room}} t}) \tag{6.3.51}$$

where $I$ is the release rate of activity to the air, $V$ is the volume of the room and $\lambda_{\mathrm{room}}$ is the rate of change in the activity in the room due to ventilation and decay.

The activity, $A_{\mathrm{Th}}$, in the thyroid is then given by

$$\frac{\mathrm{d}A_{\mathrm{Th}}}{\mathrm{d}t} = U_{\mathrm{Th}} - \lambda_{\mathrm{eff}} A_{\mathrm{Th}} \tag{6.3.52}$$

$U_{\mathrm{Th}}$ is the uptake in the thyroid per time unit and is given by

$$U_{\mathrm{Th}} = \frac{Ihf}{V\lambda_{\mathrm{room}}}(1 - e^{-\lambda_{\mathrm{room}} t}) \tag{6.3.53}$$

where $h$ is the breathing rate, $f$ is part of activity taken up in the thyroid, $\lambda_{\mathrm{eff}}$ is the effective decay constant in the thyroid, and $\lambda_{\mathrm{room}}$ is the effective ventilation rate constant in the room.

Inserting $U_{\mathrm{Th}}$ in the differential equation (6.3.52), rearranging and integrating gives

$$\int \left( \frac{\mathrm{d}A_{\mathrm{Th}}}{\mathrm{d}t} + \lambda_{\mathrm{eff}} A_{\mathrm{Th}} \right) \mathrm{d}t = \int \frac{Ihf}{V\lambda_{\mathrm{room}}}(1 - e^{-\lambda_{\mathrm{room}} t})\,\mathrm{d}t \tag{6.3.54}$$

Multiplying both sides with the integrating factor $\mathrm{e}^{\lambda_{\mathrm{eff}} t}$ and solving the integrals give

$$A_{\mathrm{Th}} \cdot e^{\lambda_{\mathrm{eff}} t} = \frac{Ihf}{V\lambda_{\mathrm{room}}}\left(\frac{e^{\lambda_{\mathrm{eff}} t}}{\lambda_{\mathrm{eff}}} - \frac{e^{(\lambda_{\mathrm{eff}} - \lambda_{\mathrm{room}})t}}{\lambda_{\mathrm{eff}} - \lambda_{\mathrm{room}}}\right) + C \tag{6.3.55}$$

The constant $C$ is obtained by assuming that the activity $A_{\mathrm{Th}}$=0, when $t$=0.

$$C = \frac{Ihf}{V\lambda_{\mathrm{room}}}\left(-\frac{1}{\lambda_{\mathrm{eff}}} + \frac{1}{\lambda_{\mathrm{eff}} - \lambda_{\mathrm{room}}}\right) \tag{6.3.56}$$

Inserting $C$ in the equation and rearranging gives

$$A_{\mathrm{Th}} = \frac{Ihf}{V\lambda_{\mathrm{room}}}\left(\frac{1-e^{-\lambda_{\mathrm{eff}}t}}{\lambda_{\mathrm{eff}}} + \frac{e^{-\lambda_{\mathrm{eff}}t}-e^{-\lambda_{\mathrm{room}}t}}{\lambda_{\mathrm{eff}}-\lambda_{\mathrm{room}}}\right) \tag{6.3.57}$$

Data:
$I$=0.20 MBq h$^{-1}$ (release of activity to air)
$V$=20 m$^3$ (volume of the room)
$h$=0.020·60 m$^3$ h$^{-1}$ (breathing rate)
$f$=0.30 (uptake in the thyroid)
$t$=8.0 h (working time)
$T_{\mathrm{b}}$=75 d (biological half life)
$T_{\mathrm{f}}$=8.04 d (physical half life)
$\lambda_{\mathrm{v}}$=0.50 h$^{-1}$ (room ventilation rate)

$\lambda_{\mathrm{room}} = 0.50 + \frac{\ln 2}{8.04\cdot 24} = 0.504\ \mathrm{h}^{-1}$

$\lambda_{\mathrm{eff}} = \frac{\ln 2}{75\cdot 24} + \frac{\ln 2}{8.04\cdot 24} = 3.98\cdot 10^{-3}\ \mathrm{h}^{-1}$

Data inserted in Eq. 6.3.57 gives

$$A_{\mathrm{b}} = \frac{0.20\cdot 10^{6}\cdot 0.020\cdot 60\cdot 0.30}{20\cdot 0.504}\left(\frac{1-e^{-3.98\cdot 10^{-3}\cdot 8}}{3.98\cdot 10^{-3}} + \frac{e^{-3.98\cdot 10^{-3}\cdot 8}-e^{-0.504\cdot 8}}{3.98\cdot 10^{-3}-0.504}\right)$$

$A_{\mathrm{b}}$=42.7 kBq

**Answer:** The activity in the thyroid is 43 kBq.

### 6.3.2 Point Radioactive Sources

**Solution exercise 6.13.**
The absorbed dose rate at the calculation point P is, assuming charged particle equilibrium, given by the relation

$$\dot{D} = \frac{Af}{4\pi r^2}B(\mu_{\mathrm{en}}/\rho)_{\mathrm{water}}h\nu e^{-\mu_{\mathrm{Fe}}d_{\mathrm{Fe}}}e^{-\mu_{\mathrm{concrete}}d_{\mathrm{concrete}}} \tag{6.3.58}$$

Data:
$A$=0.63 GBq (source activity)
$B$=3.0 (dose build-up factor)
$f$=2.0 (number of photons per decay)
$h\nu$=1.25 MeV (mean photon energy for $^{60}$Co)
$r$=60 cm (distance source-calculation point)

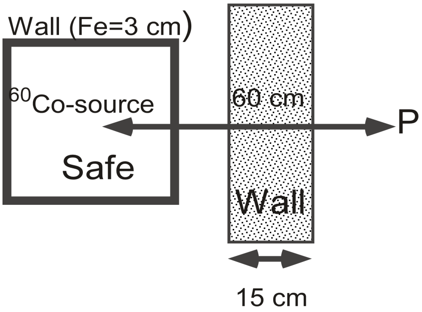


**Figure 6.10:** Illustration of the irradiation geometry in exercise 6.13.

$d_{Fe}$=3.0 cm (safe wall thickness)
$d_{concrete}$=15.0 cm (concrete wall thickness)
$(\mu/\rho)_{Fe}$=0.00535 $m^2\,kg^{-1}$ (mass attenuation coefficient for iron)
$\rho_{Fe} = 7.86 \cdot 10^3$ kg $m^{-3}$ (density of iron)
$(\mu/\rho)_{concrete}$=0.005807 $m^2\,kg^{-1}$ (mass attenuation coefficient for concrete)
$\rho_{concrete} = 2.35 \cdot 10^3$ kg $m^{-3}$ (density of concrete)
$(\mu_{en}/\rho)_{water}$=0.00296 $m^2\,kg^{-1}$ (mass energy absorption coefficient for water)

Data inserted in Eq. 6.3.58 gives

$$\dot{D} = \frac{0.63 \cdot 10^9 \cdot 2 \cdot 3.0 \cdot 0.00296 \cdot 1.25 \cdot 1.602 \cdot 10^{-13}}{4\pi \cdot 0.60^2} \times e^{-0.00535 \cdot 7.86 \cdot 10^3 \cdot 0.03} \cdot e^{-0.005807 \cdot 2.35 \cdot 10^3 \cdot 0.15} = 1.81 \cdot 10^{-8}\ \mathrm{Gy\,s^{-1}}$$

During a week the absorbed dose will be

$D = 1.8 \cdot 10^{-8} \cdot 40 \cdot 3600 = 2.6 \cdot 10^{-3}$ Gy

**Answer**: The absorbed dose during one week will be 2.6 mGy. This is not acceptable according to the ICRP recommendations.

