

3 Detectors and Measurements

3.1 Definitions and Relations

3.1.1 Counting Statistics

The Binomial Distribution

The most general statistical distribution is the Binomial distribution. The other distributions can be derived from this one. If n is the number of trials with a success probability of p , then the probability of obtaining x successful trials is given by

$$P(x) = \frac{n!}{(n-x)!x!} p^x (1-p)^{n-x} \quad (3.1.1)$$

$P(x)$ is a predicted probability distribution function defined only for integer values of n and x

The mean value of the distribution is

$$\bar{x} = pn \quad (3.1.2)$$

and the standard deviation is

$$\sigma = \sqrt{\bar{x}(1-p)} \quad (3.1.3)$$

The Poisson Distribution

If the success probability p is small and the number of trials n large then the Binomial distribution can be simplified to

$$P(x) = \frac{\bar{x}^x e^{-\bar{x}}}{x!} \quad (3.1.4)$$

where $\bar{x} = pn$. This is called the Poisson distribution.

Note that there is only one parameter, the mean value \bar{x} needed to describe the Poisson distribution. The standard deviation is given by

$$\sigma = \sqrt{\bar{x}} \quad (3.1.5)$$

The Gaussian or Normal Distribution

If besides a low value of p , the mean value is large, the binomial distribution may be approximated by the Gaussian distribution

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{(-\frac{(x-\bar{x})^2}{2\sigma^2})} \quad (3.1.6)$$

If we use the information from the Poisson distribution that $\sigma^2 = \bar{x}$, then the Gaussian distribution may be written as

$$P(x) = \frac{1}{\sqrt{2\pi\bar{x}}} e^{\left(-\frac{(x-\bar{x})^2}{2\bar{x}}\right)} \quad (3.1.7)$$

This distribution is very useful as it is symmetric and continuous, contrary to the other distributions. It is also normalized like the other distributions

$$\int_0^{\infty} P(x) dx = 1 \quad (3.1.8)$$

This distribution is also described by only one single parameter $\bar{x} = np$ and the standard deviation is $\sigma = \sqrt{\bar{x}}$, as the Poisson distribution.

In general the standard deviation, σ , of a normal distribution can be independent of the mean value, \bar{x} , and then is a need for two parameters, σ and \bar{x} to describe the function.

χ^2 -test

To test if an obtained experimental distribution agrees with e.g. a Poisson distribution, or if there are some systematic errors, the χ^2 -test can be used. The χ^2 -value is given by

$$\chi^2 = \frac{1}{\bar{x}_e} \sum_{i=1}^N (x_i - \bar{x}_e)^2 \quad (3.1.9)$$

where N is the number of measurements, \bar{x}_e is the experimental mean value and x_i are the separate independent values.

For a Poisson distribution the χ^2 -value should be close to $N - 1$. There are tables giving the probability that an obtained distribution is a Poisson distribution for a given χ^2 -value, and this test is often of great value to check the measurements for systematic errors.

Uncertainty propagation

There are often several uncertainties in an experimental determination and these uncertainties will add up. If the uncertainties are individually small and symmetric around zero, the total uncertainty can be shown to be given by

$$\sigma_u^2 = \left(\frac{\delta u}{\delta x}\right)^2 \sigma_x^2 + \left(\frac{\delta u}{\delta y}\right)^2 \sigma_y^2 + \left(\frac{\delta u}{\delta z}\right)^2 \sigma_z^2 + \dots \quad (3.1.10)$$

where $u = u(x, y, z, \dots)$ represents the derived quantity and $\sigma_x, \sigma_y, \sigma_z, \dots$ are the standard deviations of the individual variables.

Using this relation for the standard variation of the net count rate one obtains

$$\sigma_{r_n} = \sqrt{\frac{r_T}{t_T} + \frac{r_b}{t_b}} = \sqrt{\frac{N_T}{t_T^2} + \frac{N_b}{t_b^2}} \quad (3.1.11)$$

where r_T is the count rate for sample including background, r_b is the count rate for background, N_T is the number of counts for sample including background, N_b is the number of counts for background, t_T is the calculation time for sample including background and t_b is the calculation time for background.

Rate meter statistics

The relative standard deviation of a rate meter is given by

$$\frac{\sigma_r}{r} = \frac{1}{\sqrt{2rRC}} \quad (3.1.12)$$

where r is the count rate and RC is the time constant.

The equilibrium time, t_0 , of a rate meter is defined as the time that is needed to obtain a value that is equal to the saturation value subtracted with one standard deviation. This is obtained from the relation

$$t_0 = \frac{RC}{2} \ln(r2RC). \quad (3.1.13)$$

3.1.2 Detector Properties

Full width half maximum

The energy resolution of a detector is often described by the full width half maximum $w_{1/2}$. If it is assumed that the width of the energy peak is due to statistical variation following a Poisson distribution and that this may be approximated by a Gaussian distribution, $w_{1/2}$ is given by

$$w_{1/2} = 2.35E\sqrt{\frac{F}{N}} = 2.35\sqrt{EF\epsilon} \quad (3.1.14)$$

where F is the Fano factor (the ratio between the observed variance and the Poisson expected variance), N is the number of charge carriers, E is the peak energy and ϵ is the energy to produce a charge carrier.

Measurement geometry for a point source on the central axis

The geometry factor, G , is defined as the number of particles that are emitted by the source and hitting the detector. For a point source positioned at the central axis of a cylindrical detector, see Fig. 3.1, G is given by

$$G = \frac{1}{2} \left(1 - \frac{h}{\sqrt{h^2 + a^2}} \right) = \frac{1}{2} (1 - \cos(\theta/2)) \quad (3.1.15)$$

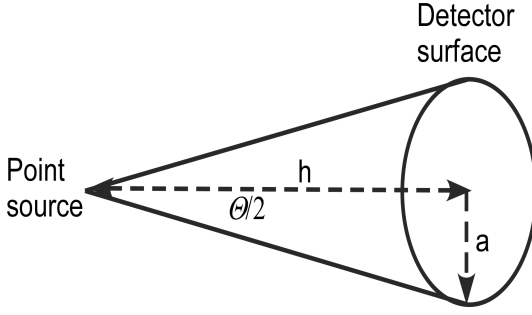


Figure 3.1: Geometry factor for a point source.

where h is the distance source-detector surface, a is the detector radius and $\theta/2$ is half of the opening angle.

Note that this definition is different from the solid angle Ω , which for a cylindrical detector surface is given by

$$\Omega = 2\pi\left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right) \quad (3.1.16)$$

Thus

$$G = \frac{\Omega}{4\pi} \quad (3.1.17)$$

When the source to surface distance is large compared to the detector radius, the geometry factor can be simplified to

$$G = \frac{a^2}{4h^2} \quad (3.1.18)$$

and the solid angle, Ω , to

$$\Omega = \frac{\pi a^2}{h^2} \quad (3.1.19)$$

Correction for resolving time

Pulse detectors and the electronic measuring devices need a finite time to register a pulse. The minimum time between two separate pulses is called the resolving time or the dead time of the counter. This dead time may result in a reduction in count rate. The correction for this is dependent if the counter is paralyzable or non-paralyzable. A paralyzable detector does not register a pulse during the dead time, but the dead time is prolonged. This prolongation will not occur for non-paralyzable counters. The equations for the two different detectors are given as:

Non paralyzable counters

$$r_0 = \frac{r}{1 - r\tau} \quad (3.1.20)$$

Paralyzable counters

$$r = r_0 e^{-r_0 \tau} \quad (3.1.21)$$

where r_0 is the corrected counting rate, r is the measured counting rate and τ is the resolving time (dead time) of the counter.

Coincidence measurements

If two particles from the same decay are absorbed by a detector their energy may be added and give rise to a “sum-peak”. The true counting rate in the sum peak is given by

$$r_{12} = A \eta_1 \eta_2 f_1 f_2 G^2 \quad (3.1.22)$$

where A is the source activity, η_1, η_2 detector efficiency for particle 1 and particle 2 respectively, f_1, f_2 are the number of particles per decay for particle 1 and particle 2 respectively and G is the geometry factor.

The detector will register two pulses as one within a certain resolving time, τ . This means that beside the true counting coincidence rate, there is a possibility that two particles from different decays hit the detector at nearly the same time and will be detected as a coincidence count. This random coincidence rate is given by

$$r_{ch} = r_1 r_2 \tau = A^2 \eta_1 \eta_2 G^2 f_1 f_2 \tau \quad (3.1.23)$$

Sometimes two detectors are used in coincidence. In a positron camera the annihilation photons are emitted in coincidence and can be measured by two detectors in opposite direction from the point of annihilation. It is also often possible to reduce background count rates by using two detectors in coincidence. Beside the true coincidences there is also here a possibility that two particles from different decays are counted. This random coincidence rate is approximately given by

$$r_{ch} = 2r_1 r_2 \tau_c \quad (3.1.24)$$

where r_1, r_2 are the counting rates of the separate detector and τ_c the resolving time of the coincidence circuit.

Detector relations

A. Gas detectors

Gas multiplication in a proportional counter

$$\ln M = \frac{V}{\ln(r_2/r_1)} \frac{\ln 2}{\Delta V} \left(\ln \frac{V}{p r_1 \ln(r_2/r_1)} - \ln K \right) \quad (3.1.25)$$

where M is the gas multiplication factor, V is the applied voltage, r_1 is the anode radius, r_2 is the cathode radius, p is the gas pressure, and K is the Diethorn-parameter

for the proportional counter.

Relative standard deviation in the charge

$$\left(\frac{\sigma_Q}{Q}\right)^2 = \left(\frac{\sigma_{n_0}}{n_0}\right)^2 + \frac{1}{n_0} \left(\frac{\sigma_A}{\bar{A}}\right)^2 \quad (3.1.26)$$

$$\left(\frac{\sigma_A}{\bar{A}}\right)^2 \approx 0.5 \text{ if } \bar{A} \text{ is large} \quad (3.1.27)$$

where A is the electron multiplication factor, \bar{A} is the mean value of A and n_0 is the number of produced ion pairs.

Pulse drift in a cylindrical gas detector

The velocity of the ions at radius r is given by

$$v^+(r) = \frac{\mu}{p} \frac{V_0}{\ln(r_2/r_1)} \frac{1}{r} \quad (3.1.28)$$

The position of the ions at time t is given by

$$r(t) = \sqrt{\frac{2\mu}{p} \frac{V_0 t}{\ln(r_2/r_1)} + r_1^2} \quad (3.1.29)$$

The time needed to collect the ions is given by

$$t^+ = \frac{(r_2^2 - r_1^2)p \ln(r_2/r_1)}{2\mu V_0} \quad (3.1.30)$$

The variation of the detector signal with time is given by

$$V_i(t) = \frac{Q}{C} \frac{1}{\ln(r_2/r_1)} \ln \left(\frac{2\mu V_0 t}{r_1^2 p \ln(r_2/r_1)} + 1 \right)^{1/2} \quad (3.1.31)$$

where r_1 is the anode radius, r_2 is the cathode radius, p is the pressure in the chamber, V_0 is the voltage over the chamber, μ is the mobility, C is the detector capacitance, and Q is the collected charge.

B. Scintillation detectors

Pulse shape from a photomultiplier tube is given by

$$V(t) = \frac{1}{\lambda - \theta} \frac{\lambda Q}{C} (e^{-\theta t} - e^{-\lambda t}) \quad (3.1.32)$$

where λ is the scintillation detector decay time, θ is the anode time constant, Q is the total charge, and C is the anode capacitance

With a large time constant ($\theta \ll \lambda$)

$$V(t) = \frac{Q}{C}(e^{-\theta t} - e^{-\lambda t}) \quad (3.1.33)$$

With a small time constant ($\theta \gg \lambda$)

$$V(t) = \frac{\lambda}{\theta} \frac{Q}{C}(e^{-\lambda t} - e^{-\theta t}) \quad (3.1.34)$$

C. Semiconductor detectors

The depletion depth d is given by

$$d \cong \sqrt{\left(\frac{2\epsilon V}{eN}\right)} \quad (3.1.35)$$

where ϵ is the dielectric constant of the material, N is the concentration of doped atoms and V is the applied voltage.

D. Activation detector

The saturation activity in activation of a material is given by

$$A_{\infty} = \frac{\lambda(C - B)}{\eta(1 - e^{-\lambda t_0})(e^{-\lambda t_1} - e^{-\lambda t_2})} \quad (3.1.36)$$

where C is the total number of pulses measured during time t_1 to t_2 , time measured from the end of irradiation. B is the number of background pulses during the same time, t_0 is the irradiation time, λ is the decay constant and η is the detector efficiency.

