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Statistics of scattered photons from a driven three-level emitter in 1D open space

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We derive the statistics of scattered photons from a Λ - or ladder-type three-level emitter (3LE) embedded in a 1D open waveguide. The weak probe photons in the waveguide are coupled to one of the two allowed transitions of the 3LE, and the other transition is driven by a control beam. This system shows electromagnetically induced transparency (EIT) which is accompanied with the Autler-Townes splitting (ATS) at a strong driving by the control beam, and some of these effects have been observed recently. We show that the nature of second-order coherence of the transmitted probe photons near two-photon resonance changes from bunching to antibunching to constant as strength of the control beam is ramped up from zero to a higher value where the ATS appears.

Strong light-matter interaction in open space at the level of single atom and a few photons can be created by coupling a real or artificial atom to photon modes confined in an open one-dimensional (1D) waveguide [1–22]. Efficient strong coupling between matter and photon field has been achieved by using highly confined propagating microwave photon modes in a 1D open superconducting transmission line and a large dipole moment of an artificial atom such as a superconducting qubit [12–14, 19–21]. A destructive interference between the emitted photons from a two-level atom and the incident photons in the waveguide yields extinction of the transmitted photons for the atom being side-coupled to a weak incident photon field. Extinction efficiencies greater than 99% have been observed in recent experiments with superconducting transmission lines and superconducting ‘transmon’ qubits [19–21]. Other systems which are currently under extensive studies include surface plasmons of a metallic nanowire coupled to quantum dots or nanocrystals [23] and line-defects in photonic crystals coupled to quantum dots [24, 25].

One of us (D.R.) has recently studied single- and two-photon scattering by a driven Λ -type three-level atom or emitter (3LE) which is coupled to a quasi 1D continuum of photon modes of the waveguide [17]. The excited state $|2\rangle$ of the emitter (see Fig.1(a)) is connected to the state $|3\rangle$ by a classical laser beam with Rabi frequency Ω_c . We set energy of the ground state $|1\rangle$ to be zero. Thus we can write the Hamiltonian of the 3LE within the rotating-wave approximation as $\mathcal{H}_{3LE} = (E_2 - i\gamma_2/2)|2\rangle\langle 2| + (E_2 - \Delta - i\gamma_3/2)|3\rangle\langle 3| + (\Omega_c/2)(|3\rangle\langle 2| + |2\rangle\langle 3|)$, where spontaneous emission loss from the 1D waveguide is modeled by including an imaginary part $-i\gamma_2/2$ and $-i\gamma_3/2$ to the energy of the respective states $|2\rangle$ and $|3\rangle$. The states $|1\rangle$ and $|3\rangle$ can be two hyperfine split states, and the transitions $|1\rangle - |2\rangle$ and $|2\rangle - |3\rangle$ would couple to different polarizations of light by selection rule. A probe beam in the photon modes of the waveguide is sent near resonant to the transition $|1\rangle - |2\rangle$. We also consider that there is no direct transition between the states $|1\rangle$ and $|3\rangle$ by selection rule. An exact single-photon scattering state of the probe beam and the corresponding transmission line-

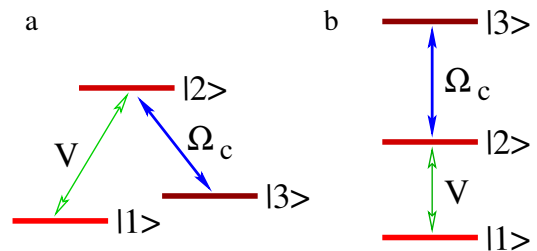


FIG. 1. Schematic of (a) Λ - and (b) ladder-type three-level emitters whose one of the two allowed transitions is coupled to probe photons by strength V and the other transition is driven by a control beam with Rabi frequency Ω_c .

shape showing electromagnetically induced transparency (EIT) [26] were calculated for this system in Ref.[16]. One of us has derived two-photon scattering state of the probe beam in this system for a weak control field and studied scaling of EIT line-shape for single and two probe photons [17]. Incident photons are in Fock-state in both the previous studies.