**Solution exercise 6.14.**

The absorbed dose rate at point P outside the safe (see Fig. 6.11) is, assuming charged particle equilibrium, given by the equation

$$\dot{D} = \frac{Af}{4\pi r^2} B(\mu d)(\mu_{en}/\rho)_{water} h\nu e^{-\mu_{Fe} d_{Fe}} \tag{6.3.59}$$

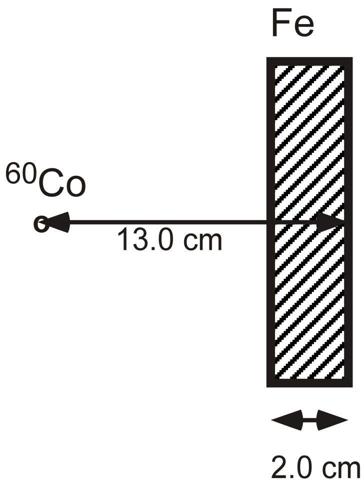


**Figure 6.11:** Illustration of the irradiation geometry in exercise 6.14.

$B(\mu d)$ is the build-up factor given by the Berger expression

$$B = 1 + a\mu d e^{b\mu d} \tag{6.3.60}$$

Data:
$\dot{D}$=10 μGy h$^{-1}$ (dose rate at point $P$)
$h\nu$=1.25 MeV (mean photon energy for $^{60}$Co)
$f$=2.0 (number of photons per decay)
$r$=13.0 cm (distance between source and calculation point)
$d_{Fe}$=2.0 cm (iron thickness)
$(\mu/\rho)_{Fe} = 0.00535$ m$^2$ kg$^{-1}$ (mass attenuation coefficient for Fe)
$\rho_{Fe} = 7.86 \cdot 10^3$ kg m$^{-3}$ (density of Fe)
$(\mu_{en}/\rho)_{water}$=0.00296 m$^2$ kg$^{-1}$ (mass energy absorption coefficient for water)
$a$=0.955 and $b$=0.024 (parameters for Berger expression)

Data inserted in Eq. (6.3.59) and (6.3.60)

$$\frac{10 \cdot 10^{-6}}{3600} = \frac{A \cdot 2.0}{4\pi(0.130)^2}(1 + 0.955 \cdot 0.020 \cdot 0.00535 \cdot 7.86 \cdot 10^3 \cdot e^{0.024 \cdot 7.86 \cdot 10^3 \cdot 0.00535 \cdot 0.020}) \times$$
$$e^{-7.86 \cdot 10^3 \cdot 0.00535 \cdot 0.020} \cdot 0.00296 \cdot 1.25 \cdot 1.602 \cdot 10^{-13}$$

Solving the equation gives

$A$=0.63 MBq

**Answer:** The maximum activity of $^{60}$Co is 0.63 MBq.

**Solution exercise 6.15.**
Method 1:

The kerma in air at P (see Fig. 6.12) is given by the equation

$$K_1 = \frac{A\sum f_i h\nu_i(\mu_{tr}/\rho)_{air,i}}{4\pi r_1^2}B(\mu d)t_1 e^{-\mu_{Pb,i}d} \tag{6.3.61}$$

$B$ is the build-up factor given by the Berger expression

$$B = 1 + a\mu d e^{b\mu d} \tag{6.3.62}$$

Method 2:

The kerma in air at P is now given by the equation

$$K_2 = \frac{A\sum f_i h\nu_i(\mu_{tr}/\rho)_{air,i}}{4\pi r_2^2}t_2 \tag{6.3.63}$$

The ratio of the kerma values is given by

$$\frac{K_1}{K_2} = \frac{\sum f_i h\nu(\mu_{tr}/\rho)_{air,i}4\pi r_2^2 B(\mu d)t_1 e^{-\mu_{Pb,i}d_{Pb}}}{A\sum f_i h\nu(\mu_{tr}/\rho)_{air,i}4\pi r_1^2 t_2} \tag{6.3.64}$$

With only one energy the relation can be simplified to

$$\frac{K_1}{K_2} = \frac{r_2^2 B(\mu d)t_1 e^{-\mu_{Pb}d_{Pb}}}{r_1^2 t_2} \tag{6.3.65}$$

Data:
$A$=200 GBq (activity of the $^{60}$Co-source)
$h\nu$=1.25 MeV (mean photon energy for $^{60}$Co)
$f$=2.0 (number of photons per decay)
$r_1$=50 cm, $r_2$=40 cm (distances between source and measuring point)
$t_1$=600 s, $t_2$=140 s (time for measurement)
$d_{Pb}$=4.0 cm (thickness of lead absorber)
$(\mu/\rho)_{Pb}$=0.00588 m$^2$ kg$^{-1}$ (mass attenuation coefficient for lead)
$\rho_{Pb} = 11.35 \cdot 10^3$ kg m$^{-3}$ (density of lead)
$(\mu_{tr}/\rho)_{air}$=0.00267 m$^2$ kg$^{-1}$ (mass energy transfer coefficient for air)
$a$=0.33 and $b$=-0.011 (parameters for Berger expression)

Data inserted in Eq. 6.3.62 gives

$$B = 1 + 0.33 \cdot 0.00588 \cdot 0.040 \cdot 11.35 \cdot 10^3 \cdot e^{-0.011 \cdot 0.00588 \cdot 0.040 \cdot 11.35 \cdot 10^3} = 1.86$$

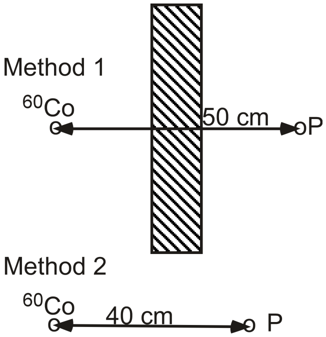


**Figure 6.12:** Illustration of the irradiation geometry in exercise 6.15.

Data inserted in Eq. 6.3.65 gives

$$\frac{K_1}{K_2} = \frac{40^2 \cdot 1.86 \cdot 600 \cdot e^{-0.00588 \cdot 0.040 \cdot 11.35 \cdot 10^3}}{50^2 \cdot 140} = 0.35$$

Method 1 gives the lowest kerma.

The kerma rate in air for method 1 is

$$\dot{K}_1 = \frac{200 \cdot 10^9 \cdot 2 \cdot 1.25 \cdot 1.602 \cdot 10^{-13} \cdot 0.00267 \cdot 1.86 \cdot e^{-0.00588 \cdot 0.040 \cdot 11.35 \cdot 10^3}}{4\pi \cdot 0.5^2}$$

$\dot{K}_1 = 8.752 \cdot 10^{-6}$ Gy s$^{-1}$ = 32 m Gy h$^{-1}$

**Answer:** Method 1 gives lower kerma. The kerma rate in air for this method is 32 m Gy h$^{-1}$.

**Solution exercise 6.16.**
The air kerma at the father is obtained both from scattered radiation and from leakage radiation.

I. Scattered radiation

The air kerma for scattered radiation is given by

$$K_S = \frac{\dot{K} t S}{d^2} \qquad (6.3.66)$$

where
$t$=5 · 4 · 150= 3000 s (total treatment time)
$S$=0.001 (fraction of scattered radiation 1.0 m from the center of the scatterer)

$d$=4.0 m (distance from scatterer to the father)

$\dot{K}$=air kerma rate at the center of the patient given by

$$\dot{K} = \frac{21.8 \cdot 10^{-3} \cdot 1.0^2}{0.9^2}\,\mathrm{Gy\,s^{-1}}$$

Data inserted gives for the scattered radiation

$$K_S = \frac{21.8 \cdot 10^{-3} \cdot 1.0^2 \cdot 3000 \cdot 0.001}{0.9^2 \cdot 4.0^2}\,\mathrm{Gy}$$

The relation, effective dose to air kerma, is 1.09 (given in the exercise text). Thus, the effective dose $E_S$ to the father from the scattered radiation is

$$E_S = K_S \cdot 1.09 \tag{6.3.67}$$

The permissible effective dose $E_P$ to the father shall be below 100 $\mu$Sv.

The necessary reduction of the kerma is then given by the transmission factor

$$B = \frac{E_P}{E_S} \tag{6.3.68}$$

With inserted data

$$B = \frac{0.9^2 \cdot 4^2 \cdot 100 \cdot 10^{-6}}{21.8 \cdot 10^{-3} \cdot 1.0^2 \cdot 3000 \cdot 0.001 \cdot 1.09} = 1.82 \cdot 10^{-2}$$

From transmission data (Fig 26, ICRP 33 (ICRP, 1982)) a thickness of 1.4 cm Pb is obtained.

II. Leakage radiation.

According to ICRP 33 the air kerma rate 1.0 m from the source should be below 0.1% of the primary air kerma rate at 1.0 m.

$$\dot{K}_L = 21.8 \cdot 10^{-3} \cdot 0.1 \cdot 10^{-2}$$

The effective dose to the father from leakage radiation without shielding during the total treatment is then given by

$$E_L = \frac{21.8 \cdot 10^{-3} \cdot 0.1 \cdot 10^{-2} \cdot 3000 \cdot 1.09}{4^2} = 4.46\,\mathrm{mSv}$$

To calculate the necessary absorber thickness, the number of TVT (tenth value thicknesses) is calculated.

The number of TVT is obtained by

$$N_{TVT} = {}^{10}\log \frac{4.46 \cdot 10^{-3}}{100 \cdot 10^{-6}} = 1.65$$

According to Table 4 (ICRP 33) 1 TVT=4.0 cm Pb.