E. Neutron scintillation detector.

The detector efficiency for an organic scintillation detector made of carbon and hydrogen irradiated with neutrons is given by

$$\eta = \frac{N_H \sigma_H}{N_H \sigma_H + N_C \sigma_C} \left(1 - e^{-(N_H \sigma_H + N_C \sigma_C)d}\right) \quad (3.1.37)$$

where N_H , N_C is the number of target nuclides per volume unit in hydrogen and carbon respectively. σ_H , σ_C is the reaction cross section in hydrogen and carbon respectively. d is the detector thickness.

3.2 Exercises in Detectors and Measurements

3.2.1 Counting Statistics

Exercise 3.1. Derive an expression that optimizes data acquisition lengths between measurement of a sample plus background and background alone, respectively, for a specified total acquisition length. Then calculate the optimal acquisition time for sample with background and background alone, when given a total acquisition time of 0.50 h and count rates of 100 and 20 min^{-1} , respectively. Calculate also the relative standard deviation of the net count rate.

Exercise 3.2. The current from an ion-chamber exposed to γ -radiation is $1.0 \cdot 10^{-13}$ A. The output from the electrometer is connected to a printing system. What will the expected relative standard deviation of the printout be and what will the equilibrium time be, knowing that the system time constant $\tau = 2.0$ s and the average path-length of secondary electrons under radiation equilibrium is 0.20 m air at NTP? Assume that the energy loss of secondary electrons through collision processes equals 0.20 $\text{MeV m}^2 \text{kg}^{-1}$.

Exercise 3.3. A radiation detector is measuring at the sarcophagus at Chernobyl to check possible emission of radioactivity. The mean dose rate is $12.5 \mu\text{Gy h}^{-1}$, which corresponds to a current in the detector of 7.5 nA. The time constant of the detector is 5.0 s. The detector shall alarm if the dose rate is increasing with more than 3.0 standard deviations. Which dose rate is this corresponding to? Every pulse in the detector has a charge of 0.19 nC.

Exercise 3.4. A dairy uses a scintillation detector to check if the activity concentration of ^{137}Cs in the milk is lower than 50 Bq l^{-1} . The staff at the dairy is not used with this type of measurements and they have a feeling that the background is higher if there is a person present in the detector room. The following result as shown in the table was obtained. The detector measured photon energies in the energy range 600-700 keV.

Sample	Measuring time	Number of counts
Milk sample (10 ml)	20.0 min	456
Background (without person)	40.0 min	210
Background (with person)	30.0 min	165

The efficiency of the detector in the measuring region was determined to be 0.62 ± 0.02 counts (1.0 SD) per photon in the energy range 600-700 keV.

a) Determine the standard deviation of the difference in background count rate with or without a person. Discuss if the difference in background count rate with or

without a person is significant?

b) Calculate the concentration of the activity in Bq l^{-1} and the standard deviation of the concentration. Should one consider the milk as radioactive according to the limit 50 Bq l^{-1} ?

Exercise 3.5. You are responsible of an air sample unit, placed outside a plant using radioactive nuclides. There is a suspicion of a leakage of ^{239}Pu . The air sample unit has an air sampling rate of $1.56 \text{ m}^3 \text{ min}^{-1}$ and the air filter has an efficiency of 0.80. The air sampling time is 60.0 min.

After the sampling, there is a delay period of 24.0 h, resulting in that all natural radioactive nuclides from Rn have decayed. Then the filter is measured during 600 s and 220 counts are registered. Calculate the activity concentration of ^{239}Pu in Bq m^{-3} and the standard deviation in the determination of the concentration. Only counting statistics need to be considered.

The self absorption of the alpha-particles in the filter implies that only 40% of the emitted alpha-particles leave the filter in the direction of the detector and can be detected. The filter is placed close to an alpha detector with an active detector area of 60.0 cm^2 . The background is 20.0 counts during 100 min. The detector efficiency for alpha-particles that hit the detector is 0.30 and the area of the active filter is 500 cm^2 .

Exercise 3.6. The half life of a radioactive source shall be determined. The radiation source is measured twice, each time with the measuring time 600 s and with 24 h between the measurements. In the first measurement 1683 counts are obtained and in the second 914 counts, both including background counts. The count rate for the background is 21.0 min^{-1} , determined with a measurement time of 20.0 min. Determine the half life of the source and the uncertainty in the determination.

Exercise 3.7. The linear attenuation coefficient for an unknown material shall be determined. The following data are obtained in a measurement made in “narrow beam geometry”.

Material thickness: $2.53 \pm 0.02 \text{ cm}$ (1 standard deviation).

Measurement without material: 35 000 counts during 300 s (including background).

Measurement with material: 25 700 counts during 300 s (including background).

Measurement of the background (both measurements): 2350 counts during 600 s.

The uncertainty in the measurement time can be neglected.

Determine the linear attenuation coefficient and the standard deviation.

Exercise 3.8. At a company that produces ethanol, there is a suspicion that somebody steals the ethanol and replaces it with water. To check this without a need

to open the closed bottles, they are irradiated with γ -rays from a ^{137}Cs -source with an activity of 5.0 MBq, in a collimated beam with the diameter at the detector of 20 mm. This can be regarded as a narrow beam geometry. The distance between the source and the detector is 60.0 cm. A NaI-detector is used for the measurements. The diameter of the detector is 50.0 mm and the intrinsic efficiency for the used photon energy is 0.73. The transmission through a bottle filled with ethanol is compared with the transmission through a bottle with suspected content. The bottles are square shaped with a thickness of the content of 80.0 mm. Determine the measurement time for each bottle in order to be able to obtain a difference in count rate of 2.0 standard deviations between a bottle with water and a bottle with ethanol. Neglect the attenuation in the wall of the bottle. The density of ethanol is $0.791 \cdot 10^3 \text{ kg m}^{-3}$. The count rate for the background has been determined to be 17.0 s^{-1} during a measurement time of 100 s. The correction for dead time may be neglected.

Exercise 3.9. The activity of a point source placed 0.50 m below the ground level is to be determined by the use of a detector located on the ground level, facing downwards. The area of the detector is 10.0 cm^2 . The intrinsic efficiency of the detector is determined to 0.60 ± 0.01 . The build-up factor of the secondary photons is 1.10 ± 0.1 . The linear attenuation coefficient of ground material is $2.83 \pm 0.050 \text{ m}^{-1}$. A total of 10 000 counts is achieved during 60.0 s. The number of background counts is $1.20 \cdot 10^4$ during 20 min. Determine the activity of the source and the uncertainty in the activity assuming one emitted photon per decay. Neglect the uncertainty in the distance and the detector diameter and assume that given uncertainties are one standard deviation.

Exercise 3.10. The activity of a ^{137}Cs -source is going to be determined with a scintillation detector. The detector has a diameter of 5.08 cm. The radiation source can be positioned at an effective distance of 20 or 30 cm from the detector. The standard deviation in the determination of the distance is estimated to be 0.50 cm. The radiation source is small and can be regarded as a point source. The efficiency of the detector for the γ -radiation from ^{137}Cs is 0.76. When positioning the source at an effective distance of 30 cm, 3562 counts are obtained during 60.0 s. The number of background counts is 3826 obtained during 600 s. Calculate the activity of the source and the standard deviation in the activity if only counting statistics and the uncertainty in the distance are included. The approximate estimation of the geometry factor may be used. Neglect the attenuation in the air.

If the radioactive source instead is positioned at an effective distance of 20 cm and the measurement is made with the same measuring time, will this result in a smaller or larger uncertainty?

3.2.2 Detector Properties

Exercise 3.11. In a measuring set up, the average count rate r of a radioactive source of ^{11}C is determined by measuring the number of counts during 30.0 min. The measurement is regarded as representative of the activity of the sample after 15.0 min, i.e. at the center of the counting period. This value will however differ from the real count rate at this point of time. Calculate the ratio of the two count rates.

Exercise 3.12. The possible angular dependence of the two photons emitted from a ^{60}Co -source shall be determined. Two NaI-scintillation detectors are positioned according to Fig. 3.2 and the coincidence rate of the two detectors is measured. The pulse height analyzer is open for energies between 1.0 and 1.5 MeV. The efficiency of the detectors is 0.45 in this energy range. Which coincidence rate is expected if there is no angular dependence between the two gamma-rays? The resolving time for the coincidence circuit is $2.0\ \mu\text{s}$ and the coincidence background count rate is $2.0\ \text{s}^{-1}$. The activity of the radioactive source is 3.90 MBq. The diameter of the scintillation detector is 50 mm.

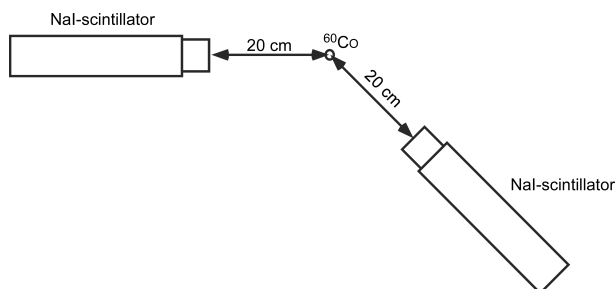


Figure 3.2: Geometry of the coincidence measurement in exercise 3.12.

Exercise 3.13. A radionuclide is emitting two coincident gamma photons with 100% exchange per decay. A sample of this radionuclide is placed at the central axis 10.0 cm from the surface of a cylindrical detector with a radius of 5.0 cm. The efficiency in the total-absorption peak is 50% for one of the photons and 30% for the other one. The total count rate in the sum peak is $129\ \text{s}^{-1}$. Calculate the activity of the source if the detector has a resolving time of $2.0\ \mu\text{s}$, and the background count rate in the sum peak is $12\ \text{s}^{-1}$.

Exercise 3.14. Calculate the largest dead time for a GM-counter in order to have dead time losses less than 1.0%, if the detector is used to measure the count rate

of a ^{14}C radioactive source with the activity 3.0 kBq. The radioactive source can be regarded as a point and is placed at the central axis at a distance of 40 mm from the detector surface. The effective diameter of the detector is 30 mm. The transmission through the detector window is 0.87. The counting efficiency of the particles that pass through the window is 1.0.

Exercise 3.15. A cylindrical GM-counter with a diameter of 40 mm and an anode diameter of 0.16 mm is filled with argon and ethyl alcohol. The mean free path of the electrons between two collisions at the pressure 101.3 kPa and temperature 20°C is $6.4 \cdot 10^{-4}$ mm. The maximal radius at which secondary electrons can be produced is 0.65 mm. The voltage over the tube is 1500 V. The ionization energy is 15.7 eV. Calculate the gas pressure in the tube if the temperature is 20°C .

Exercise 3.16. A cylindrical gas detector has a central anode with a radius of $25\text{ }\mu\text{m}$ and a cathode with a radius of 25 mm. The voltage over the tube is 1000 V. At what distance from the center will an electron get enough energy, when traveling a mean free path length of 1.0 mm, to be able to ionize helium, which has an ionization energy of 23 eV?

Exercise 3.17. A GM-counter has an anode radius of $20\text{ }\mu\text{m}$ and a cathode radius of 20.0 mm. The pressure in the chamber is 10.7 kPa. If the voltage over the counter is 600 V, how far have the positive ions traveled when the pulse amplitude has increased to 25% of its maximum? The time constant of the external circuit is assumed to be long compared to the collection time. The ions are assumed to be produced at the anode. Their mobility is $1.40 \cdot 10^4 (\text{mm s}^{-1})(\text{V mm}^{-1})^{-1}$ kPa.

Exercise 3.18. A cylindrical gas detector with a cathode radius of 20 mm and an anode radius of 0.50 mm can theoretically be used both as an ionization chamber and as a GM-counter. A ^{59}Fe point radiation source with the activity 2.76 MBq is positioned centrally at a distance of 50 mm from the tube end window. The window has a thickness of 1.5 mg cm^{-2} and is made of Mica. The electrons emitted from the source are assumed to be totally absorbed in the detector, but the contribution from the photons may be neglected. The absorption of the electrons in the window may also be neglected. The gas in the detector is argon with a mean energy to produce an ion pair, $\bar{W}=35.0\text{ eV}$.

When the voltage over the chamber is low the detector is working as an ionization chamber. Calculate the produced ionization current.