In a set of recent experiments [14, 19] it has been claimed to observe EIT line-shape for a driven ladder-type three-level superconducting qubit embedded in an open transmission line. It is also shown in the experiments that the Autler-Townes splitting (ATS) [27] appears when the one of the two allowed transitions of the qubit is driven by a strong control beam [14, 19]. One interesting feature of a system of an emitter coupled to an open waveguide is strong photon-photon interactions generated by the two- or multi-level emitter by preventing multiple occupancy of photons locally at the emitter. It is known that scattered states from a two-level emitter embedded in a 1D waveguide can be non-classical [1, 10, 15]. The antibunching of the reflected photons and superbunching of the transmitted photons have been recently demonstrated by measuring second-order coherence of the scattered fields from a two-level emitter [20]. However we are not aware of any theoretical or experimental study on photon-photon correlations of the scattered probe beam from a 3LE in the presence of an ar-

bitrary strong control beam. Therefore, we here study scattering of multiple probe photons from a Λ -type 3LE for a general value of Ω_c . In particular, we derive exact two- and multi-photon scattering states of the probe beam for an arbitrary Ω_c and calculate second-order coherence of the reflected and transmitted probe photons.

The scattering of photons from a driven 3LE embedded in a 1D photonic waveguide can be described by the following Hamiltonian [17]

$$\mathcal{H} = \mathcal{H}_{wg} + \mathcal{H}_{3LE} + \mathcal{H}_c, \quad (1)$$

where \mathcal{H}_{wg} represents free probe photons in the waveguide and \mathcal{H}_{3LE} for a driven 3LE is already introduced. The local coupling of the probe photons with the 3LE is given by \mathcal{H}_c . We consider a linear energy-momentum dispersion ($E_k = v_g k$) for the free probe photons, and divide the positive and negative momentum photons as right- and left-moving modes. Thus we write

$$\mathcal{H}_{wg} = -iv_g \int dx [a_R^\dagger(x) \partial_x a_R(x) - a_L^\dagger(x) \partial_x a_L(x)], \quad (2)$$

where v_g is the group velocity of the photons and $a_R(x)$ [$a_L(x)$] is the annihilation operator of a right-(left-) moving photon at position x . In our model the 3LE is side-coupled to the propagating light fields locally at $x = 0$; thus we write

$$\mathcal{H}_c = V|2\rangle\langle 1|(a_R(0) + a_L(0)) + h.c., \quad (3)$$

where V is coupling strength between the emitter and the probe photons. We set here $v_g = \hbar = 1$.

The single-photon transmission and reflection line-shapes for a driven Λ -type 3LE coupled to a 1D waveguide have been reported earlier in Refs.[16, 17]. The single-photon transmission and reflection amplitudes are given respectively by $\tilde{t}_k = (t_k + 1)/2 = \chi/(\chi + i\Gamma/2)$ and $\tilde{r}_k = (t_k - 1)/2 = -0.5i\Gamma/(\chi + i\Gamma/2)$ where $\Gamma = 2V^2$ and

$$\chi = E_k - E_2 + i\gamma_2/2 - \frac{\Omega_c^2}{4(E_k - E_2 + \Delta + i\gamma_3/2)}. \quad (4)$$

In Fig.2 we plot the transmission coefficient $T_k = |\tilde{t}_k|^2$ with detuning $(E_k - E_2)$ of the incident probe photon for different values of the control beam Rabi frequency Ω_c . Here we set the loss γ_3 very small compared to Γ , i.e., the state $|3\rangle$ is metastable. In the absence of the control beam a probe photon is strongly reflected by the emitter due to $|1\rangle - |2\rangle$ transition, and a Lorentzian dip around $E_k = E_2$ in the transmission line-shape in Fig.2(a) reveals it. Thus the 3LE in the absence of a control beam acts as a perfect reflector, and it has been observed in the recent experiments as shown in Fig.2(a) of Refs.[14, 19]. A narrow transmission window which is much narrower than the Lorentzian dip in the transmission appears at two-photon resonance $E_k - E_2 = -\Delta$ as we switch on a weak control beam, $\Omega_c < \Gamma$ in Fig.2(b). This induced transparency by the control beam is known as EIT. The EIT is developed due to destructive Fano interference between two allowed atomic transitions which leads to

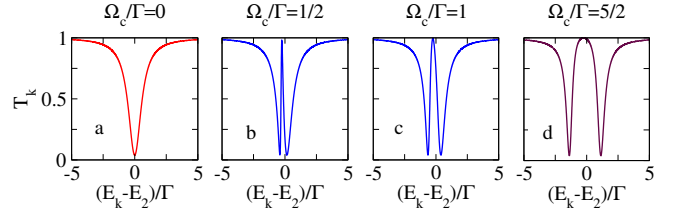


FIG. 2. Appearance of electromagnetically induced transparency (EIT) at two-photon resonance, $E_k - E_2 = -\Delta$ when a weak control beam ($\Omega_c < \Gamma$) is switched on, and the Autler-Townes splitting (ATS) appears at a relatively strong control beam ($\Omega_c > \Gamma$). The splitting between the Autler-Townes doublet is Ω_c . The parameters are $\Delta/\Gamma = \gamma_2/\Gamma = 1/4$, $\gamma_3/\Gamma = 1/40$ for a Λ -type emitter.