This gives the necessary thickness of lead for the leakage radiation to 4.0·1.65=6.60 cm.

The necessary thickness for the scattered radiation is 1.4 cm Pb. This is more than 1 TVT smaller than for leakage radiation. Thus 6.60 cm is enough according the recommendations of ICRP.

**Answer**: The necessary thickness of lead is 6.6 cm.

**Solution exercise 6.17.**
Wall at point A:

The wall is hit by primary radiation and the transmission is obtained by the relation

$$B = \frac{Pd_\mathrm{p}^2}{W} \tag{6.3.69}$$

where
$P = 10 \cdot 10^{-6}\,\mathrm{Gy\,h^{-1}}$ (permissible absorbed dose rate taking use and occupancy factors into account)
$d_\mathrm{P}$=5.5 m (distance target-point A)
$W$=4.0 Gy min$^{-1}$ (primary absorbed dose rate)

Data inserted in Eq. 6.3.69 gives

$$B = \frac{10 \cdot 10^{-6} \cdot 5.5^2}{60 \cdot 4.0} = 1.26 \cdot 10^{-6} \tag{6.3.70}$$

From Fig. 13 in ICRP 33 (ICRP, 1982) the necessary wall thickness is obtained and around 190 cm concrete.

Wall at point B:

This wall is not hit by primary radiation but only scattered and leakage radiation. The transmission for scattered radiation is obtained from the relation

$$B = \frac{Pd_\mathrm{s}^2}{WS} \tag{6.3.71}$$

where
$P = 100 \cdot 10^{-6}\,\mathrm{Gy\,h^{-1}}$ (permissible absorbed dose rate taking use and occupancy factors into account)
$d_\mathrm{S}$=5.0 m (distance isocenter-point B)
$S = 0.03 \cdot 10^{-2}$ (fraction of scattered radiation)

Data inserted in Eq. 6.3.71 gives

$$B = \frac{100 \cdot 10^{-6} \cdot 5.0^2}{60 \cdot 4.0 \cdot 0.03 \cdot 10^{-2}} = 3.47 \cdot 10^{-2} \tag{6.3.72}$$

From Fig. 27 in ICRP 33 the necessary wall thickness is obtained and around 32 cm concrete.

The number of tenth value thicknesses for the leakage radiation is given by

$$N_{\mathrm{TVT}} = {}^{10}\log \frac{W_{\mathrm{L}}}{d_{\mathrm{S}}^2 P} \tag{6.3.73}$$

where $W_{\mathrm{L}} = 0.1 \cdot 10^{-2} \cdot 4.0$ Gy min$^{-1}$ (absorbed dose rate 1.0 m from isocenter). In principle the distance should be calculated from the target, but often the distance from isocenter to calculation point is used.
The other parameters are the same as above. Data inserted gives

$$N_{\mathrm{TVT}} = {}^{10}\log \frac{0.001 \cdot 4.0}{5.0^2 \cdot 100 \cdot 10^{-6}/60} = {}^{10}\log 96 = 1.98 \tag{6.3.74}$$

Table 3 in ICRP 33 gives that 1 TVT=33.8 cm concrete. Thus the needed wall thickness is 1.98x33.8=67 cm

The needed wall thickness for leakage radiation is more than 1 TVT larger than the wall thickness for scattered radiation. Thus 67 cm is enough.

**Answer:** The wall thickness at point A should be 190 cm and for point B 67 cm concrete.

**Solution exercise 6.18.**
The kerma in water at the patient is given by

$$K = \frac{A\Gamma}{r^2} B \cdot (1 - f) \cdot (\mu_{\mathrm{tr}}/\rho)_{\mathrm{water,air}} \cdot t \tag{6.3.75}$$

$t$=50·3600 s (treatment time)
$f$=0.30 (absorption in the brachytherapy patient)
$E$=0.02 mSv (effective dose during one week)
$K$=0.02 mGy (kerma during one week as $w_{\mathrm{R}}$=$w_{\mathrm{T}}$=1)
$\Gamma$=20.3·10$^{-18}$ Gy s$^{-1}$ Bq$^{-1}$ m$^2$ (air kerma rate constant)
$(\mu_{\mathrm{tr}}/\rho)_{\mathrm{air}}$=0.00294 m$^2$ kg$^{-1}$ (mass energy transfer coefficient for air)
$(\mu_{\mathrm{tr}}/\rho)_{\mathrm{water}}$=0.00327 m$^2$ kg$^{-1}$ (mass energy transfer coefficient for water)
$B$=transmission through the wall
$A$=2.60 GBq (activity of the $^{137}$Cs source)

Data inserted gives

$$0.02 \cdot 10^{-3} = \frac{2.60 \cdot 10^{9} \cdot 20.3 \cdot 10^{-18} \cdot 0.00327}{2^2 \cdot 0.00294} B \cdot (1 - 0.30) \cdot 50 \cdot 3600$$

$B$=1.08·10$^{-2}$

Fig 18 in ICRP 33 (ICRP, 1982) gives 42 cm concrete.

**Answer:** The concrete wall thickness shall be 42 cm.

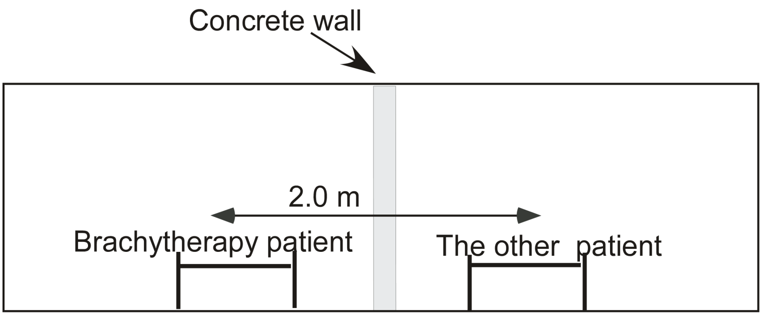


**Figure 6.13:** Illustration of the ward rooms with beds.

### 6.3.3 Extended Radioactive Sources

**Solution exercise 6.19.** The fluence rate $\dot{\Phi}$ from a line source (see Fig. 6.14) is given by

$$\dot{\Phi} = \frac{S_L f}{4\pi h}[F(|\theta_2|, \sum(\mu_i x_i)) + F(|\theta_1|, \sum(\mu_i x_i))] \tag{6.3.76}$$

where

$$S_L = \frac{A}{l}$$

is the line source strength and

$$F(|\theta|, \sum(\mu_i x_i))$$

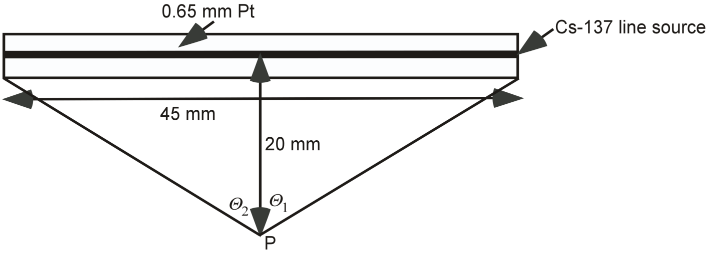


**Figure 6.14:** Brachytherapy line source in exercise 6.19.

is the Sievert integral.

If $P$ is centrally located then $|\theta_2|$=$|\theta_1|$.

If charged particle equilibrium (CPE) is assumed, the absorbed dose rate $\dot{D}$ to water is given by

$$\dot{D} = \dot{\Phi} h\nu(\mu_{en}/\rho)_{water} \tag{6.3.77}$$

Data:
$A$=57.5 GBq (source activity)
$l$=45 mm (length of the brachytherapy source)
$f$=0.85 (number of 0.662 MeV photons per decay)
$h\nu$=0.662 MeV (photon energy)
$h$=20 mm (shortest distance to line source)
$\theta_2 = \theta_1$=0.844 rad (48.4°) (opening angle)
$(\mu_{en}/\rho)_{water}$=0.00325 m$^2$ kg$^{-1}$ (mass energy absorption coefficient for water)
$\mu_{Pt} = (\mu/\rho)_{Pt} \cdot \rho_{Pt} = 0.01062 \cdot 21.45 \cdot 10^3$ m$^{-1}$ (linear attenuation coefficient for Pt)
$\mu_{water}$=8.60 m$^{-1}$ (linear attenuation coefficient for water)
$d_{water}$=20-0.65=19.35 mm (water thickness)
$d_{Pt}$=0.65 mm (platina thickness)
$\sum(\mu_i d_i) = 8.60 \cdot 19.35 \cdot 10^{-3} + 0.01062 \cdot 21.45 \cdot 10^3 \cdot 0.65 \cdot 10^{-3}$=0.3145

$F$ is obtained from Table 6.4 which gives F(0.844,0.3145)=0.593

Data inserted in Eq. (6.3.76) gives

$$\dot{\Phi} = \frac{57.5 \cdot 10^9 \cdot 0.85 \cdot 2 \cdot 0.593}{4\pi \cdot 0.045 \cdot 0.020} = 5.125 \cdot 10^{12}\ \mathrm{m^{-2}s^{-1}}$$

The absorbed dose rate is then

$$\dot{D} = 5.125 \cdot 10^{12} \cdot 0.662 \cdot 1.602 \cdot 10^{-13} \cdot 0.00325 = 1.766 \cdot 10^{-3}\ \mathrm{Gy\,s^{-1}}$$

The required absorbed dose is 2.0 Gy.