The voltage is then increased and the detector is now working as a GM-counter. Which voltage is necessary if the maximal radius for which secondary electrons can be produced is 0.65 mm? The ionization energy is 15.7 eV and the mean free path of the electrons at 101.3 kPa at room temperature is $6.4 \cdot 10^{-4}$ mm. The pressure in the chamber is 1.50 kPa.

How many counts per second are expected if the dead time is 130 μs ?

Exercise 3.19. In a plane parallel ionization chamber the distance between the electrodes is 2.0 mm. In order to have a fast collection of the ions it is necessary to have a high field strength. However, if the voltage is too high, there is a risk of increased charge leakage and flash-over. The maximum practical value of the voltage is then estimated to 400 V. Calculate the maximal and the minimal collection time for this voltage. Assume that the liberated electrons do not produce negative ions. Calculate also the pulse voltage obtained if an electron with the energy 0.7 MeV is absorbed in the chamber, and the produced charge is measured over a capacitance of 13 pF.

The ionization chamber is filled with air. The temperature and the pressure in the chamber is 21°C and 100 kPa, respectively. The mobility of the ions is $1.32 \cdot 10^4 \text{ (mm s}^{-1}\text{)(V mm}^{-1}\text{)}^{-1}\text{ kPa}$ and $1.62 \cdot 10^7 \text{ (mm s}^{-1}\text{)(V mm}^{-1}\text{)}^{-1}\text{ kPa}$ for electrons.

Exercise 3.20. Calculate the necessary amplification that is needed in order that the most energetic photon from a ^{60}Co radioactive source that is totally absorbed in a NaI-scintillator, will give a pulse in channel 500 in a multichannel analyzer, where channel 1024 corresponds to a voltage of 10.0 V. The following data are given for the detector:

Light conversion efficiency=12%.

Light photon wave length=415 nm.

Light collection efficiency=50%.

Photocathode efficiency=20%.

80% of the produced electrons in the photocathode are collected at the first dynode in the PM-tube. The PM-tube has 10 dynodes, with a multiplication factor of $\delta=2.5$. The anode has a load resistor of 100 k Ω and a capacitance of 100 pF. The decay constant of the NaI-scintillator is $4.35 \cdot 10^6 \text{ s}^{-1}$.

Exercise 3.21. A HpGe-detector is used to detect 90° Compton scattered radiation. A voltage pulse of 1.25 V is obtained after the preamplifier, which has an amplification of 1500 and an input capacitance of 7.5 pF. If the scattered photon is totally absorbed in the HpGe-detector, what is the energy of the primary photon? If the energy spread in the total absorption peak only depends on statistics in the electron-hole production what will the FWHM (full width half maximum) be? The Fano factor is 0.10.

Exercise 3.22. A NaI-scintillator connected to a rate meter is used to check if a material contains ^{65}Zn . Calculate the smallest activity that can be determined with the following conditions.

The NaI-scintillator has a thickness of 50 mm. The radiation source is considered to be a point source centrally positioned at a distance of 200 mm from the

surface of the detector. In front of the detector a collimator with a diameter opening of 20 mm is positioned. This means that for calculation of absorbed energy it is possible to assume a parallel impinging beam of photons. Neglect escape of characteristic x rays. Assume that Compton scattered photons are not absorbed in the scintillator. To produce a photoelectron at the photocathode 100 eV is needed on average. Assume that all photoelectrons produced at the photocathode reach the first dynode. The PM-tube has 10 dynodes with a multiplication factor of 3.5. There is a background current of 124 μA . The size of the pulses from the background radiation is $2.39 \cdot 10^{-11} \text{ C}$. The time constant of the rate meter is 20.0 s. The measured total current shall differ from the background current with 2.0 standard deviations of the background current. Assume that the decay of the pulses can be neglected.

Exercise 3.23. In radiotherapy with high photon energies, there will be (γ, n) reactions that will produce a fluence rate of neutrons in the treatment room. To determine the fluence of thermal neutrons, ^{115}In -foils are placed at different positions in the room. The irradiation time is 10.0 min. A measurement of an $^{116\text{m}}\text{In}$ -foil activity starts 10.0 minutes after finishing the irradiation. The measurement time is 20.0 min. A total amount of 76 800 counts is obtained. The background count rate have been determined to 13 s^{-1} . Calculate the neutron fluence rate, if the absolute efficiency of the detector is 0.30 pulses per photon and the number of photons per decay is 0.48 in the relevant measurement region. The activation cross section for ^{115}In for the energy range of interest is $2640 \cdot 10^{-28} \text{ m}^2$. The mass of the foil is 2.0 g.

Exercise 3.24. There is an interest to determine how much silver there is in an old coin. The coin is irradiated with a fluence of thermal neutrons with a fluence rate of $10.2 \cdot 10^7 \text{ m}^{-2} \text{ s}^{-1}$. The coin is irradiated during 600 s and then the electrons, which are emitted from the activated coin, are measured during a 300 s period, with a break of 60 s between the irradiation and the measurement. A total amount of 25 000 counts is obtained. The background count rate has been determined to 3.0 s^{-1} . Calculate the mass of silver in the coin. Neglect the self absorption of the electrons in the coin and assume that the detector efficiency for electrons is 0.10 in the geometry used. The activation cross sections per nucleus are: $\sigma(^{107}\text{Ag}, ^{108}\text{Ag}) = 30 \cdot 10^{-28} \text{ m}^2$, $\sigma(^{109}\text{Ag}, ^{110}\text{Ag}) = 110 \cdot 10^{-28} \text{ m}^2$. Natural silver contains 51.35% ^{107}Ag and 48.65% ^{109}Ag .

Exercise 3.25. A spherical proportional counter "without walls" and with the diameter 10 mm is placed in a vacuum tank. This is filled with tissue equivalent gas according to Rossi (64.4% CH_4 , 32.4% CO_2 , 3.2% N_2) at a pressure of 9.44 kPa. The detector is irradiated with α -particles from ^{241}Am in a parallel beam. The fluence rate is very low and each event is registered separately. The gas multiplication is $1.0 \cdot 10^3$. The capacitance of the preamplifier is 1.0 pF and the signal is then amplified 10 times before it is analysed in a multichannel analyser with 1024 channels and with the pulse height interval 0-10 V. Determine the shape of the spectrum. The measurement

is made at 293 K. The density of the gas at 273 K and 101.3 kPa is 1.128 kg m^{-3} . $\bar{W}=31.1 \text{ eV}$ (mean energy to produce an ion pair).

3.3 Solutions in Detectors and Measurements

3.3.1 Counting Statistics

Solution exercise 3.1.

The standard deviation of the net counting rate is given by

$$s_n = \sqrt{\frac{r_T}{t_T} + \frac{r_b}{t_b}} \quad (3.3.1)$$

where r_T =count rate for sample including background, r_b =count rate for background, t_T =calculation time for sample including background and t_b =calculation time for background. Assume that the total time for measurement is $\tau = t_T + t_b$.

Inserting this in Eq. (3.3.1) and squaring the result, the following relation is obtained.

$$s_n^2 = \frac{r_T}{t_T} + \frac{r_b}{\tau - t_T} \quad (3.3.2)$$

Derivation gives

$$\frac{d(s_n^2)}{dt_T} = -\frac{r_T}{t_T^2} + \frac{r_b}{(\tau - t_T)^2} \quad (3.3.3)$$

Reorganizing Eq. (3.3.3) and setting $d(s_n^2)/dt_T$ equal to zero gives the optimal distribution of measurement times for a minimal variance or standard deviation.

$$\frac{r_T}{t_T^2} = \frac{r_b}{(\tau - t_T)^2} \quad (3.3.4)$$

and

$$\frac{t_T}{t_b} = \sqrt{\frac{r_T}{r_b}} \quad (3.3.5)$$

Use this relation for the following data:

$$\tau=30 \text{ min}, r_T=100 \text{ min}^{-1}, r_b=20 \text{ min}^{-1}$$

This inserted in Eq. (3.3.5) gives

$$\frac{t_T}{30 - t_T} = \sqrt{\frac{100}{20}}$$

This gives $t_T=20.7 \text{ min}$ and $t_b=30-20.7=9.3 \text{ min}$

The relative standard deviation is then

$$\frac{s_n}{r_n} = \frac{\sqrt{\frac{100}{20.7} + \frac{20}{9.3}}}{100 - 20} = 0.033$$

Answer: The optimal measurement times are 20.7 min for sample including background and 9.3 min for background only. The relative standard deviation in the net count rate is 3.3%.

Solution exercise 3.2.

The number of ion pairs, N , produced by a passage of a secondary electron is given by

$$N = \frac{d\rho(\frac{dE}{\rho dl})}{\bar{W}} \quad (3.3.6)$$

The charge per pulse is $q = Ne$. The measured current is $I = rq$. Thus the count rate is given by the equation

$$r = \frac{I}{q} = \frac{I\bar{W}}{d\rho(\frac{dE}{\rho dl})e} \quad (3.3.7)$$

where

$d=0.20$ m (average path length of the secondary electron)

$\rho=1.293$ kg m⁻³ (density of air at NTP)

$(dE/\rho dl)=0.20$ MeV m² kg⁻¹ (mass stopping power of the secondary electrons)

$\bar{W}=33.97$ eV (mean energy to ionize an ion pair in air)

$e=1.602 \cdot 10^{-19}$ C (charge of an electron)

$I=1.0 \cdot 10^{-13}$ A (measured current)

r =count rate

Data inserted in Eq. (3.3.7) gives

$$r = \frac{1.0 \cdot 10^{-13} \cdot 33.97}{0.20 \cdot 0.20 \cdot 10^6 \cdot 1.293 \cdot 1.602 \cdot 10^{-19}} = 410 \text{ s}^{-1}$$

The standard deviation in the count rate is given by

$$\frac{s_r}{r} = \frac{1}{\sqrt{r2RC}} \quad (3.3.8)$$

where $RC=2.0$ s (time constant of the system).

Data inserted gives

$$\frac{s_r}{r} = \frac{1}{\sqrt{410 \cdot 2 \cdot 2}} = 2.47 \cdot 10^{-2}$$

The equilibrium time, t_0 , of the printer is obtained from the relation

$$t_0 = \frac{RC}{2} \ln(r2RC) \quad (3.3.9)$$

Data inserted gives

$$t_0 = \frac{2}{2} \ln(410 \cdot 2 \cdot 2) = 7.4 \text{ s}$$

Answer: The relative standard deviation is 2.5% and the equilibrium time is 7.4 s.

Solution exercise 3.3.

The standard deviation of a rate meter is given by

$$s_r = \frac{r}{\sqrt{r2RC}} \quad (3.3.10)$$

where RC is the time constant of the detector (5.0 s) and r is the count rate.

The count rate is given by the relation $r=I/q$ where

$I = 7.50 \cdot 10^{-9} \text{ A}$ (measured current)

$q = 0.19 \cdot 10^{-9} \text{ C}$ (charge per pulse)

Thus

$$s_r = \frac{(7.5 \cdot 10^{-9} / 0.19 \cdot 10^{-9})}{\sqrt{2 \cdot (7.5 \cdot 10^{-9} / 0.19 \cdot 10^{-9}) \cdot 5}} = 1.987 \text{ s}^{-1} \quad (3.3.11)$$

Three standard deviations are then 5.96 s^{-1} . This corresponds to an increase in current of

$$\Delta I = 5.96 \cdot 0.19 \cdot 10^{-9} = 1.13 \text{ nA}.$$

This means that the current after increase should be $7.50 + 1.13 = 8.63 \text{ nA}$.

This corresponds to a dose rate of $8.63 \cdot (12.5 / 7.50) = 14.4 \text{ } \mu\text{Gy h}^{-1}$.

Answer: The dose rate after the increase is $14.4 \text{ } \mu\text{Gy h}^{-1}$.

Solution exercise 3.4.

The difference in the background count rate in the measurements with and without a person in the room is given by

$$\Delta r_b = \frac{N_2}{t_2} - \frac{N_1}{t_1} \quad (3.3.12)$$

The standard deviation in the difference is given by

$$s_{\Delta r_b} = \sqrt{\frac{N_1}{t_1^2} + \frac{N_2}{t_2^2}} \quad (3.3.13)$$

where

$N_1=210$ (number of background counts without a person)

$N_2=165$ (number of background counts with a person)

$t_1=40$ min (time for measurement of background counts without a person)

$t_2=30$ min (time for measurement of background counts with a person)

Data inserted gives

$$\Delta r_b = \frac{165}{30} - \frac{210}{40} = 0.25 \text{ min}^{-1}$$

and

$$s_{\Delta r_b} = \sqrt{\frac{165}{30^2} + \frac{210}{40^2}} = 0.56 \text{ min}^{-1}$$

The difference Δr_b is less than half of one standard deviation. It is unlikely that there is a difference between the two background counting measurements. The total background is used in the further calculations.