cancellation of the population of the state $|2\rangle$, i.e., formation of the ‘dark state’. As the transition $|1\rangle - |2\rangle$ gets suppressed at two-photon resonance due to formation of the dark state, the probe photons pass the emitter without being scattered. The width of the transparency window near two-photon resonance increases with an increasing strength of the control beam which is shown in Fig.2(c). Finally the ATS appears at a relatively stronger control field, $\Omega_c \geq \Gamma$ (depending on the loss terms) and the splitting between the Autler-Townes doublet is given by the control beam Rabi frequency Ω_c (see Fig.2(d)). The Autler-Townes doublet forms due to Rabi splitting of the states $|2\rangle$ and $|3\rangle$.

A ladder-type 3LE (see Fig.1(b)) made of a superconducting qubit was used in two recent experiments [14, 19] with transmission lines. In these experiments, the lower transition $|1\rangle - |2\rangle$ of the two allowed transitions of the ladder-type 3LE is coupled to a weak probe beam and the upper transition $|2\rangle - |3\rangle$ is driven by a control beam. A formula for the transmission amplitude of the probe beam was derived in Ref.[14] using the Markovian master equation for the density matrix. We find that their transmission amplitude formula for the driven ladder-type 3LE is exactly similar to our single photon transmission amplitude \tilde{t}_k in the driven Λ -type 3LE when we replace their probe and control beam detunings $\delta\omega_p$ and $\delta\omega_c$ by our $(E_k - E_2)$ and Δ respectively, and their loss terms γ_{21} and γ_{31} by our $(\gamma_2 + \Gamma)/2$ and $\gamma_3/2$ respectively. We also identify the probe beam coupling Γ_{21} in Ref.[14] with our Γ . This similarity is not surprising as the two 3LEs are identical except the loss rates are practically very different in the two 3LEs. The authors of Refs.[14, 19] have demonstrated an induced transparency by a strong control beam. However, an EIT transmission line-shape for an arbitrarily weak value of the control beam, $\Omega_c \ll \Gamma$ has not been observed in both the experiments. Thus it is not quite clear whether these experiments demonstrate EIT or they only see the ATS at a strong driving field. A recent theoretical study [28] concludes after an objective test of the experimental data that the ATS is preferred to be observed than EIT in these experiments. The state $|3\rangle$ is not metastable for a ladder-type 3LE

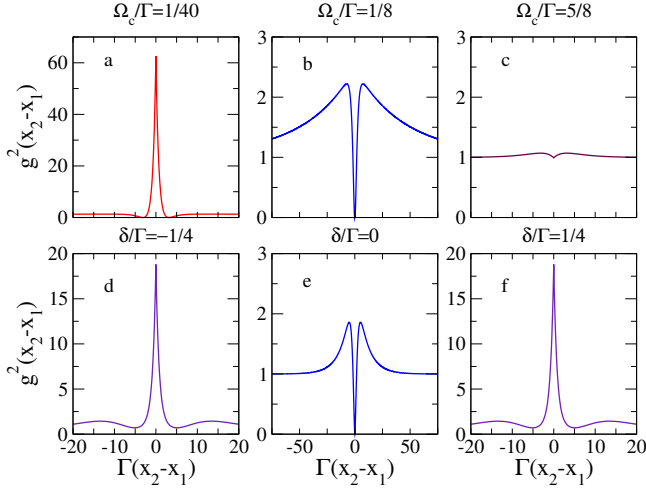


FIG. 3. Second-order coherence $g^2(x_2 - x_1)$ of the transmitted probe photons from a driven Λ -type emitter at various control beam driving Ω_c (first row) and two-photon detuning δ (second row). The probe beam is on two-photon resonance $\delta = (E_k - (E_2 - \Delta)) = 0$ and $\gamma_3/\Gamma = 1/40$ in the first row, and the control beam strength $\Omega_c/\Gamma = 3/10$ and $\gamma_3/\Gamma = 1/8$ in the second row. The other parameters are $\Delta = 0$, $\gamma_2/\Gamma = 0.31$.

and it has fast pure dephasing or loss, $\gamma_3 > \Gamma$. Therefore it has not been possible to observe a transmission window near two-photon resonance at a weak Ω_c in the experiments[14].