This gives the treatment time

$$t = \frac{2.0}{1.766 \cdot 10^{-3}} = 1133\,\text{s}$$

**Answer:** The treatment time is 18.9 min.

**Solution exercise 6.20.**
The fluence rate at the center of the ring is given by

$$\dot{\Phi} = \int_0^{2\pi R} \frac{S_L f\,\mathrm{d}x}{4\pi R^2} e^{-\mu d} = \frac{S_L f}{2R} e^{-\mu d} \tag{6.3.78}$$

The kerma rate to water is

$$\dot{K} = \dot{\Phi} h\nu (\mu_{\text{tr}}/\rho)_{\text{water}} \tag{6.3.79}$$

The total kerma during the time $T$ is

$$K = \int_0^T \dot{K} e^{-\lambda t}\,\mathrm{d}t = \frac{\dot{K}}{\lambda}(1 - e^{-\lambda T}) \tag{6.3.80}$$

Combining Eq. (6.3.78), Eq. (6.3.79) and Eq. (6.3.80) gives

$$K = \frac{S_L f}{2R} e^{-\mu d} h\nu (\mu_{\text{tr}}/\rho)_{\text{water}} \frac{1}{\lambda}(1 - e^{-\lambda T}) \tag{6.3.81}$$

$^{15}O$ decays by $\beta^+$-decay and the positrons are supposed to be annihilated, giving two annihilation photons in opposite directions.

Data:
$S_L$ = 1.30 GBq m$^{-1}$ (line source strength)
$f$=2.0 (number of photons per annihilation)
$h\nu$=0.511 MeV (energy of the annihilation photons)
$(\mu_{\text{tr}}/\rho)_{\text{water}}$=0.00330 m$^2$ kg$^{-1}$ (mass energy transfer coefficient for water)
$(\mu/\rho)_{\text{Fe}}$=0.00834 m$^2$ kg$^{-1}$ (mass attenuation coefficient for iron)
$\rho_{\text{Fe}} = 7.87 \cdot 10^3$ kg m$^{-3}$ (density of iron)
$\lambda$=ln2/122.2 s$^{-1}$ (decay constant for $^{15}O$)
$T$=30.0 min (measurement time)
$R$=0.75 m (betatron radius)
$d$=10 mm (iron thickness)

Data inserted gives

$$K = \frac{1.30 \cdot 10^9 \cdot 2 \cdot 0.511 \cdot 1.602 \cdot 10^{-13} \cdot 0.00330 \cdot 122.2 \cdot e^{-0.00834 \cdot 0.010 \cdot 7.87 \cdot 10^3}}{2 \cdot 0.75 \cdot \ln 2}$$
$$\times (1 - e^{-\frac{30 \cdot 60 \cdot \ln 2}{122.2}})\ \text{Gy}$$

$K$=4.28·10$^{-5}$ Gy

**Answer:** The kerma to a small mass of water at the center of the betatron is 0.043 mGy.

**Solution exercise 6.21.**
The air kerma rate at $x$ (see Fig. 6.15) is given by

$$\dot{K} = \dot{\Phi} h\nu (\mu_{tr}/\rho)_{air} \tag{6.3.82}$$

where

$$\dot{\Phi} = \frac{S_L f}{4\pi h}[F(|\theta_2|, \sum(\mu_i x_i)) + F(|\theta_1|, \sum(\mu_i x_i))] \tag{6.3.83}$$

$^{18}$F decays by $\beta^+$-decay and the positrons are supposed to be annihilated, giving two

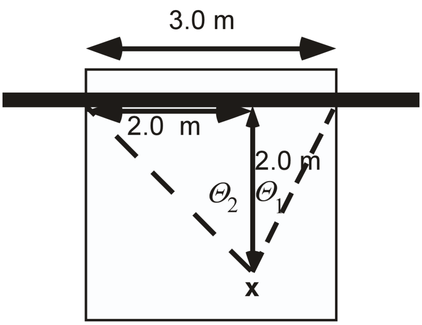


**Figure 6.15:** Sketch of the room with a tube containing a radioactive gas.

annihilation photons in opposite directions.

Data:
$S_L$= 1.20·10$^9$ Bq m$^{-1}$ (line source strength)
$f$=2.0 (number of photons per annihilation)

$h\nu$=0.511 MeV (annihilation photon energy)
$\theta_1$=26.56°=0.4635 rad, $\theta_2$=45°=0.7854 rad (opening angles)
$d_1$=2.0 mm Fe, $d_2$=2.0 m air
$\mu_{Fe}=0.00834{\cdot}7.86 \cdot 10^3\ \mathrm{m}^{-1}$
$\mu_{air}=0.00865{\cdot}1.20\ \mathrm{m}^{-1}$(assuming $T$=293 K and $p$=101.3 kPa)
$\sum(\mu_i d_i)$=0.131+0.021=0.152 (total attenuation thickness)
$(\mu_{tr}/\rho)_{air}=0.00295\ \mathrm{m}^2\ \mathrm{kg}^{-1}$ (mass energy transfer coefficient for air)

$F$ is obtained from Table 6.4 which gives

$F$(45, 0.152)=0.663

$F$(26.56, 0.152)=0.398

Data inserted in Eq. 6.3.82 gives

$$\dot{K} = \frac{1.20 \cdot 10^9 \cdot 2 \cdot 0.511 \cdot 1.602 \cdot 10^{-13} \cdot 0.00295}{4\pi \cdot 2.0}(0.663 + 0.398)\ \mathrm{Gy\,s^{-1}}$$

$\dot{K} = 2.45 \cdot 10^{-8}\ \mathrm{Gy\,s^{-1}}$=88 μGy h$^{-1}$

**Answer:** The air kerma rate is 88 μGy h$^{-1}$. This kerma rate is too high.

**Solution exercise 6.22.**
The fluence rate decreases due to decay of the activity.

The kerma to tissue is thus given by integrating over time

$$K = \int_0^T \dot{\Phi}_0 h\nu(\mu_{tr}/\rho)_{tissue} e^{-\lambda t}\,\mathrm{d}t \tag{6.3.84}$$

$$K = \dot{\Phi}_0 \cdot h\nu(\mu_{tr}/\rho)_{tissue}\frac{1}{\lambda}(1 - e^{-\lambda T}) \tag{6.3.85}$$

The initial fluence rate for an infinite surface area is given by

$$\dot{\Phi}_0 = \frac{S_A f}{2}E_1(\mu d) \tag{6.3.86}$$

where $E_1(\mu d)$ is the exponential integral of the first order.