The activity of the sample is given by

$$A = \frac{r_n}{\eta f} \quad (3.3.14)$$

where

$\eta=0.62\pm0.02$ (efficiency of the detector)

$f=0.946\cdot0.898$ (number of 0.662 keV photons per decay)

The net count rate, r_n , is

$$r_n = \frac{456}{20} - \frac{375}{70} = 17.44 \text{ min}^{-1}$$

The standard deviation in r_n is

$$s_{r_n} = \sqrt{\frac{456}{20^2} + \frac{375}{70^2}} = 1.029 \text{ min}^{-1}$$

Data inserted in Eq. (3.3.14) gives the activity for a 10 ml sample

$$A = \frac{17.44}{60 \cdot 0.62 \cdot 0.946 \cdot 0.898} = 0.55 \text{ Bq}$$

Thus the activity concentration C is 55 Bq^{-1} .

The standard deviation of the activity concentration is given by the relation

$$s_C = C \sqrt{\left(\frac{1}{r_n}\right)^2 s_{r_n}^2 + \left(\frac{1}{\eta}\right)^2 s_\eta^2} \quad (3.3.15)$$

Data inserted in Eq. (3.3.15) gives

$$s_C = 55 \sqrt{\left(\frac{1.03}{17.44}\right)^2 + \left(\frac{0.02}{0.62}\right)^2} = 3.90 \text{ Bq}^{-1}$$

This means that the concentration of ^{137}Cs -activity is larger than 50 Bq^{-1} with slightly more than one standard deviation. The milk should be considered as radioactive.

Answer: The standard deviation in the difference in the two background measurements is 0.56 min^{-1} . The concentration of ^{137}Cs -activity is $55 \pm 4 \text{ Bq}^{-1}$ (One standard deviation).

Solution exercise 3.5.

The net count rate r_n of the detector is given by

$$r_n = \frac{C \eta_d \eta_f S_A F t S_d f}{S_f} \quad (3.3.16)$$

The concentration of ^{239}Pu , C , is then

$$C = \frac{r_n S_f}{\eta_d \eta_f S_A F t S_d f} \quad (3.3.17)$$

where

$$r_n = \frac{N_f}{t_f} - \frac{N_b}{t_b} \quad (3.3.18)$$

Data:

$N_f=220$ (number of counts with filter)

$t_f=600 \text{ s}$ (time for filter measurement)

$N_b=20$ (number of counts of background)

$t_b=6000 \text{ s}$ (time for background measurement)

$S_f=500 \text{ cm}^2$ (filter area)

$\eta_d=0.30$ (detector efficiency)

$\eta_f=0.80$ (filtration efficiency)

$S_A=0.40$ (self absorption in the filter)

$F=1.56 \text{ m}^3 \text{ min}^{-1}$ (air sampling rate)

$t=60.0$ min (sampling time)

$S_d=60$ cm² (detector area)

$f=1.0$ (number of α -particles per decay)

The count rate, r_n , is

$$r_n = \frac{220}{600} - \frac{20}{6000} = 0.363 \text{ s}^{-1}$$

Data inserted in Eq. (3.3.17) gives

$$C = \frac{0.363 \cdot 500}{0.3 \cdot 0.8 \cdot 0.4 \cdot 1.56 \cdot 60 \cdot 60 \cdot 1.0} = 0.337 \text{ Bq/m}^3$$

The standard deviation of the net count rate r_n is given by

$$s_{r_n} = \sqrt{\frac{N_f}{t_f^2} + \frac{N_b}{t_b^2}} \quad (3.3.19)$$

Data inserted gives

$$s_{r_n} = \sqrt{\frac{220}{600^2} + \frac{20}{6000^2}} = 2.47 \cdot 10^{-2} \text{ min}^{-1}$$

This results in a standard deviation in the concentration

$$s_C = 2.47 \cdot 10^{-2} \cdot 0.337 / 0.363 = 0.023 \text{ Bq m}^{-3}$$

Answer: The activity concentration of ²³⁹Pu in the air is 0.34 Bq m⁻³. The standard deviation is 0.02 Bq m⁻³.

Solution exercise 3.6.

The half life is obtained from the relation

$$r = r_0 e^{-t \ln 2 / T_{1/2}} \quad (3.3.20)$$

and thus

$$T_{1/2} = -\frac{t \ln 2}{\ln(r/r_0)} \quad (3.3.21)$$

where r_0 is the count rate at time 0 and r is the count rate at time 24 h.

Data:

$N_1=1683$ (number of total counts at time 0)

$r_b=21$ min⁻¹ (background count rate)

$N_1=914$ (number of total counts at time 24 h)

$t_1=600$ s (time for measurement of total number of counts)

$t_2=20$ min (time for measurement of background counts)

Then

$$r_0 = \frac{1683}{600} - \frac{21}{60} = 2.455 \text{ s}^{-1}$$

$$r = \frac{914}{600} - \frac{21}{60} = 1.173 \text{ s}^{-1}$$

Data inserted gives

$$T_{1/2} = -\frac{24 \ln 2}{\ln(1.173/2.455)} = 22.5 \text{ h}$$

Uncertainty analysis:

The total standard deviation is given by

$$s_{T_{1/2}} = \sqrt{\left(\frac{\delta T_{1/2}}{\delta r}\right)^2 s_r^2 + \left(\frac{\delta T_{1/2}}{\delta r_0}\right)^2 s_{r_0}^2} \quad (3.3.22)$$

where

$$s_r^2 = \frac{914}{600^2} + \frac{21}{60 \cdot 20 \cdot 60} = 2.831 \cdot 10^{-3} \text{ s}^{-2}$$

and

$$s_{r_0}^2 = \frac{1683}{600^2} + \frac{21}{60 \cdot 20 \cdot 60} = 4.967 \cdot 10^{-3} \text{ s}^{-2}$$

Differentiation of $T_{1/2}$ gives

$$\left| \frac{\delta T_{1/2}}{\delta r} \right| = \frac{t \ln 2}{r(\ln(r/r_0))^2} = \frac{24 \cdot 3600 \ln 2}{1.173(\ln(1.173/2.455))^2} = 9.360 \cdot 10^4 \text{ s}^2$$

and

$$\left| \frac{\delta T_{1/2}}{\delta r_0} \right| = \frac{t \ln 2}{r_0(\ln(r/r_0))^2} = \frac{24 \cdot 3600 \ln 2}{2.455(\ln(1.173/2.455))^2} = 4.472 \cdot 10^4 \text{ s}^2$$

Data inserted in Eq. (3.3.22) gives

$$s_{T_{1/2}} = \sqrt{(9.360 \cdot 10^4)^2 \cdot 2.831 \cdot 10^{-3} + (4.472 \cdot 10^4)^2 \cdot 4.967 \cdot 10^{-3}} = 5.894 \cdot 10^3 \text{ s}$$

Answer: The half life is 22.5 h and the standard deviation is 1.6 h.

Solution exercise 3.7.

The attenuation in the material is given by

$$r_d = r_0 e^{-\mu d} \quad (3.3.23)$$

This gives

$$\mu = -\ln(r_d/r_0)/d \quad (3.3.24)$$

where μ is the linear attenuation coefficient, r_0 is the net count rate before attenuation and r_d is the net count rate after attenuation.

Data:

$d=2.53 \pm 0.02$ cm (material thickness)

$N_1=35000$ (number of total counts before attenuation)

$N_2=25700$ (number of total counts after attenuation)

$N_b=2350$ (number of background counts)

$t_1=300$ s (time for measurement of total number of counts)

$t_b=600$ s (time for measurement of background counts)

$$r_0 = \frac{35000}{300} - \frac{2350}{600} = 112.75 \text{ s}^{-1}$$

$$r = \frac{25700}{300} - \frac{2350}{600} = 81.75 \text{ s}^{-1}$$

Data inserted in Eq. (3.3.24) gives

$$\mu = \frac{1}{2.53} \ln \left(\frac{112.75}{81.75} \right) = 0.127 \text{ cm}^{-1}$$

Uncertainty analysis:

The total uncertainty is obtained by differentiating the parameters in Eq. (3.3.24) separately.

$$\frac{\delta\mu}{\delta d} = \frac{\mu}{d} \quad (3.3.25)$$

$$\frac{\delta\mu}{\delta r_d} = \frac{1}{r_d d} \quad (3.3.26)$$

$$\frac{\delta\mu}{\delta r_0} = \frac{1}{r_0 d} \quad (3.3.27)$$

By adding the different differentials in square multiplied with their variances Eq. (3.3.28) is obtained.

$$s_\mu = \sqrt{\left(\frac{\mu}{d}\right)^2 s_d^2 + \left(\frac{1}{r_0 d}\right)^2 s_{r_0^2} + \left(\frac{1}{r_d d}\right)^2 s_{r_d}^2} \quad (3.3.28)$$

Data inserted gives

$$s_{r_0} = \sqrt{\frac{35000}{300^2} + \frac{2350}{600^2}} = 0.63$$

$$s_{r_d} = \sqrt{\frac{25700}{300^2} + \frac{2350}{600^2}} = 0.54$$

Data inserted in Eq. (3.3.28) gives

$$s_\mu = \sqrt{\left(\frac{0.02 \cdot 0.127}{2.53}\right)^2 + \left(\frac{0.63}{112.75 \cdot 2.53}\right)^2 + \left(\frac{0.54}{81.75 \cdot 2.53}\right)^2} = 3.56 \cdot 10^{-3} \text{ cm}^{-1}$$

Answer: The linear attenuation coefficient is 0.127 cm^{-1} with the standard deviation 0.004 cm^{-1} .

Solution exercise 3.8.

Count rate of the detector is given by

$$r = Af\eta Ge^{-\mu d} \quad (3.3.29)$$

where

$A=5.0 \text{ MBq}$ (source activity)

$f=0.946 \cdot 0.898$ (number of 662 keV photons per decay)

$d=80 \text{ mm}$ (liquid thickness)

$\eta=0.73$ (detector efficiency)

$(\mu/\rho)_{\text{water}}=0.00856 \text{ m}^2 \text{ kg}^{-1}$ (mass attenuation coefficient for water)

$\rho_{\text{water}}=1.0 \cdot 10^3 \text{ kg m}^{-3}$ (density of water)

$(\mu/\rho)_{\text{ethanol}}=0.00871 \text{ m}^2 \text{ kg}^{-1}$ (mass attenuation coefficient for ethanol)

$\rho_{\text{ethanol}}=0.791 \cdot 10^3 \text{ kg m}^{-3}$ (density of ethanol)

The geometry factor, G , is obtained from the relation

$$G = \frac{1}{2} \left(1 - \frac{h}{\sqrt{h^2 + a^2}}\right)$$

with

$h=60.0 \text{ cm}$ (source-detector distance)

$a=1.0 \text{ cm}$ (beam radius)

$$G = \frac{1}{2} \left(1 - \frac{60}{\sqrt{60^2 + 1^2}}\right) = 6.943 \cdot 10^{-5}$$

1) Bottle filled with water

Count rate with inserted data gives

$$r_{\text{water}} = 5.0 \cdot 10^6 \cdot 0.946 \cdot 0.898 \cdot 0.73 \cdot 6.943 \cdot 10^{-5} \cdot e^{-0.00856 \cdot 1.0 \cdot 10^3 \cdot 0.080} = 108.6 \text{ s}^{-1}$$

2) Bottle filled with ethanol

Count rate with inserted data gives

$$r_{\text{ethanol}} = 5.0 \cdot 10^6 \cdot 0.946 \cdot 0.898 \cdot 0.73 \cdot 6.943 \cdot 10^{-5} \cdot e^{-0.00871 \cdot 0.791 \cdot 10^3 \cdot 0.080} = 124.1 \text{ s}^{-1}$$

Background count rate

$$r_b = 17 \text{ s}^{-1} \text{ obtained during } t_b = 100 \text{ s.}$$

This implies that the total count rate including background is

$$1) \text{ Water: } r_{\text{water}, T} = 108.6 + 17 = 125.6 \text{ s}^{-1}$$

$$2) \text{ Ethanol: } r_{\text{ethanol}, T} = 124.1 + 17 = 141.1 \text{ s}^{-1}$$

The difference in count rate is

$$r_{\text{diff}} = 141.1 - 125.6 = 15.5 \text{ s}^{-1}$$

The standard deviation in the count rate is given by

$$s_r = \sqrt{\frac{r_T}{t_T} + \frac{r_b}{t_b}} \quad (3.3.30)$$

1) Water:

$$s_{r_{\text{water}}} = \sqrt{\frac{125.6}{t_T} + \frac{17}{100}}$$

2) Ethanol:

$$s_{r_{\text{ethanol}}} = \sqrt{\frac{141.1}{t_T} + \frac{17}{100}}$$

The standard deviation in the difference is given by

$$s_{\text{diff}} = \sqrt{s_{r_{\text{water}}}^2 + s_{r_{\text{ethanol}}}^2} \quad (3.3.31)$$

The difference in count rate should be equal to two standard deviations of the difference. Thus

$$15.5 = 2 \sqrt{\frac{125.6}{t_T} + \frac{17}{100} + \frac{141.1}{t_T} + \frac{17}{100}}$$

$$t_T = 4.5 \text{ s.}$$

Answer: The measurement time needed to obtain the wanted confidence level is 4.5 s.