To calculate the statistics of the scattered probe field from a driven 3LE we need to derive multi-photon scattering state of the probe field. A two-photon scattering state of the probe field for a weak control beam was derived in Ref.[17]. However the two-photon state in Ref.[17] is not sufficient to understand how the statistics of scattered field changes with an increasing value of the

control beam Rabi frequency, specially at a Ω_c where the ATS appears. We here derive an exact two-photon scattering state of the probe field for an arbitrary strength of Ω_c using a method developed recently for an atomic ensemble [29]. We are also able to derive a multi-photon scattering state in the present system. This is done in a similar spirit of Ref.[15]. In the multi-photon scattering state we consider scattering processes with inelastic exchange of momentum between one pair of photons and elastic exchange of momentum between other all possible pairs. One general way to quantify the statistics is by measuring second-order coherence of the scattered photons. We define second-order coherence by

$$g^2(x_2 - x_1) = \frac{\langle \psi | a_m^\dagger(x_1) a_m^\dagger(x_2) a_m(x_2) a_m(x_1) | \psi \rangle}{\langle \psi | a_m^\dagger(x_1) a_m(x_1) | \psi \rangle \langle \psi | a_m^\dagger(x_2) a_m(x_2) | \psi \rangle}, \quad (5)$$

where $m = R$ for the transmitted photons and $m = L$ for the reflected photons for an incident probe beam from the left. Here $|\psi\rangle$ is a N -photon scattering Fock state with incident momenta k_1, k_2, \dots, k_N . A single emitter becomes saturated by a single photon as one emitter can absorb only one photon at a time. Therefore, a strong photon-photon nonlinearity is created by an emitter for two incident photons. However the relative strength of photon-photon nonlinearity created by an emitter falls with an increasing number of photons of more than two, and most of the incident photons pass by the emitter without interacting with it. Thus we would be able to capture main features of the statistics of scattered photons due to photon-photon nonlinearity and control beam driving by considering a scattering state of the probe beam with minimum two incident photons. We here find after keeping higher order contributions in the numerator and denominator of Eq.5

$$g^2(x_2 - x_1) = \frac{|\sum_P (t_{k_{P_1}} \pm 1)(t_{k_{P_2}} \pm 1) h_{k_{P_1}}(x_1) h_{k_{P_2}}(x_2) + 2i \sum_{PQ} V \beta (t_{k_{P_1}} - 1) \Xi_{k_{P_2}}(x_{Q_{12}}) h_{k_{P_1}}(x_{Q_1}) h_{k_{P_2}}(x_{Q_1}) \theta(x_{Q_{12}})|^2}{|(t_{k_1} \pm 1)(t_{k_2} \pm 1)|^2}, \quad (6)$$

where $+$ sign for the transmitted probe beam and $-$ sign for the reflected probe beam. Here $\Xi_k(x_1 - x_2) = \sum_{j=\pm} d_j \varepsilon_j(k) e^{i(s+j\Omega_c/4\beta)|x_1-x_2|}$, $h_k(x) = e^{ikx} \theta(x)/\sqrt{2}$, $\varepsilon_\pm(k) = V/(E_k + s \pm \Omega_c/4\beta)$, $s = -(E_2 - \Delta/2) + i(\gamma_2 + \gamma_3 + \Gamma)/4$, $\beta = \Omega_c/\sqrt{\epsilon^2 + 4\Omega_c^2}$, $d_\pm = (1/(2\beta) \pm \epsilon/(2\Omega_c))$ and $\epsilon = -2\Delta + i(\gamma_2 + \Gamma - \gamma_3)$. We use $P = (P_1, P_2)$ and $Q = (Q_1, Q_2)$ for permutation of $(1, 2)$ and $x_{Q_{12}} = x_{Q_1} - x_{Q_2}$.

Next we discuss nature of second-order coherence at two-photon resonance, i.e., $E_{k_1} = E_{k_2} = E_2 - \Delta$. We find from Eq.4 that the transmission amplitude $\tilde{t}_k = (t_k + 1)/2 \approx 0$ when $\Omega_c \sim 0$ and $\tilde{t}_k \approx 1$ when $\Omega_c \sim \Gamma$.