Data:
$S_A$=450 MBq m$^{-2}$ (area source strength)

$f$=0.812 (number of photons per decay)
$T$=14.0 d (integration time)
$h\nu$=0.364 MeV (main photon energy of $^{131}$I decay)
$(\mu/\rho)_{\mathrm{air}}$=0.00995 m$^2$ kg$^{-1}$ (mass attenuation coefficient for air)
$\rho_{\mathrm{air}} = (1.293 \cdot 100 \cdot 273)/(101.3 \cdot 288)$ kg/m$^{-3}$(density of air at $T$=288 K, $p$=100 kPa)
$d$=1.50 m (air thickness)
$\lambda_{\mathrm{I131}} = \ln 2/(8.04 \cdot 24 \cdot 3600)$ s$^{-1}$
$(\mu_{\mathrm{tr}}/\rho)_{\mathrm{tissue}}$=0.00322 m$^2$ kg$^{-1}$
$(\mu d)_{\mathrm{air}} = 0.0181$
$E_1$(0.0181)=3.462 (Table 6.6)

Data inserted in Eq. (6.3.85) gives

$$K = \frac{450 \cdot 10^6 \cdot 0.812 \cdot 3.462 \cdot 8.04 \cdot 24 \cdot 3600}{2 \cdot \ln 2} \cdot 0.00322 \cdot 0.364$$
$$\times 1.602 \cdot 10^{-13}(1 - e^{-\frac{14 \cdot \ln 2}{8.04}}) = 0.0834\,\mathrm{Gy}$$

**Answer:** The kerma to tissue is 83 mGy.

**Solution exercise 6.23.**
The fluence rate centrally above a circular radioactive area is given by the equation

$$\dot{\Phi} = \frac{S_A f}{2}[E_1(\sum \mu_i x_i) - E_1(\sum \mu_i x_i \sec\theta)] \tag{6.3.87}$$

where $E_1$ is the exponential integral of first order. $\beta$ is the opening angle (see Fig. 6.16). The corresponding kerma rate in air is given by

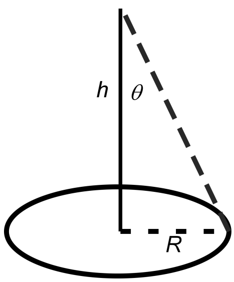


**Figure 6.16:** Circular surface area source.

$$\dot{K} = \dot{\Phi} \sum h\nu_i (\mu_{\mathrm{tr}}/\rho)_{\mathrm{i,air}} \tag{6.3.88}$$

Data:
$S_A$=80000 Bq m$^{-2}$ (area source activity)
$f$=0.898·0.946 (number of 0.662 MeV photons per decay)
$h\nu$=0.662 MeV (photon energy)
$\rho_{\mathrm{air}}=1.293\frac{101.3\cdot 273.1}{101.3\cdot 293.1}=1.205$ kg m$^{-3}$ ($T$=293.1 K, $p$=101.3 kPa)
$(\mu/\rho)_{\mathrm{air}} = 0.007752$ m$^2$ kg$^{-1}$ (mass attenuation coefficient for air)
$(\mu_{\mathrm{tr}}/\rho)_{\mathrm{air}}$=0.00294 m$^2$ kg$^{-1}$ (mass energy transfer coefficient for air)
$h$=1.5 m (height of calculation point above ground)
$(\mu h)_{\mathrm{air}} = 0.007752 \cdot 1.5 \cdot 1.205$
$\theta$=89.57 (opening angle)
sec$\beta$=133.3

Data inserted in Eq. (6.3.87) gives

$$\dot{\Phi} = \frac{80000 \cdot 0.898 \cdot 0.946}{2}[E_1(7.752{\cdot}10^{-3}{\cdot}1.5{\cdot}1.205) - E_1(7.752{\cdot}10^{-3}{\cdot}1.5{\cdot}1.205{\cdot}133.3)]$$

$$\dot{\Phi} = 33980[E_1(1.395 \cdot 10^{-2}) - E_1(1.868)]\ \mathrm{m^{-2}\,s^{-1}}$$

$E_1$ is obtained from Table 6.5. This gives

$E_1$(0.01395)=3.721 and $E_1$(1.868)=0.059

This inserted gives $\dot{\Phi} = 33980(3.721 - 0.059)\ \mathrm{m^{-2}\,s^{-1}}$

and

$\dot{K} = 33980(3.721 - 0.059) \cdot 0.662 \cdot 1.602 \cdot 10^{-13} \cdot 0.00294 = 38.8 \cdot 10^{-12}$ Gy s$^{-1}$ =0.140 μGy h$^{-1}$

Answer: The maximal air kerma rate is 0.14 μGy h$^{-1}$.

**Solution exercise 6.24.**
The fluence rate, due to primary photons centrally above the circular area with infinite radius covered with snow, is given by the equation

$$\dot{\Phi}_{\mathrm{P,A}} = \frac{S_A f}{2} E_1(\mu_{\mathrm{snow}} x_{\mathrm{snow}} + \mu_{\mathrm{air}} x_{\mathrm{air}}) \tag{6.3.89}$$

where $E_1$ is the exponential integral of first order.

To calculate the fluence rate from secondary photons the build-up factor given by the Berger expression

$$B = 1 + a\mu x e^{b\mu x} \tag{6.3.90}$$

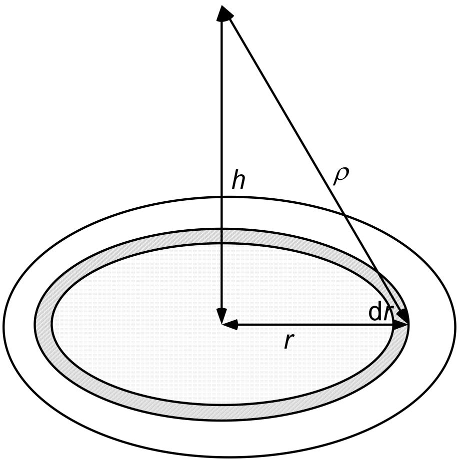


**Figure 6.17:** Circular surface area source for exercise 6.24.

is used. The second factor gives the ratio of the secondary to the primary photons. Thus the contribution from the secondary photons is given by

$$\dot{\Phi}_{\mathrm{S,A}} = S_A f \int_0^{\infty} \frac{a\mu x\rho e^{-(1-b)\mu x(\rho/h)} 2\pi r\,\mathrm{d}r}{4\pi\rho^2 h} \tag{6.3.91}$$

where $\mu x$ is total attenuation thickness and $r$ is the radius and $\rho = \sqrt{h^2 + r^2}$ (see Fig. 6.17).

Substituting $r$ for $\rho$ and rearranging, the integral can be rewritten as

$$\dot{\Phi}_{\mathrm{S,A}} = \frac{S_{\mathrm{A}} f a 2\pi\mu\, x}{4\pi h} \int_h^{\infty} \frac{\rho\rho e^{-(1-b)\mu x(\rho/h)}\,\mathrm{d}\rho}{\rho^2} \tag{6.3.92}$$

Solving the integral gives

$$\dot{\Phi}_{\mathrm{S,A}} = \frac{S_{\mathrm{A}} f a}{2(1-b)} e^{-(1-b)\mu x} \tag{6.3.93}$$

The corresponding kerma rate in air for a fluence rate $\dot{\Phi}$ is given by

$$\dot{K} = \dot{\Phi} \sum h\nu_i (\mu_{\mathrm{tr}}/\rho)_{\mathrm{i,air}} \tag{6.3.94}$$

In principle there should be different kerma factors for primary and secondary photons as they have different energies. However, assuming that the Berger expression holds for kerma or dose, the same kerma factor will be applied for primary and secondary photons.

Data:
$S_A$ = 10000 Bq m$^{-2}$ (area source strength)
$f$=0.80 (number of photons per decay)
h$\nu$=0.80 MeV (photon energy)
$(\mu_{\mathrm{tr}}/\rho)_{\mathrm{air}} = 0.00289$ m$^2$ kg$^{-1}$ (mass energy transfer coefficient for air)
$\rho_{\mathrm{snow}} = 0.25 \cdot 10^3$ kg m$^{-3}$ (density for snow)
$\mu_{\mathrm{snow}} = 0.00787 \cdot 0.25 \cdot 10^3$ m$^{-1}$ (linear attenuation coefficient for snow)
$\rho_{\mathrm{air}} = 1.297$ kg m$^{-3}$ (density for air if T=268 K, p=100 kPa)
$\mu_{\mathrm{air}} = 0.00707 \cdot 1.297$ m$^{-1}$ (linear attenuation coefficient for air)
$x_{\mathrm{snow}}$=50 cm (snow thickness)
$x_{\mathrm{air}}$=100 cm (thickness of air)
$a$=1.74, $b$=0.045 (obtained for Berger parameters for water)

***Kerma rate without snow***

Data inserted gives:

Primary photons
$E_1(1.50 \cdot 0.00707 \cdot 1.297) = E_1(1.375 \cdot 10^{-2})$

Table 6.5 gives $E_1$=3.74

The primary photon fluence rate is then given by Eq. 6.3.89

$\dot{\Phi}_{\mathrm{P}} = 10000 \cdot 0.8 \cdot 3.74/2 = 1.496 \cdot 10^4$ m$^{-2}$ s$^{-1}$

Secondary photons
Eq. 6.3.93 gives

$$\dot{\Phi}_{\mathrm{S}} = \frac{10000 \cdot 0.8 \cdot 1.74}{2(1-0.045)} e^{-(1-0.045)1.375 \cdot 10^{-2}} = 7.19 \cdot 10^3 \mathrm{m}^{-2}\,\mathrm{s}^{-1}$$

Total photon fluence rate is then

$\dot{\Phi} = 1.496 \cdot 10^4 + 7.19 \cdot 10^3 = 2.215 \cdot 10^4$ m$^{-2}$ s$^{-1}$