Solution exercise 3.9.

The net count rate at the ground surface is given by the equation

$$r_n = \frac{ASB\eta e^{-\mu d}}{4\pi d^2} \quad (3.3.32)$$

Thus the activity is given by

$$A = \frac{4\pi d^2 r_n}{SB\eta e^{-\mu d}} \quad (3.3.33)$$

where

A =activity (Bq)

S =10.0 cm² (detector area)

μ =2.83±0.05 m⁻¹ (linear attenuation coefficient)

d =0.50 m (distance source-ground surface)

η =0.60±0.01 (detector efficiency)

B =1.10±0.01 (build up factor for secondary photons)

The net count rate r_n is obtained from the relation

$$r_n = \frac{N_s}{t_s} - \frac{N_b}{t_b} \quad (3.3.34)$$

where

N_s =10 000 (number of pulses with source)

N_b =12 000 (number of background pulses)

t_s =60 s (time for measurement of source)

t_b =20.0 min (time for measurement of background)

Data inserted in Eq. (3.3.34) gives

$$r_n = \frac{10000}{60} - \frac{12000}{20.0 \cdot 60} = 156.67 \text{ s}^{-1}$$

Data inserted in Eq. (3.3.33) gives

$$A = \frac{4\pi \cdot 0.50^2 \cdot 156.67}{0.001 \cdot 1.10 \cdot 0.60 \cdot e^{-2.83 \cdot 0.5}} = 3.07 \text{ MBq}$$

To calculate the uncertainty in the determination of the activity, take the logarithm of Eq. (3.3.33) and differentiate.

$$\ln A = \ln 4\pi + 2 \ln d + \ln r_n - \ln S - \ln B + \mu d - \ln \eta \quad (3.3.35)$$

and

$$dA/A = (2/d + \mu) dd + dr_n/r_n - dS/S - dB/B + d\mu - d\eta/\eta \quad (3.3.36)$$

where the differentials can be expressed as the uncertainties.

The uncertainties can be assumed to be added in square. The standard deviation for the count rate is obtained by

$$s_{r_n} = \sqrt{\frac{10000}{60^2} + \frac{12000}{(20 \cdot 60)^2}} = 1.669 \text{ s}^{-1}$$

Inserting this value and the given uncertainties (standard deviations) in the problem, neglecting uncertainties in the detector area and the distance, gives

$$dA/A = \sqrt{(1.67/156.7)^2 + (0.01/1.10)^2 + (0.5 \cdot 0.05)^2 + (0.01/0.60)^2} = 0.0314$$

$$dA = 3.14 \cdot 10^{-2} \cdot 3.07 = 0.096 \text{ MBq}$$

Answer: The source activity is 3.1 MBq and the standard deviation 0.1 MBq.

Solution exercise 3.10.

The net count rate, r_n , at the detector is given by

$$r_n = Af\eta G \quad (3.3.37)$$

Thus the equation for the activity is

$$A = \frac{r_n}{f\eta G} \quad (3.3.38)$$

where

$f=0.898 \cdot 0.946$ (number of 0.662 MeV photons per decay)

$\eta=0.76$ (detector efficiency)

$\Phi=5.08 \text{ cm}$ (detector diameter)

$h=20 \text{ or } 30 \text{ cm}$ (distance detector-source)

G =geometry factor

According to the problem text the simplified relation for the geometry factor may be used. This means that G is given by ($h=30 \text{ cm}$)

$$G = \frac{\Phi^2}{4 \cdot 4 \cdot h^2} = \frac{5.08^2}{16 \cdot 30^2} = 1.792 \cdot 10^{-3} \quad (3.3.39)$$

With $h=20 \text{ cm}$, $G = 4.032 \cdot 10^{-3}$

The net count rate is

$$r_n = \frac{N_T}{t_T} - \frac{N_b}{t_b}$$

where

$N_T=3562$ (total number of pulses, when $h=30$ cm)

$t_T=60.0$ s (measurement time for total number of pulses)

$N_b=3826$ (number of background pulses)

$t_b=600$ s (measurement time for number of background pulses)

This gives

$$r_{30} = \frac{3562}{60} - \frac{3826}{600} = 52.99 \text{ s}^{-1}$$

Data inserted in Eq. (3.3.38) gives the activity

$$A = \frac{52.99}{0.898 \cdot 0.946 \cdot 0.76 \cdot 1.792 \cdot 10^{-3}} = 45.80 \text{ kBq}$$

The relative uncertainty is given by the relation

$$\left(\frac{s_A}{A}\right)^2 = \left(\frac{s_r}{r}\right)^2 + \left(\frac{s_G}{G}\right)^2 \quad (3.3.40)$$

where s_r is

$$s_{r,30} = \sqrt{\frac{3562}{60^2} + \frac{3826}{600^2}} = 1.000 \text{ s}^{-1}$$

and s_G is given by

$$\frac{dG}{dh} = \frac{\Phi^2 2h}{16h^4} = \frac{\Phi^2}{8h^3} \quad (3.3.41)$$

and

$$\frac{dG}{G} = \frac{\Phi^2}{8h^3} \frac{16h^2 dh}{\Phi^2} = \frac{2 dh}{h} \quad (3.3.42)$$

Data inserted gives

$$\left(\frac{s_A}{A}\right)^2 = \left(\frac{1.0}{52.99}\right)^2 + \left(\frac{2 \cdot 0.5}{30}\right)^2 = 1.467 \cdot 10^{-3}$$

This gives the uncertainty in the activity $s_A=1.75$ kBq.

With a source-detector distance of 20 cm the net count rate is obtained using the inverse square law

$$r_{20} = r_{30} \cdot 30^2 / 20^2 = 119.23 \text{ s}^{-1} \quad (3.3.43)$$

The count rate including background is then 125.60 s^{-1} . Then the standard deviation in the count rate s_r is

$$s_r = \sqrt{\frac{7536}{60^2} + \frac{3826}{600^2}} = 1.45 \text{ s}^{-1}$$

The relative uncertainty in the activity then is

$$\left(\frac{s_A}{A}\right)^2 = \left(\frac{1.45}{119.2}\right)^2 + \left(\frac{2 \cdot 0.5}{20}\right)^2 = 2.65 \cdot 10^{-3}$$

This gives an uncertainty in the activity $s_A = 2.36$ kBq.

Thus the uncertainty in the activity is larger with the 20 cm source-detector distance as compared with 30 cm.

Answer: The source activity is 45.8 kBq with a standard deviation of 1.7 kBq. The standard deviation is larger with a source detector distance of 20 cm as compared with 30 cm.

3.3.2 Detector Properties

Solution exercise 3.11.

The counting rate, r , will decrease with time according to the decay of the radioactive source.

$$r(t) = r_0 e^{-\lambda t} \quad (3.3.44)$$

where λ is the decay constant and r_0 is the count rate at time $t = 0$.

The average count rate during a measurement period T is given by

$$\bar{r} = \frac{\int_0^T r_0 e^{-\lambda t} dt}{T} = \frac{r_0(1 - e^{-\lambda T})}{\lambda T} \quad (3.3.45)$$

The ratio of the count rate at the center of the measuring period ($T/2$) and the average count rate is thus

$$r(T/2)/\bar{r} = \frac{\lambda T r_0 e^{-\lambda T/2}}{r_0(1 - e^{-\lambda T})} \quad (3.3.46)$$

Data:

$$T = 30.0 \text{ min}$$

$$\lambda = \ln 2 / 20.38 \text{ min}^{-1} \text{ (decay constant for } ^{11}\text{C)}$$

Data inserted in Eq. (3.3.46) gives

$$r(15)/\bar{r} = \frac{\ln 2 \cdot 30.0 \cdot r_0 \cdot e^{-15 \cdot \ln 2 / 20.38}}{20.38 \cdot r_0 (1 - e^{-30 \cdot \ln 2 / 20.38})} = 0.958$$

Answer: The ratio between the count rate at the center of the measurement period and the average count rate is 0.96.

Solution exercise 3.12.

The total count rate is given by

$$r_{\text{tot}} = r_{12} + r_{\text{ch}} + r_{\text{b}} \quad (3.3.47)$$

where r_{12} is the true coincidence rate given by

$$r_{12} = A\eta^2 G^2 f_1 f_2 \quad (3.3.48)$$

and r_{ch} is the random coincidence rate given by

$$r_{\text{ch}} = 2r_1 r_2 \tau = 2A^2 \eta^2 G^2 f_1 f_2 \tau \quad (3.3.49)$$

r_1, r_2 are count rates of the separate detectors

Data:

$A=3.90$ MBq (source activity)

$\eta=0.45$ (detector efficiency)

$f_1, f_2=1.0$ (number of photons per decay)

$\tau=2.0 \mu\text{s}$ (resolving time of the coincidence circuit)

$r_b=2.0 \text{ s}^{-1}$ (coincidence background count rate)

G is the geometry factor given by

$$G = \frac{1}{2} \left(1 - \frac{h}{\sqrt{h^2 + a^2}} \right) \quad (3.3.50)$$

where

$h=20$ cm (distance detector surface-source)

$a=2.5$ cm (detector radius)

Data inserted in Eq. (3.3.50) gives

$$G = \frac{1}{2} \left(1 - \frac{20}{\sqrt{20^2 + 2.5^2}} \right) = 3.86 \cdot 10^{-3} \quad (3.3.51)$$

Data inserted in Eq. (3.3.47) gives

$$r = 3.90 \cdot 10^6 \cdot 0.45^2 \cdot (3.86 \cdot 10^{-3})^2 \cdot 1.0^2 + 2 \cdot (3.90 \cdot 10^6 \cdot 0.45 \cdot 3.86 \cdot 10^{-3})^2 \cdot 1 \cdot 1 \cdot 2.0 \cdot 10^{-6} + 2.0$$

$$r=197 \text{ s}^{-1}$$

Answer: The total coincidence rate is $0.20 \cdot 10^3 \text{ s}^{-1}$.

Solution exercise 3.13.

The true coincidence rate in the sum peak is given by

$$r_{12} = A\eta_1 \eta_2 f_1 f_2 G^2 \quad (3.3.52)$$

The random coincidence rate is given by

$$r_{\text{ch}} = A^2 \eta_1 \eta_2 f_1 f_2 G^2 \tau_r \quad (3.3.53)$$

The total count rate r_T is given by

$$r_T = r_{12} + r_{ch} + r_b \quad (3.3.54)$$

Data:

A =source activity

$\eta_1=0.30$ (detector efficiency for photon 1)

$\eta_2=0.50$ (detector efficiency for photon 2)

$f_1, f_2=1.0$ (number of photons per decay)

$\tau_r=2.0 \mu s$ (resolving time of the coincidence circuit)

$r_b=12.0 s^{-1}$ (coincidence background count rate)

$r_T=129 s^{-1}$ (total coincidence count rate)

G is the geometry factor given by

$$G = \frac{1}{2} \left(1 - \frac{h}{\sqrt{h^2 + a^2}} \right) \quad (3.3.55)$$

where

$h=10$ cm (distance detector surface-source)

$a=5.0$ cm (detector radius)

Data inserted in Eq. (3.3.55) gives

$$G = \frac{1}{2} \left(1 - \frac{10}{\sqrt{10^2 + 5^2}} \right) = 5.28 \cdot 10^{-2}$$

Data inserted in Eq. (3.3.54) gives

$$129 = A \cdot 0.3 \cdot 0.5 \cdot 1.0 \cdot 1.0 \cdot (5.28 \cdot 10^{-2})^2 + A^2 \cdot 0.3 \cdot 0.5 \cdot 1.0 \cdot 1.0 \cdot (5.28 \cdot 10^{-2})^2 \cdot 2.0 \cdot 10^{-6} + 12$$

Reorganizing the equation gives

$$A^2 + \frac{A \cdot 4.180 \cdot 10^{-4}}{8.359 \cdot 10^{-10}} = \frac{117}{8.359 \cdot 10^{-10}}$$

and

$$A = -\frac{4.180 \cdot 10^{-4}}{2 \cdot 8.359 \cdot 10^{-10}} \pm \sqrt{\left(\frac{4.180 \cdot 10^{-4}}{2 \cdot 8.359 \cdot 10^{-10}} \right)^2 + \frac{117}{8.359 \cdot 10^{-10}}}$$

$$A = -2.500 \cdot 10^5 \pm 4.500 \cdot 10^5$$

$$A = 2.000 \cdot 10^5 \text{ Bq}$$

Answer: The source activity is 200 kBq.

Solution exercise 3.14.

The true count rate, neglecting background is given by

$$r_0 = A\eta ftG \quad (3.3.56)$$

where

$A=3.0$ kBq (source activity)

$f=1.0$ (number of β -particles per decay)

$\eta=1.0$ (detector efficiency)

$t=0.87$ (transmission through detector window)

$h=40$ mm (distance detector surface-source)

$a=15.0$ mm (detector radius)

and

G is the geometry factor given by

$$G = \frac{1}{2} \left(1 - \frac{h}{\sqrt{h^2 + a^2}} \right) \quad (3.3.57)$$

Data inserted gives

$$G = \frac{1}{2} \left(1 - \frac{40}{\sqrt{40^2 + 15^2}} \right) = 3.18 \cdot 10^{-2}$$

Data inserted in Eq. (3.3.56) gives

$$r_0 = 3.0 \cdot 10^3 \cdot 1.0 \cdot 1.0 \cdot 0.87 \cdot 3.18 \cdot 10^{-2} = 83.0 \text{ s}^{-1}$$

The relation between measured and true count rate is given by

$$r_0 = \frac{r}{1 - r\tau} \quad (3.3.58)$$

According to the problem $r = 0.99r_0$. Thus

$$r_0 = \frac{0.99r_0}{1 - 0.99r_0\tau}$$

and

$$\tau = (1 - 0.99)/(0.99 \cdot 83.0) = 1.22 \cdot 10^{-4} \text{ s}$$

Answer: The dead time is 0.12 ms.

Solution exercise 3.15.

The field strength in the cylindrical counter at radius r is given by (see Fig. 3.3)

$$\epsilon(r) = \frac{V}{r \ln(r_2/r_1)} \quad (3.3.59)$$

The ionization energy is then given by

$$dE = q \frac{V}{r \ln(r_2/r_1)} dr \quad (3.3.60)$$

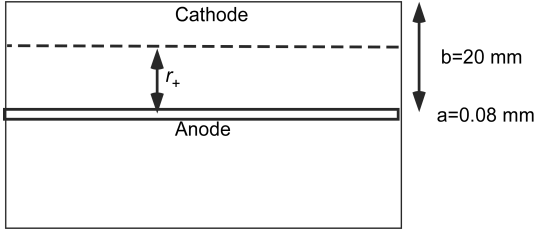


Figure 3.3: Sketch of the GM-counter in exercise 3.15.

dE is the energy transferred to the electron when passing a distance dr . The electron has to obtain an energy of $W=15.7$ eV in a distance, l , equal to the free mean path.

Integration of Eq. (3.3.60) gives

$$E = \int_r^{r+l} \frac{qV dr}{r \ln(r_2/r_1)} = \frac{qV}{\ln(r_2/r_1)} \ln\left(\frac{r+l}{r}\right) \quad (3.3.61)$$

Solving V gives

$$V = \frac{E \ln(r_2/r_1)}{q \ln((r+l)/r)} \quad (3.3.62)$$

Data:

$r=0.65$ mm (the maximal radius where secondary electrons can be produced)

$r_1=0.08$ mm (anode radius)

$r_2=20$ mm (cathode radius)

$V=1500$ V (voltage over the counter)

$q=1$ (charge per electron if energy W is measured in eV)

$l=6.4 \cdot 10^{-4} \cdot 101.3/p$ mm, where p is the gas pressure in the counter.

Data inserted in Eq. (3.3.62) gives

$$1500 = \frac{15.7 \ln(20.0/0.08)}{\ln\left(\frac{0.65 + \frac{6.4 \cdot 10^{-4} \cdot 101.3}{p}}{0.65}\right)}$$

Rearranging the equation gives

$$\frac{15.7 \ln(20.0/0.08)}{1500} = \ln\left(\frac{0.65 + \frac{6.4 \cdot 10^{-4} \cdot 101.3}{p}}{0.65}\right)$$

and

$$0.65 \cdot e^{\frac{15.7 \ln(20.0/0.08)}{1500}} = 0.65 + \frac{6.4 \cdot 10^{-4} \cdot 101.3}{p}$$

Solving p gives

$$p = 1.68 \text{ kPa}$$

Answer The gas pressure in the counter is 1.7 kPa.

Solution exercise 3.16.

The field strength in the cylindrical counter at radius r is given by

$$\epsilon(r) = \frac{V}{r \ln(r_2/r_1)} \quad (3.3.63)$$

The electron shall in the path length 1.0 mm obtain the energy $E=23 \text{ eV}$, where E is given by

$$E = q \int_r^{r+1} E(r) dr = q \int_r^{r+1} \frac{V}{r \ln(r_2/r_1)} dr = \frac{qV}{\ln(r_2/r_1)} (\ln(r+1) - \ln r) \quad (3.3.64)$$

Data:

$r_1 = 25 \mu\text{m}$ (anode radius)

$r_2 = 25 \text{ mm}$ (cathode radius)

$V = 1000 \text{ V}$ (voltage over the counter)

$q=1$ (charge per electron if energy E is measured in eV)

Data inserted in Eq. (3.3.64) gives

$$23 = \frac{1 \cdot 1000}{\ln(25 \cdot 10^{-3}/25 \cdot 10^{-6})} (\ln(r+1) - \ln r) \quad (3.3.65)$$

$$0.15888 = \ln\left(\frac{r+1}{r}\right) \quad (3.3.66)$$

$$r = 5.81 \text{ mm}$$

Answer: The radius where the electron gets enough energy is 5.8 mm.

Solution exercise 3.17.

The pulse height from a GM-counter at a time t after the pulse is initiated is given by the relation

$$V_r(t) = \frac{Q}{C} \frac{1}{\ln(r_2/r_1)} \ln \left(\frac{2\mu V_0 t}{r_1^2 p \ln(r_2/r_1)} + 1 \right)^{1/2} \quad (3.3.67)$$

where

$r_1=20\ \mu\text{m}$ (anode radius)

$r_2=20\ \text{mm}$ (cathode radius)

$V_0=600\ \text{V}$ (voltage over the counter)

$p=10.7\ \text{kPa}$ (gas pressure)

$\mu=1.40\cdot 10^4\ (\text{mm s}^{-1})(\text{V mm}^{-1})^{-1}\text{kPa}$ (ion mobility)

Q =produced charge

C =detector capacitance

Data inserted in Eq. (3.3.67) gives

$$V_r(t) = \frac{Q}{C} \frac{1}{\ln(20/20 \cdot 10^{-3})} \ln \left(\frac{2 \cdot 1.40 \cdot 10^4 \cdot 600 \cdot t}{(20 \cdot 10^{-3})^2 \cdot 10.7 \ln(20/20 \cdot 10^{-3})} + 1 \right)^{1/2}$$

$V_r(t)=0.25(Q/C)$ according to information.

Inserting this in the equation and solve for t gives

$$t=5.39\cdot 10^{-8}\ \text{s}.$$

The radius where the ions are positioned at a certain time t is given by the relation

$$r(t) = \sqrt{\frac{2\mu}{p} \frac{V_0 t}{\ln(r_2/r_1)} + r_1^2} \quad (3.3.68)$$

Data inserted in Eq. (3.3.68) gives

$$r(t) = \sqrt{\frac{2 \cdot 1.40 \cdot 10^4}{10.7} \frac{600 \cdot 5.39 \cdot 10^{-8}}{\ln(20/20 \cdot 10^{-3})} + (20 \cdot 10^{-3})^2} = 0.11\ \text{mm}$$

The anode radius is $20\ \mu\text{m}$. This means that the ions have traveled a distance of

$$d=0.11-20\cdot 10^{-3}=0.09\ \text{mm}$$

Answer: The ions have traveled a distance of 0.09 mm.

Solution exercise 3.18.

The current obtained when the detector is used as an ionization chamber is given by

$$I = \frac{\sum A y_i \bar{E}_{\beta_i} G q}{\bar{W}} \quad (3.3.69)$$

where

$A=2.76\ \text{MBq}$ (source activity)

$\bar{W}=35$ eV (mean energy to produce an ion pair in argon)

$q=1.602 \cdot 10^{-19}$ C (charge of the electron)

y_i =number of β -particles per decay with the mean energy \bar{E}_{β_i} . See Table 3.1.

G is the geometry factor given by

$$G = \frac{1}{2} \left(1 - \frac{h}{\sqrt{h^2 + a^2}} \right) \quad (3.3.70)$$

Data inserted for the geometry ($h=50$ mm, $a=20$ mm) gives the value of G

$$G = \frac{1}{2} \left(1 - \frac{5.0}{\sqrt{5.0^2 + 2.0^2}} \right) = 0.0358$$

The data for the decay of ^{59}Fe gives the total β -energy $E_\beta=0.117$ MeV.

Table 3.1: Mean energy and frequency of β -particles from ^{59}Fe decay.

E_{β_i} (Energy/MeV)	y_i (frequency)	$y_i \cdot E_{\beta_i}$
0.036	0.0127	$4.54 \cdot 10^{-4}$
0.081	0.456	$3.69 \cdot 10^{-2}$
0.149	0.528	$7.88 \cdot 10^{-2}$
0.636	0.0018	$1.14 \cdot 10^{-3}$
Total		0.117

Data inserted in Eq. (3.3.69) gives

$$I = \frac{2.76 \cdot 10^6 \cdot 0.117 \cdot 10^6 \cdot 0.0358 \cdot 1.602 \cdot 10^{-19}}{35.0} = 5.26 \cdot 10^{-11} \text{ A}$$

The number of pulses per second when the detector is used as a GM-counter is obtained from the relation

$$r_0 = A \sum y_i G \eta \quad (3.3.71)$$

where $\eta=1.0$ is the detector efficiency

Data inserted gives

$$r_0 = 2.76 \cdot 10^6 \cdot 0.0358 \cdot 0.9985 \cdot 1.0 = 9.866 \cdot 10^4 \text{ s}^{-1}$$

This is the true count rate. The measured count rate is less due to dead time losses. The relation between measured and true count rate is given by

$$r = \frac{r_0}{1 + r_0 \tau} \quad (3.3.72)$$

With the dead time $\tau=130\ \mu\text{s}$ the measured count rate, neglecting background count rate, is

$$r = \frac{9.866 \cdot 10^4}{1 + 9.866 \cdot 10^4 \cdot 130 \cdot 10^{-6}} = 7.136 \cdot 10^3\ \text{s}^{-1}$$

The relation between the ionization energy (W) and the voltage in the detector is given by the relation

$$W = \frac{eV}{\ln(r_2/r_1)} (\ln x_2 - \ln x_1) \quad (3.3.73)$$

where

$W=15.7\ \text{eV}$

$r_1=0.08\ \text{cm}$ (anode radius)

$r_2=2.0\ \text{cm}$ (cathode radius)

$x_2 - x_1=6.4 \cdot 10^{-4}\ \text{mm}$ (mean free path at 101.33 kPa)

At a pressure of 1.5 kPa, $x_2 - x_1=0.0432\ \text{mm}$. With $x_2=0.65\ \text{mm}$ according to the problem, $x_1=0.65-0.0432=0.6068\ \text{mm}$

Data inserted in Eq. (3.3.73) gives

$$15.7 = \frac{1.0 \cdot V}{\ln(2.0/0.08)} (\ln 0.65 - \ln 0.6068)$$

$V=735\ \text{V}$.