The air kerma rate is

$\dot{K} = 2.215 \cdot 10^4 \cdot 0.00289 \cdot 0.8 \cdot 1.602 \cdot 10^{-13} = 8.204 \cdot 10^{-12}$ Gy s$^{-1}$

$\dot{K}$ =0.26 mSv (year)$^{-1}$

***Kerma rate with snow***

Data inserted gives

Primary photons
$E_1(100 \cdot 0.0707 \cdot 1.297 \cdot 10^{-3} + 0.0787 \cdot 0.25 \cdot 50) = E_1(0.993)$

Table 6.5 gives $E_1$=0.220

The primary photon fluence rate is then given by

$\dot{\Phi}_{\mathrm{P,A}} = 10000 \cdot 0.8 \cdot 0.220/2 = 880\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}$

Secondary photons

$$\dot{\Phi}_{\mathrm{S,A}} = \frac{10000 \cdot 0.8 \cdot 1.74}{2(1-0.045)} e^{-(1-0.045)0.993} = 2.823 \cdot 10^3\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}$$

Total photon fluence rate is then

$\dot{\Phi}_A = 880 + 2.823 \cdot 10^3 = 3.703 \cdot 10^3\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}$

The air kerma rate is

$\dot{K} = 3.703 \cdot 10^3 \cdot 0.00289 \cdot 0.8 \cdot 1.602 \cdot 10^{-13} = 1.371 \cdot 10^{-12}\ \mathrm{Gy\ s}^{-1}$

$\dot{K} = 0.043\ \mathrm{mGy\ (year)}^{-1}$

**Answer:** The air kerma rate without snow is 0.26 mGy $(\mathrm{year})^{-1}$ and with snow 0.043 mGy $(\mathrm{year})^{-1}$

**Solution exercise 6.25.**
The primary photon fluence rate at P is given by (see Fig. 6.18)

$$\dot{\Phi}_{\mathrm{P}} = \int_{H}^{H+L} \frac{S_L f e^{-\mu_{\mathrm{water}} H} e^{-\mu_{\mathrm{rod}}(x-H)}}{4\pi x^2}\, \mathrm{d}x \tag{6.3.95}$$

where $\mu_{\mathrm{rod}}$ is the linear attenuation coefficient in the uranium rod, $\mu_{\mathrm{water}}$ is the linear attenuation coefficient in water, and $S_L$ is linear source strength and $f$ is the number of photons per decay.

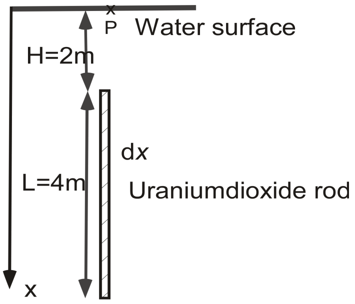


**Figure 6.18:** Uraniumdioxide rod placed in a water basin.

Rewriting the equation gives

$$\dot{\Phi}_{\mathrm{P}} = \frac{S_L f e^{-H(\mu_{\mathrm{water}}-\mu_{\mathrm{rod}})}}{4\pi} \int_{H}^{H+L} \frac{e^{-\mu_{\mathrm{rod}} x}}{x^2}\,\mathrm{d}x \tag{6.3.96}$$

Substitute $\mu_{\mathrm{rod}} x$=y, dx=dy/$\mu_{\mathrm{rod}}$

$$\dot{\Phi}_{\mathrm{P}} = \frac{S_L f e^{-H(\mu_{\mathrm{water}}-\mu_{\mathrm{rod}})}}{4\pi} \int_{\mu_{\mathrm{rod}} H}^{\mu_{\mathrm{rod}}(H+L)} \frac{\mu_{\mathrm{rod}} e^{-y}}{y^2}\,dy \tag{6.3.97}$$

Using the definition of exponential integrals the solution is

$$\dot{\Phi}_{\mathrm{P}} = \frac{S_L f e^{-H(\mu_{\mathrm{water}}-\mu_{\mathrm{rod}})}}{4\pi} \left[\frac{E_2(\mu_{\mathrm{rod}} H)}{H} - \frac{E_2(\mu_{\mathrm{rod}}(H+L))}{H+L}\right] \tag{6.3.98}$$

For large z, $E_2(z) \approx \frac{e^{-z}}{z}$ and thus $E_2(\mu_{\mathrm{rod}} H) \approx \frac{e^{-\mu_{\mathrm{rod}} H}}{\mu_{\mathrm{rod}} H}$

Also, $\frac{E_2(\mu_{\mathrm{rod}} H)}{H}$ is much larger than $\frac{E_2(\mu_{\mathrm{rod}}(H+L))}{H+L}$. This means that the second term may be neglected.

This gives

$$\dot{\Phi}_{\mathrm{P}} = \frac{S_L f e^{-H(\mu_{\mathrm{water}}-\mu_{\mathrm{rod}})}}{4\pi} \frac{e^{-\mu_{\mathrm{rod}} H}}{\mu_{\mathrm{rod}} H H} = \frac{S_L f e^{-H\mu_{\mathrm{water}}}}{4\pi \mu_{\mathrm{rod}} H^2} \tag{6.3.99}$$

Data:
To calculate $(\mu/\rho)_{\mathrm{rod}}$ for uraniumdioxide ($UO_2$) use the Bragg additivity rule.

$$(\mu/\rho)_{\mathrm{UO_2}} = \omega_{\mathrm{U}}(\mu/\rho)_{\mathrm{U}} + \omega_{\mathrm{O}}(\mu/\rho)_{\mathrm{O}} \tag{6.3.100}$$

Mass fraction: $\omega_U$ : $238.2/270.3 = 0.8816$; $\omega_O$ : $32/270.3 = 0.1184$

$(\mu/\rho)_U = 0.00754\,m^2\,kg^{-1}$, $(\mu/\rho)_O = 0.00637\,m^2\,kg^{-1}$
$(\mu/\rho)_{rod}$=0.8816·0.00754+0.1184·0.00637=0.00740 $m^2\,kg^{-1}$
$\rho_{UO_2}$=10.96·$10^3$ kg $m^{-3}$ (density of uraniumdioxide)
$(\mu/\rho)_{water}$=0.00707 $m^2\,kg^{-1}$ (mass attenuation coefficient for water)
$\rho_{water}$=1.00·$10^3$ kg $m^{-3}$ (density of water)
$S_L$=3.7·$10^{16}$/4 Bq $m^{-1}$ (line source strength)
$f$=1.0 (number of photons per decay)
$H$ = 2.0 m (distance from line source to water surface)
$L$ = 4.0 m (length of line source)

Data inserted gives

$$\dot{\Phi}_P = \frac{3.7 \cdot 10^{16} \cdot 1 \cdot e^{-2 \cdot 0.00707 \cdot 1000}}{4 \cdot 4\pi \cdot 0.00740 \cdot 10.96 \cdot 10^3 \cdot 2^2} = 1.640 \cdot 10^6\,m^{-2}s^{-1}$$

This solution however probably underestimates the fluence rate at the water surface. This fluence rate is obtained only just above the line source and photons emitted nearly parallel with the source will only be absorbed in water. An estimate of the maximum fluence rate may be obtained if the attenuation coefficient for uranium dioxide is exchanged for the attenuation coefficient for water. Then the relation will be

$$\dot{\Phi}_P = \frac{S_L \cdot e^{-0}}{4\pi}\left[\frac{E_2(\mu_{water}(2))}{2} - \frac{E_2(\mu_{water}(6))}{6}\right] \qquad (6.3.101)$$

This will give the fluence rate

$\dot{\Phi}_P = 6.63 \cdot 10^7\,m^{-2}\,s^{-1}$

This is 40 times larger than when using the self absorption in the uranium dioxide rod.

**Answer:** The fluence rate directly above the uranium rod is 1.64·$10^6\,m^{-2}\,s^{-1}$. However close to this point the fluence rate is probably much higher due to less attenuation in water, compared to in uranium dioxide.