Answer The ionization current is $5.3 \cdot 10^{-11}\ \text{A}$. The measured count rate is $7.14 \cdot 10^3\ \text{s}^{-1}$. The voltage over the counter is 735 V.

Solution exercise 3.19.

The measured voltage is obtained from the relation

$$V = \frac{Q}{C} = \frac{Eq}{\bar{W}C} \quad (3.3.74)$$

where

$E=0.70\ \text{MeV}$ (electron energy)

$q=1.602 \cdot 10^{-19}\ \text{C}$ (charge of an electron)

$C=13.0\ \text{pF}$ (capacitance over the chamber)

$\bar{W}=33.97\ \text{eV}$ (mean energy to produce an ion pair in air)

Data inserted in Eq. (3.3.74) then gives

$$V = \frac{0.70 \cdot 10^6 \cdot 1.602 \cdot 10^{-19}}{33.97 \cdot 13.0 \cdot 10^{-12}} = 2.54 \cdot 10^{-4}\ \text{V}$$

The time to collect the ions is given by the distance traveled and the velocity. The velocity is given by the relation

$$v = \frac{\mu V}{dp} \quad (3.3.75)$$

where

$\mu_i = 1.32 \cdot 10^4 \text{ (mm s}^{-1}\text{)(V mm}^{-1}\text{)}^{-1}\text{kPa}$ (ion mobility)

$\mu_e = 1.62 \cdot 10^7 \text{ (mm s}^{-1}\text{)(V mm}^{-1}\text{)}^{-1}\text{kPa}$ (electron mobility)

$V=400 \text{ V}$ (voltage over the chamber)

$p=100 \text{ kPa}$ (gas pressure in the chamber)

$d=2.0 \text{ mm}$ (distance between electrodes)

The time needed to cross the chamber is given by

$$t = \frac{d}{v} = \frac{d^2 p}{\mu V} \quad (3.3.76)$$

The shortest time is obtained when the ions are produced close to the negative electrode resulting in transport only by electrons. The longest time is then obtained when the ions are produced close to the positive electrode and the transport is only by ions.

a) Shortest time

$$t = \frac{2^2 \cdot 100}{1.62 \cdot 10^7 \cdot 400} = 6.17 \cdot 10^{-8} \text{ s}$$

b) Longest time

$$t = \frac{2^2 \cdot 100}{1.32 \cdot 10^4 \cdot 400} = 7.58 \cdot 10^{-5} \text{ s}$$

Answer The shortest collection time is 62 ns and the longest 76 μs . The pulse voltage is 0.25 mV

Solution exercise 3.20.

The pulse height at a time t is given by the relation

$$V(t) = \frac{1}{\Lambda - \theta} \frac{\Lambda Q}{C} (e^{-\theta t} - e^{-\Lambda t}) \quad (3.3.77)$$

and the time t_{\max} when the maximum voltage is obtained, is given by solving the equation

$$\frac{dV}{dt} = \frac{1}{\Lambda - \theta} \frac{\Lambda Q}{C} (-\theta e^{-\theta t} + \Lambda e^{-\Lambda t}) = 0 \quad (3.3.78)$$

Thus

$$\theta e^{-\theta t} - \Lambda e^{-\Lambda t} = 0 \quad (3.3.79)$$

and

$$t_{\max} = \frac{\ln \frac{\theta}{\Lambda}}{\theta - \Lambda} \quad (3.3.80)$$

where Λ is the decay constant of the light from the NaI-scintillator and θ the time constant of the anode circuit.

The charge Q is given by the relation

$$Q = \frac{E_0 \eta \epsilon p \alpha \delta^N q}{E_\lambda} \quad (3.3.81)$$

where

$E_0=1.33$ MeV (energy of the photon)

$\eta=0.12$ (light conversion efficiency)

$\epsilon=0.50$ (light collection efficiency)

$p=0.20$ (photo cathode efficiency)

$\alpha=0.80$ (collection efficiency at the first dynode)

$\delta=2.5$ (amplification at each dynode)

$N=10$ (number of dynodes)

$q = 1.602 \cdot 10^{-19}$ C (charge of an electron)

E_λ is obtained from the knowledge of the wavelength of the light photons. The relation between energy and wavelength is

$$E_\lambda = \frac{hc}{\lambda} \quad (3.3.82)$$

where

$h = 6.6256 \cdot 10^{-34}$ Js (Planck's constant)

$c = 3.0 \cdot 10^8$ m s⁻¹ (velocity of light)

$\lambda=415$ nm (wavelength of the light photons)

Data inserted in Eq. (3.3.82) gives

$$E_\lambda = \frac{6.6256 \cdot 10^{-34} \cdot 3.0 \cdot 10^8}{415 \cdot 10^{-9} \cdot 1.602 \cdot 10^{-19}} = 2.99 \text{ eV}$$

Data inserted in Eq. (3.3.81) gives

$$Q = \frac{1.33 \cdot 10^6 \cdot 0.12 \cdot 0.50 \cdot 0.20 \cdot 0.8 \cdot 2.5^{10} \cdot 1.602 \cdot 10^{-19}}{2.99} = 6.5245 \cdot 10^{-12} \text{ C}$$

θ in Eq. (3.3.77) is given by $\theta = 1/RC$, where RC is the time constant of the anode circuit. With $R=100$ k Ω and $C=100$ pF, $\theta=1.0 \cdot 10^5$ s⁻¹.

$\Lambda = 4.35 \cdot 10^6$ s⁻¹ (decay constant of the NaI-scintillator)

Inserting θ and Λ in Eq. (3.3.80) gives

$$t_{\max} = \frac{\ln\left(\frac{1.0 \cdot 10^5}{4.35 \cdot 10^6}\right)}{1.0 \cdot 10^5 - 4.35 \cdot 10^6} = 0.888 \cdot 10^{-6} \text{ s}$$

t_{\max} inserted in Eq. (3.3.77) gives

$$V(t_{\max}) = \frac{1}{(4.35 \cdot 10^6 - 1.0 \cdot 10^5)} \frac{4.35 \cdot 10^6 \cdot 6.524 \cdot 10^{-12}}{100 \cdot 10^{-12}} \\ \times (e^{-0.888 \cdot 10^{-6} \cdot 10^5} - e^{-0.888 \cdot 10^{-6} \cdot 4.35 \cdot 10^6})$$

$$V(t_{\max}) = 5.97 \cdot 10^{-2} \text{ V}$$

The voltage corresponding to a channel in the multichannel analyzer is obtained from the knowledge of the voltage at the maximum channel number. Thus the voltage at channel 500, when 10.0 V corresponds to channel 1024 is $500 \cdot 10.0 / 1024 = 4.88 \text{ V}$. The voltage from the PM-tube is $5.97 \cdot 10^{-2} \text{ V}$. The needed amplification G is then

$$G = 4.88 / 5.97 \cdot 10^{-2} = 81.8$$

Answer: The amplification is 82.

Solution exercise 3.21.

The number of produced electron-hole pairs in the detector is given by

$$N = \frac{E}{\epsilon} \quad (3.3.83)$$

The voltage after amplification is given by

$$V = \frac{EGq}{\epsilon C} \quad (3.3.84)$$

where

E = the absorbed energy in the HpGe-detector in eV.

ϵ = 2.98 eV (energy per electron-hole pair)

G = 1500 (amplification in the amplifier)

$C = 7.5 \cdot 10^{-12} \text{ F}$ (input capacitance)

$q = 1.602 \cdot 10^{-19} \text{ C}$ (charge of an electron)

V = 1.25 V

Solving Eq. (3.3.84) for E and inserting data gives

$$E = \frac{1.25 \cdot 7.5 \cdot 10^{-12} \cdot 2.98}{1500 \cdot 1.602 \cdot 10^{-19}} = 1.165 \cdot 10^5 \text{ eV} \quad (3.3.85)$$

The energy of the primary photon is obtained from the Compton relation

$$h\nu' = \frac{h\nu_0}{1 + \frac{h\nu_0}{m_e c^2} (1 - \cos \theta)} \quad (3.3.86)$$

The photon is scattered in $\theta = 90^\circ$ and thus $\cos \theta = 0$. Inserting $h\nu' = 0.1165$ MeV gives

$$0.1165 = \frac{h\nu_0}{1 + \frac{h\nu_0}{0.511}}$$

$$h\nu_0 = 0.1505 \text{ MeV}$$

The FWHM is given by the relation $w_{1/2} = 2.35\sqrt{EF\epsilon}$, where $F=0.10$ (Fano factor)

Thus

$$w_{1/2} = 2.35\sqrt{0.1165 \cdot 10^6 \cdot 0.1 \cdot 2.98} = 0.44 \text{ keV}$$

Answer: The primary photon energy is 0.15 MeV and the FWHM is 0.44 keV.

Solution exercise 3.22.

The absorbed energy in a layer dx at depth x is given by

$$dE = \sum (\mu_{\text{en}}/\rho)_i \Psi_{0,i} S \rho e^{-\mu_i x} dx \quad (3.3.87)$$

Integration over the whole detector thickness d gives

$$E = \int_0^d \sum (\mu_{\text{en}}/\rho)_i \Psi_{0,i} S \rho e^{-\mu_i x} dx = \sum \frac{(\mu_{\text{en}}/\rho)_i \Psi_{0,i} S \rho}{\mu_i} (1 - e^{-\mu_i d}) \quad (3.3.88)$$

where

μ_{en}/ρ = mass energy absorption coefficient for NaI

μ/ρ = mass attenuation coefficient in NaI

$\Psi_{0,i} = AE_i Y_i / (4\pi l^2)$ (impinging energy fluence for energy E_i)

A = activity

$l = 200$ mm (distance source detector surface)

$d = 50$ mm (detector thickness)

$S = \pi r^2 = \pi \cdot 0.01^2 \text{ m}^2$ (irradiated detector area)

^{65}Zn decays and emits photons of two energies, which can contribute to the energy deposition. See Table 3.2 which also includes data for the interaction coefficients.

Table 3.2: Decay data and interaction coefficients in NaI for photons from ^{65}Zn .

Energy (MeV)	γ (frequency)	$(\mu/\rho)_{\text{NaI}}(\text{m}^2 \text{ kg}^{-1})$	$(\mu_{\text{en}}/\rho)_{\text{NaI}}(\text{m}^2 \text{ kg}^{-1})$
0.511	0.029	0.00935	0.004085
1.116	0.507	0.00560	0.00257

Calculation of absorbed energy.

a) $E=0.511 \text{ MeV}$

$$E_{0.511} = \frac{0.004085 \cdot 0.029 \cdot 0.511 \cdot \pi \cdot 0.01^2 \cdot A}{4 \cdot \pi \cdot 0.2^2 \cdot 0.00935} (1 - e^{-0.00935 \cdot 0.05 \cdot 3.67 \cdot 10^3})$$

$$E_{0.511} = 3.38 \cdot 10^{-6} \text{ A MeV}$$

b) $E=1.116 \text{ MeV}$

$$E_{1.116} = \frac{0.00257 \cdot 0.507 \cdot 1.116 \cdot \pi \cdot 0.01^2 \cdot A}{4 \cdot \pi \cdot 0.2^2 \cdot 0.0056} (1 - e^{-0.00560 \cdot 0.05 \cdot 3.67 \cdot 10^3})$$

$$E_{1.116} = 1.042 \cdot 10^{-4} \text{ A MeV}$$

The total deposited energy is $E_{0.511} + E_{1.116} = 1.076 \cdot 10^{-4} \text{ A MeV}$

The energy to produce a photoelectron at the photocathode is $\epsilon=100 \text{ eV}$. Thus the number of photoelectrons produced is

$$N = \frac{E}{\epsilon} = \frac{1.076 \cdot 10^{-4} \cdot 10^6 \cdot A}{100} = 1.076A$$

The current at the output of the photomultiplier is given by

$$I = NqM \quad (3.3.89)$$

where

$q=1.602 \cdot 10^{-19} \text{ C}$ (charge of an electron)

$M = \delta^L = 3.5^{10}$ (δ is the electron amplification per dynode and L is the number of dynodes)

Data inserted in Eq (3.3.89) gives

$$I = 1.076 \cdot 1.602 \cdot 10^{-19} \cdot 3.5^{10} \cdot A = 4.755 \cdot 10^{-14} \cdot A$$

This is the current induced by the radiation. There is also a background current of $124 \mu\text{A}$. The standard deviation in the background current is given by

$$\frac{\sigma_r}{r} = \frac{1}{\sqrt{2rRC}} \quad (3.3.90)$$

where r is the pulse rate, and $RC=20.0$ s is the time constant.

r is obtained from the knowledge of the current $I_b = 124 \mu\text{A}$ and the charge per pulse $Q = 2.39 \cdot 10^{-11}$ C.