**Solution exercise 6.26.**
***1) Primary photons. Infinite extension of the wall.***

a) Without Pb

According to the equation for an infinite volume source, the fluence rate from primary photons at the surface is given by

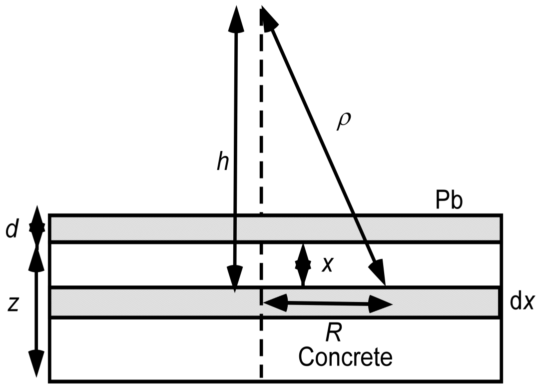


**Figure 6.19:** Radioactive concrete wall with lead shielding.

$$\dot{\Phi}_{P,V} = \frac{S_V f}{2\mu_s}[1 - E_2(\mu_s z)] \tag{6.3.102}$$

Data:
$\mu_s = 6.495 \cdot 10^{-3} \cdot 2.34 \cdot 10^3 = 15.20\ m^{-1}$ (linear attenuation coefficient for the concrete wall)
$z = 0.40$ m (wall thickness)
$S_V = 7.4 \cdot 10^3\ Bq\ m^{-3}$ (volume source strength)
$f = 2$ (number of photons per decay)

Data inserted in Eq. (6.3.102) gives (see Table 6.5)

$$\dot{\Phi}_{P,V} = \frac{2 \cdot 7.4 \cdot 10^3}{2 \cdot 15.20}[1 - E_2(15.20 \cdot 0.4)] = 4.868 \cdot 10^2(1 - 2.91 \cdot 10^{-4}) = 487\ m^{-2}s^{-1}$$

$\dot{\Phi}_{P,V} = 487\ m^{-2}\ s^{-1}$

b) With Pb

With extra shielding material the equation for the fluence rate of primary photons becomes

$$\dot{\Phi}_{P,V} = \frac{S_V f}{2\mu_s}[E_2(\mu_{Pb} d) - E_2(\mu_{Pb} d + \mu_s z)] \tag{6.3.103}$$

Data:
$\mu_{Pb} = 7.102 \cdot 10^{-3} \cdot 11.35 \cdot 10^3 = 80.54\ m^{-1}$ (linear attenuation coefficient for Pb)
d=0.020 m (Pb thickness)

Data inserted in Eq. 6.3.103 gives

$$\dot{\Phi}_{\mathrm{P,V}} = \frac{2\cdot 7.4\cdot 10^3}{2\cdot 15.20}[E_2(80.54\cdot 0.02) - E_2(80.54\cdot 0.02 + 15.20\cdot 0.4)]\,\mathrm{m}^{-2}\mathrm{s}^{-1}$$

$$\dot{\Phi}_{\mathrm{P,V}} = 4.87\cdot 10^2[E_2(1.611) - E_2(7.69)] = 4.87\cdot 10^2[0.064 - 5.51\cdot 10^{-5}]{=}31\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$$

***2) Secondary photons.***

Use the expression by Berger for the build-up factor.

$$B = 1 + a\mu x e^{b\mu x}$$

For a plane source with radius $R$ the fluence rate from secondary photons at a height $h$ is without external attenuation then given by (see Fig. 6.19).

$$\dot{\Phi}_{\mathrm{S}} = S_A f\int_0^R \frac{a\mu_s x\rho e^{-(1-b)\mu_s x\frac{\rho}{h}}2\pi r\,\mathrm{d}r}{h4\pi\rho^2} = \frac{S_A f}{2}\int_h^{\sqrt{h^2+R^2}}\frac{a\mu_s x\rho\rho e^{-(1-b)\mu_s x\frac{\rho}{h}}\,\mathrm{d}\rho}{h\rho^2}$$

The equation is reduced to

$$\dot{\Phi}_{\mathrm{S,A}} = \frac{S_A f}{2}\int_h^{\sqrt{h^2+R^2}}\frac{a\mu_s x e^{-(1-b)\mu_s x\frac{\rho}{h}}\,\mathrm{d}\rho}{h} \tag{6.3.104}$$

Substitute $y = \frac{\mu_s x\rho}{h}$ ; $\mathrm{d}y = \frac{\mu_s x\,\mathrm{d}\rho}{h}$

$$\dot{\Phi}_{\mathrm{S,A}} = \frac{S_A fa}{2}\int_{\mu_s x}^{\mu_s x\sqrt{1+\frac{R^2}{h^2}}}\frac{h\mu_s x e^{-(1-b)y}}{\mu_s x h}\,\mathrm{d}y = \frac{S_A fa}{2}\int_{\mu_s x}^{\mu_s x\sqrt{1+\frac{R^2}{h^2}}} e^{-(1-b)y}\,\mathrm{d}y \tag{6.3.105}$$

$$\dot{\Phi}_{\mathrm{S,A}} = \frac{S_A fa}{2(b-1)}[e^{-(1-b)\mu_s x\sqrt{1+\frac{R^2}{h^2}}} - e^{-(1-b)\mu_s x}] \tag{6.3.106}$$

Assume infinite extension of the wall. Then $R\to\infty$ and

$$\dot{\Phi}_{\mathrm{S}} = \frac{S_A fa}{2(1-b)}e^{-(1-b)\mu_s x} \tag{6.3.107}$$

The plane source is integrated over the wall thickness $z$ and including the Pb-shield, the fluence rate will be given by

$$\dot{\Phi}_{\mathrm{S,V}} = \frac{S_V fa}{2(1-b)}\int_0^z e^{-(1-b)(\mu_{\mathrm{Pb}}d+\mu_s x)}\,\mathrm{d}x = \frac{S_V fae^{-(1-b)\mu_{\mathrm{Pb}}d}}{2(1-b)}\int_0^z e^{-(1-b)\mu_s x}\,\mathrm{d}x \tag{6.3.108}$$

Solving the integral gives

$$\dot{\Phi}_{\mathrm{S,V}} = \frac{S_V f a e^{-(1-b)\mu_{\mathrm{Pb}} d}}{2(1-b)^2 \mu_s}[1 - e^{-(1-b)\mu_s z}] \tag{6.3.109}$$

a) Without Pb

Then $d$=0 and Eq. 6.3.109 is reduced to

$$\dot{\Phi}_{\mathrm{S,V}} = \frac{S_V f a}{2(1-b)^2 \mu_s}[1 - e^{-(1-b)\mu_s z}] \tag{6.3.110}$$

Data:

a=1.27, b=0.032 (parameters for Berger equation for concrete)

Data inserted in Eq. (6.3.110) gives

$$\dot{\Phi}_{\mathrm{S,V}} = \frac{2 \cdot 7.4 \cdot 10^3 \cdot 1.27}{2(1-0.032)^2 15.20}[1 - e^{-(1-0.032)15.20 \cdot 0.4}] = 658\,\mathrm{m}^{-2}\mathrm{s}^{-1}$$

$\dot{\Phi}_{\mathrm{S,V}}$=658 m$^{-2}$ s$^{-1}$

b) With Pb

In the Berger expression use data for lead over the whole thickness as lead is the outermost material ($a$=0.30, $b$=-0.015).

Data inserted in Eq. (6.3.109) gives

$$\dot{\Phi}_{\mathrm{S,V}} = \frac{2 \cdot 7.4 \cdot 10^3 \cdot 0.30 \cdot e^{-(1+0.015)0.02 \cdot 80.51}}{2(1+0.015)^2 \cdot 15.20}[1 - e^{-(1+0.015)15.20 \cdot 0.4}]$$

$\dot{\Phi}_{\mathrm{S,V}}$=27.6 m$^{-2}$ s$^{-1}$

This probably underestimates the contribution. If instead the Berger parameters for concrete are used the secondary fluence rate is 139 m$^{-2}$ s$^{-1}$. This is then instead an overestimation.

Total fluence rate is thus

a) Without Pb: 487+658=1145 m$^{-2}$ s$^{-1}$

b) With Pb: 31+28=59 m$^{-2}$ s$^{-1}$

***Discussion of the approximation of infinite wall area.***

Consider only primary photons.

I. Infinite area source
$\Phi_\infty = (S_A/2)E_1(\mu_s x)$

II. Finite area source
$\dot{\Phi}_{\mathrm{area}} = (S_A/2)[E_1(\mu_s x) - E_1(\mu_s x(\rho/h))]$

where $x$ is the thickness of the absorber of lead, $h$ is the height from area source to calculation point, and $\rho$ is the distance from calculation point to outer radius of the source.

Assume that the wall may be approximated with a circular area equal to the real area (6.0x2.0=12.0 m$^2$). Then the radius will be R=1.95 m. This gives $\rho/h = \sqrt{1 + 1.95^2}/1$=2.19 and $\mu_{\mathrm{Pb}} x \cdot \rho/h$=3.53

This gives the fluence rates

$$\dot{\Phi}_\infty = (S_A/2)E_1(1.61) = (S_A/2) \cdot 8.57 \cdot 10^{-2}\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}$$

$$\dot{\Phi}_{\mathrm{area}} = (S_A/2)(8.57 \cdot 10^{-2} - 6.78 \cdot 10^{-3}) = (S_{\mathrm{A}}/2) \cdot 7.89 \cdot 10^{-2}\ \mathrm{m}^{-2}\ \mathrm{s}^{-1}$$

The ratio between the two fluences is then 1.08. Assuming a similar relation for the secondary photons the approximation of an infinite area then overestimates the fluence with around 10%.

**Answer:** Assuming infinite extension the fluence rate is without Pb 1.01·10$^3$ m$^{-2}$ s$^{-1}$ and with Pb 1.8·10$^2$ m$^{-2}$ s$^{-1}$. The assumption of infinite extension overestimates the fluence rate with about 10%.

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**Table 6.4:** Data for the Sievert integral (Sievert, 1921) for various values of the parameters $\Theta$(5° – 90°)and $\Sigma\mu_i x_i$ (0-10).

| $\Sigma\mu_i x_i/\Theta$ | 5° | 10° | 15° | 20° | 30° | 40° |
|---|---|---|---|---|---|---|
| 0.00 | 0.08727 | 0.17453 | 0.26180 | 0.34906 | 0.52359 | 0.69812 |
| 0.05 | 0.08300 | 0.16597 | 0.24888 | 0.33169 | 0.49684 | 0.66102 |
| 0.10 | 0.07895 | 0.15784 | 0.23661 | 0.31519 | 0.47261 | 0.62588 |
| 0.20 | 0.07143 | 0.14275 | 0.21384 | 0.28445 | 0.42452 | 0.56116 |
| 0.40 | 0.05847 | 0.11675 | 0.17467 | 0.23204 | 0.34421 | 0.45120 |
| 0.60 | 0.04786 | 0.09549 | 0.14268 | 0.18919 | 0.27912 | 0.36289 |
| 0.80 | 0.03917 | 0.07826 | 0.11655 | 0.15426 | 0.22635 | 0.29196 |
| 1.00 | 0.03206 | 0.06388 | 0.09520 | 0.12577 | 0.18358 | 0.23496 |
| 1.25 | 0.02496 | 0.04969 | 0.07393 | 0.09745 | 0.14131 | 0.17916 |
| 1.50 | 0.01943 | 0.03865 | 0.05741 | 0.07551 | 0.10878 | 0.13668 |
| 1.75 | 0.01513 | 0.03006 | 0.04458 | 0.05851 | 0.08375 | 0.10431 |
| 2.00 | 0.01178 | 0.02357 | 0.03462 | 0.04533 | 0.06449 | 0.07964 |
| 2.50 | 0.00714 | 0.01431 | 0.02088 | 0.02722 | 0.03825 | 0.04649 |
| 3.00 | 0.00433 | 0.00856 | 0.01259 | 0.01635 | 0.02270 | 0.02718 |
| 3.50 | 0.00262 | 0.00518 | 0.00760 | 0.00982 | 0.01348 | 0.01591 |
| 4.00 | 0.00159 | 0.00313 | 0.00458 | 0.00590 | 0.00800 | 0.00933 |
| 5.00 | 0.00058 | 0.00115 | 0.00167 | 0.00213 | 0.00283 | 0.00322 |
| 6.00 | 0.00021 | 0.00042 | 0.00061 | 0.00077 | 0.00100 | 0.00112 |
| 8.00 | 0.00003 | 0.00006 | 0.00008 | 0.00010 | 0.00013 | 0.00014 |
| 10.00 | 0.0000 | 0.00001 | 0.00001 | 0.00001 | 0.00002 | 0.00002 |

| $\Sigma\mu_i x_i/\Theta$ | 50° | 60° | 70° | 80° | 90° |
|---|---|---|---|---|---|
| 0.00 | 0.87265 | 1.04718 | 1.22171 | 1.39624 | 1.57077 |
| 0.05 | 0.82359 | 0.98346 | 1.13830 | 1.28119 | 1.36517 |
| 0.10 | 0.77725 | 0.92378 | 1.06115 | 1.17832 | 1.22863 |
| 0.20 | 0.69256 | 0.81547 | 0.92368 | 1.00282 | 1.02368 |
| 0.40 | 0.55015 | 0.63677 | 0.70403 | 0.74075 | 0.74521 |
| 0.60 | 0.43743 | 0.49850 | 0.54043 | 0.55782 | 0.55889 |
| 0.80 | 0.348 11 | 0.39120 | 0.38776 | 0.42578 | 0.42797 |
| 1.00 | 0.27727 | 0.30768 | 0.32411 | 0.32867 | 0.32829 |
| 1.25 | 0.20865 | 0.22858 | 0.23888 | 0.23948 | 0.23949 |
| 1.50 | 0.15755 | 0.17033 | 0.17548 | 0.17621 | 0.17621 |
| 1.75 | 0.11898 | 0.12727 | 0.13018 | 0.13049 | 0.13049 |
| 2.00 | 0.09049 | 0.09534 | 0.09699 | 0.09712 | 0.09712 |
| 2.50 | 0.05159 | 0.05387 | 0.05440 | 0.05442 | 0.05442 |
| 3.00 | 0.02970 | 0.03067 | 0.03084 | 0.03085 | 0.03085 |
| 3.50 | 0.01716 | 0.01758 | 0.01763 | 0.01763 | 0.01763 |
| 4.00 | 0.00995 | 0.01013 | 0.01015 | 0.01015 | 0.01015 |
| 5.00 | 0.00337 | 0.00341 | 0.00341 | 0.00341 | 0.00341 |
| 6.00 | 0.00116 | 0.00116 | 0.00116 | 0.00116 | 0.00116 |
| 8.00 | 0.00014 | 0.00014 | 0.00014 | 0.00014 | 0.00014 |
| 10.00 | 0.00017 | 0.00017 | 0.00017 | 0.00017 | 0.00017 |

**Table 6.5:** Exponential integral functions, $E_1(x)$ and $E_2(x)$. for $x$ between 0 and 10. Data taken from Handbook of Mathematical Functions (Abramowitz and Stegun Eds, 1964)

| x | $E_1(x)$ | $E_2(x)$ |
|---|---|---|
| 0.0 | $\infty$ | 1.0000 |
| 0.01 | 4.03793 | 0.94967 |
| 0.015 | 3.63743 | 0.93053 |
| 0.02 | 3.35471 | 0.91310 |
| 0.025 | 3.31365 | 0.89688 |
| 0.03 | 2.95912 | 0.88166 |
| 0.04 | 2.68127 | 0.85353 |
| 0.05 | 2.46790 | 0.82783 |
| 0.06 | 2.29531 | 0.80405 |
| 0.07 | 2.15084 | 0.78183 |
| 0.08 | 2.02695 | 0.76095 |
| 0.09 | 1.91875 | 0.74124 |
| 0.10 | 1.82292 | 0.72254 |
| 0.15 | 1.46447 | 0.64103 |
| 0.20 | 1.22265 | 0.57419 |
| 0.25 | 1.04428 | 0.51773 |
| 0.30 | 0.90568 | 0.46911 |
| 0.35 | 0.79419 | 0.42671 |
| 0.40 | 0.70238 | 0.38936 |
| 0.45 | 0.62533 | 0.35622 |
| 0.50 | 0.55977 | 0.32664 |
| 0.60 | 0.45437 | 0.27618 |
| 0.70 | 0.37376 | 0.23494 |
| 0.80 | 0.31059 | 0.20085 |
| 0.90 | 0.26018 | 0.17240 |
| 1.00 | 0.21938 | 0.14849 |
| 1.10 | 0.18599 | 0.12828 |
| 1.20 | 0.15840 | 0.11110 |
| 1.30 | 0.13545 | 0.09644 |
| 1.40 | 0.11621 | 0.08388 |
| 1.50 | 0.10001 | 0.07310 |
| 1.60 | 0.08630 | 0.06380 |
| 1.70 | 0.07465 | 0.05577 |
| 1.80 | 0.06471 | 0.04881 |
| 2.00 | 0.04890 | 0.03753 |
| 2.50 | 0.02492 | 0.01980 |
| 3.00 | 0.01305 | 0.01064 |
| 3.50 | 0.00697 | 0.00580 |
| 4.00 | 0.00378 | 0.00320 |
| 5.00 | 0.00115 | 0.00010 |
| 6.00 | 0.00036 | 0.00032 |
| 7.00 | 1.15E-4 | 1.04E-4 |
| 8.00 | 3.77E-5 | 3.41E-5 |
| 9.00 | 1.24E-5 | 1.14E-5 |
| 10.00 | 4.16E-6 | 3.83E-6 |