This gives

$$r = 124 \cdot 10^{-6} / 2.39 \cdot 10^{-11} = 5.188 \cdot 10^6 \text{ s}^{-1}.$$

Data inserted in Eq (3.3.90) gives

$$\sigma_r = \frac{5.188 \cdot 10^6}{\sqrt{2 \cdot 5.188 \cdot 10^6 \cdot 20}} = 360.1 \text{ s}^{-1}$$

This corresponds to a standard deviation in the current of

$$\sigma_I = 360.1 \cdot 2.39 \cdot 10^{-11} = 8.608 \cdot 10^{-9} \text{ A}.$$

The net current should be equal to twice the standard deviation of the background current, i. e. $17.22 \cdot 10^{-9}$ A.

The activity can now be obtained from the relation

$$I = 4.755 \cdot 10^{-14} \cdot A = 17.22 \cdot 10^{-9}$$

$$A = 0.362 \cdot 10^6 \text{ Bq}$$

Answer: The activity is 0.36 MBq.

Solution exercise 3.23.

The activity of a sample in an activation process is given by

$$A = \sigma \Phi N (1 - e^{-\lambda t}) \quad (3.3.91)$$

and

$$A_\infty = \sigma \Phi N = \sigma \Phi \frac{m N_A}{m_a} \quad (3.3.92)$$

The saturation activity A_∞ , may be obtained from Eq (3.3.93) by measuring the irradiated sample (see Fig. 3.4).

$$A_\infty = \frac{\lambda(C - B)}{y\eta(1 - e^{-\lambda t_0})(e^{-\lambda t_1} - e^{-\lambda t_2})} \quad (3.3.93)$$

Data:

$\sigma = 2640 \cdot 10^{-28} \text{ m}^2$ (activation cross section)

$C = 76800$ (total number of counts)

$B = 13 \cdot 20 \cdot 60$ (number of background counts)

$t_0 = 10.0 \text{ min}$, $t_1 = 10.0 \text{ min}$, $t_2 = 30.0 \text{ min}$ (See Fig. 3.4)

$\eta = 0.30$ (detector efficiency)

$\gamma = 0.48$ (number of photons/disintegration)

$\lambda = \ln 2 / 54.1 = 0.0128 \text{ min}^{-1}$ (decay constant for ^{116m}In)

$m = 2.0 \text{ g}$ (sample mass)

$N_A = 6.022 \cdot 10^{23} \text{ mol}^{-1}$ (Avogadro's number)

$m_a = 115$ (atomic mass of ^{115}In)

Data inserted in Eq. (3.3.93) gives

$$A_\infty = \frac{0.0128(76800 - 15600)}{60 \cdot 0.48 \cdot 0.30(1 - e^{-0.0128 \cdot 10})(e^{-0.0128 \cdot 10} - e^{-0.0128 \cdot 30})}$$

$A_\infty = 3797 \text{ Bq}$

Inserting this in Eq. (3.3.92) and solving for $\dot{\Phi}$ gives

$$\dot{\Phi} = \frac{3797 \cdot 115}{2640 \cdot 10^{-28} \cdot 2.0 \cdot 6.022 \cdot 10^{23}} = 1.373 \cdot 10^6 \text{ m}^{-2} \text{ s}^{-1}$$

Answer: The fluence rate is $1.37 \cdot 10^6 \text{ m}^{-2} \text{ s}^{-1}$

Solution exercise 3.24.

The saturation activity A_∞ , is given from Eq (3.3.94) and Eq (3.3.95).

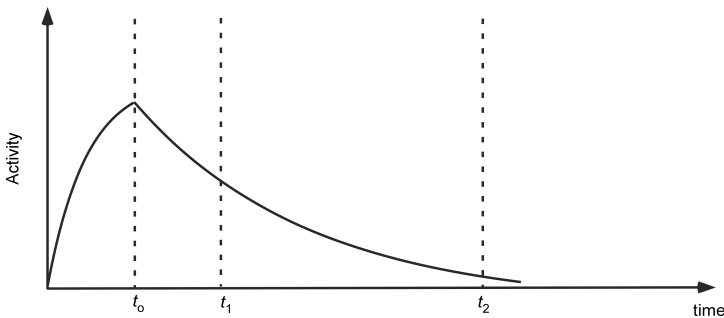


Figure 3.4: Variation of activity with time during and after activation.

$$A_{\infty} = \frac{\lambda(C - B)}{y\eta(1 - e^{-\lambda t_0})(e^{-\lambda t_1} - e^{-\lambda t_2})} \quad (3.3.94)$$

and

$$A_{\infty} = \sigma \dot{\Phi} N = \sigma \dot{\Phi} \frac{m N_A}{m_a} \quad (3.3.95)$$

Two isotopes of silver are produced during the activation process, $^{108}\text{Ag}(1)$ and $^{110}\text{Ag}(2)$. The net count obtained C-B is then given by

$$C - B = \dot{\Phi} m N_A \eta \left[\frac{f_1 \sigma_1 y_1 (1 - e^{-\lambda_1 t_0})(e^{-\lambda_1 t_1} - e^{-\lambda_1 t_2})}{\lambda_1 m_{a,1}} + \frac{f_2 \sigma_2 y_2 (1 - e^{-\lambda_2 t_0})(e^{-\lambda_2 t_1} - e^{-\lambda_2 t_2})}{\lambda_2 m_{a,2}} \right] \quad (3.3.96)$$

Data:

$C=25000$ (total amount of counts)

$B=3 \cdot 300=900$ (background counts)

$N_A=6.022 \cdot 10^{23} \text{ mol}^{-1}$ (Avogadro's number)

$\eta=0.10$ (detector efficiency)

$\dot{\Phi} = 10.2 \cdot 10^7 \text{ m}^{-2} \text{ s}^{-1}$ (neutron fluence rate)

$f_1=0.5135$ (mass fraction of ^{107}Ag)

$f_2=0.4865$ (mass fraction of ^{109}Ag)

$\lambda_1 = \ln 2 / (2.37 \cdot 60) = 4.874 \cdot 10^{-3} \text{ s}^{-1}$ (decay constant for ^{108}Ag)

$\lambda_2 = \ln 2 / (24.6) = 2.818 \cdot 10^{-2} \text{ s}^{-1}$ (decay constant for ^{110}Ag)

$m_{a,1}=107$ (atomic mass for ^{107}Ag)

$m_{a,2}=109$ (atomic mass for ^{109}Ag)

$y_1=0.977$ (frequency of emitted electrons/decay of ^{108}Ag)

$y_2=0.997$ (frequency of emitted electrons/decay of ^{110}Ag)

$\sigma_1 = 3.0 \cdot 10^{-28} \text{ m}^2$ (activation cross section for ^{107}Ag)

$\sigma_2 = 110.0 \cdot 10^{-28} \text{ m}^2$ (activation cross section for ^{109}Ag)

Data inserted in Eq. (3.3.96) gives m .

$$25000 - 900 = 6.022 \cdot 10^{23} \cdot 10.2 \cdot 10^7 \cdot 0.1 \cdot m \cdot 10^{-28} \left[\frac{0.5135 \cdot 30 \cdot 0.977 (1 - e^{-600 \cdot 4.874 \cdot 10^{-3}})(e^{-60 \cdot 4.874 \cdot 10^{-3}} - e^{-360 \cdot 4.874 \cdot 10^{-3}})}{4.874 \cdot 10^{-3} \cdot 107} + \frac{0.4865 \cdot 110 \cdot 0.997 (1 - e^{-600 \cdot 2.818 \cdot 10^{-2}})(e^{-60 \cdot 2.818 \cdot 10^{-2}} - e^{-360 \cdot 2.818 \cdot 10^{-2}})}{2.818 \cdot 10^{-2} \cdot 109} \right]$$

$m=2.08 \text{ g}$

Answer The silver mass in the coin is 2.1 g.

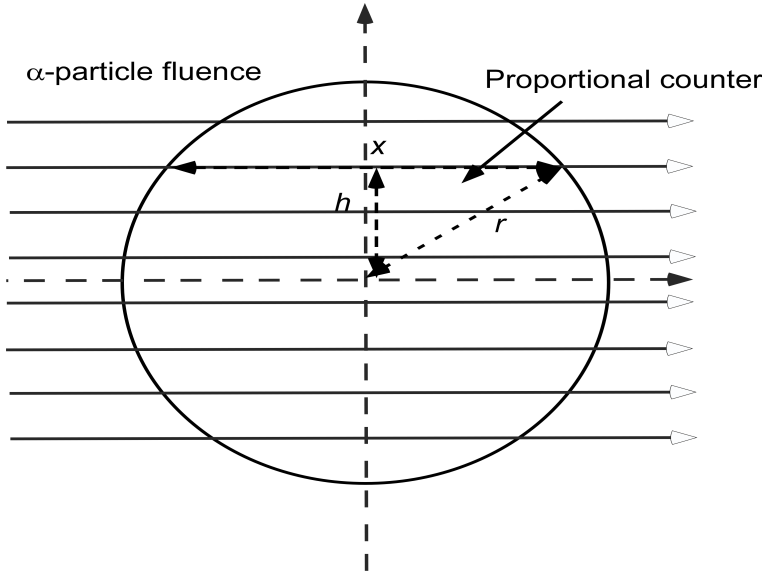
Solution exercise 3.25.

Figure 3.5: Detector geometry with the “wall less” proportional counter irradiated with a parallel fluence of α -particles.

The relative number of particles that hits the spherical counter between h and $h + dh$ (see Fig. 3.5) may be expressed as

$$P(h) dh = \frac{2\pi dh\Phi}{\pi r^2\Phi} = \frac{2h dh}{r^2} \quad (3.3.97)$$

where r is the radius of the proportional counter, and Φ is the incoming particle fluence.

The track length, x , of these particles in the spherical counter is, assuming straight particle tracks, given by

$$\left(\frac{x}{2}\right)^2 = r^2 - h^2 \quad (3.3.98)$$

Differentiation of the relation gives

$$\frac{2x dx}{4} = 2h dh \quad (3.3.99)$$

The number of particles between h and $h + dh$ is equal to the number of particles with

the track length between x and $x + dx$, i.e. $P(h) dh = P(x) dx$. Thus

$$P(x) dx = \frac{2h dh}{r^2} = \frac{x dx}{2r^2} \quad (3.3.100)$$

If it is assumed that the energy loss of the particles is constant, the probability distribution of pulse heights (V) will be equal to the probability distribution of track lengths, i.e. $P(V) \approx P(x)$, which increases linearly with x or V .

Energy loss for an α -particle passing the full diameter, d , of the counter is given by

$$E = d \cdot \rho \frac{1}{\rho} \frac{dE}{dx} \quad (3.3.101)$$

Data:

$d=10 \cdot 10^{-3}$ m (diameter of the sphere)

$\rho = 1.128 \frac{273}{293} \frac{9.44}{101.3} = 9.794 \cdot 10^{-2}$ kg m⁻³ (density of the tissue equivalent gas)

$\frac{1}{\rho} \frac{dE}{dx} = 88.33$ MeV m² kg⁻¹ (mass stopping power of the tissue equivalent gas)

Data inserted in Eq (3.3.101) gives

$$E = 10 \cdot 10^{-3} \cdot 9.794 \cdot 10^{-2} \cdot 88.33 = 0.0865 \text{ MeV}$$

The obtained pulse height is given by

$$V = \frac{EqMG}{\bar{W}C} \quad (3.3.102)$$

where

$\bar{W}=31.1$ eV (mean energy per ion pair)

$q = 1.602 \cdot 10^{-19}$ C (charge of an electron)

$M=1000$ (gas amplification)

$C = 1.0 \cdot 10^{-12}$ F (capacitance of the preamplifier)

$G=10$ (amplification in the preamplifier)

Data inserted in Eq (3.3.102) gives

$$V = \frac{0.0865 \cdot 10^6 \cdot 1.602 \cdot 10^{-19} \cdot 1000 \cdot 10}{31.1 \cdot 1.0 \cdot 10^{-12}} = 4.456 \text{ V}$$

Channel 1024 corresponds to a voltage of 10 V. Then 4.456 V corresponds to channel $4.456 \cdot 1024 / 10 = 456$.

Answer: The distribution is triangular with a maximum at channel 456.