Here we assume that $\gamma_2 < \Gamma$ for both Λ - and ladder-type 3LE. In the absence of the control beam the 3LE-waveguide system reduces to a two-level emitter coupled to a probe beam, and we find bunching of the transmitted photons due to the inelastic two-photon bound state (the second term of the numerator in Eq.6) when $E_k = E_2$. It has been demonstrated in a recent experiment [20] with a two-level emitter. When $T_k = 1$ in the presence of a strong control beam driving, the photon-photon correlation due to the inelastic two-photon bound state becomes negligible, and the second-order coherence of the transmitted probe beam is mostly determined by the first term of the numerator in Eq.6. Then the numerator and de-

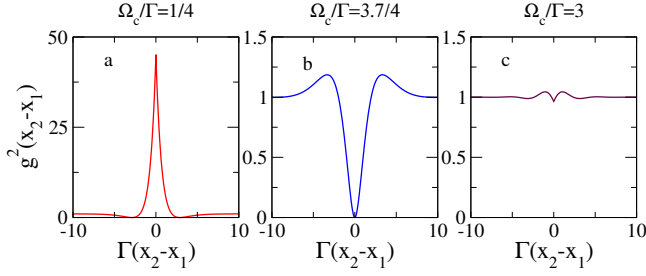


FIG. 4. Second-order coherence $g^2(x_2 - x_1)$ of the transmitted probe beam from a driven ladder-type emitter at various control beam driving Ω_c . The incident probe beam is a coherent state wave-packet with $k_0 = E_2 - \Delta \gg \Delta_k = \Gamma/40$, $\bar{n} = 1$. The other parameters are $\Delta = 0$, $\Gamma/2\pi = 11$ MHz, $\gamma_2/2\pi = 3.4$ MHz and $\gamma_3/2\pi = 13.8$ MHz.

nominator of $g^2(x_2 - x_1)$ become the same, and we have $g^2(x_2 - x_1) = 1$. At an intermediate control beam driving, $\Omega_{c0}^2 = \gamma_3(\Gamma - \gamma_2)$, t_k vanishes, and the single probe photon transmission amplitude $\tilde{t}_k = 1/2$. At Ω_{c0} the numerator in Eq.6 vanishes at $x_1 = x_2$ and the numerator is non-zero when $x_1 \neq x_2$. Therefore $g^2(x_2 - x_1)$ exhibits antibunching of the transmitted probe beam at two-photon resonance when $\Omega_c = \Omega_{c0}$. Physically the antibunching occurs due to interference between the partially transmitted probe photons and the inelastic two-photon bound state.

We show the above discussed behavior of $g^2(x_2 - x_1)$ of the transmitted probe photons for a driven Λ -type 3LE in Fig.3 where we set the loss term γ_3 to be very small compared to Γ , such that the state $|3\rangle$ is metastable. We show bunching of transmitted probe photons in Fig.3(a) for a very weak control beam when $T_k \approx 0$. We kept the incident probe beam on two-photon resonance, $E_{k_1} = E_{k_2} = E_2 - \Delta$. Next we slowly increase the strength of Rabi frequency of the control beam. We find from Fig.3(b) that $g^2(x_2 - x_1)$ shows antibunching of the transmitted probe photons when Ω_c is near $\sqrt{\gamma_3(\Gamma - \gamma_2)}$. The antibunching implies that two probe photons can not transmit through the emitter simultaneously. This happens for a Rabi frequency when a complete dark state is not yet formed. As we further increase Ω_c a dark state is formed and T_k becomes unity at the two-photon resonance. There $g^2(x_2 - x_1) = 1$ as shown in Fig.3(c), and the incident probe photons are not scattered by the driven emitter. When frequency of the incident probe beam is de-

tuned from the two-photon resonance condition of EIT, $g^2(x_2 - x_1)$ shows bunching (check Figs.3(d,f)) as one photon then gets strongly scattered by the emitter.

Finally we discuss second-order coherence of the transmitted probe beam when the incident probe beam is a coherent state wave-packet. The incident coherent state wave-packet is given by $|\alpha\rangle = e^{a_\alpha^\dagger - \bar{n}/2}|\varphi\rangle$ where $a_\alpha^\dagger = \int dk \alpha(k) a^\dagger(k)$, $|\varphi\rangle$ is vacuum state and the mean photon number is $\bar{n} = \int dk |\alpha(k)|^2$. Here $a^\dagger(k) = \int dx e^{ikx} a_R^\dagger(x)/\sqrt{2\pi}$ for an incident wave-packet from the left. We consider the mean photon number of the coherent state wave-packet $\bar{n} \leq 1$ and choose a Gaussian wave-packet [15]

$$\alpha(k) = \frac{\sqrt{\bar{n}}}{(2\pi\Delta_k^2)^{1/4}} \exp\left(-\frac{(k - k_0)^2}{4\Delta_k^2}\right), \quad (7)$$

where Δ_k is the width of the wave-packet and k_0 is mean momentum (or energy) of the wave-packet. We choose $k_0 = E_2 - \Delta \gg \Delta_k = \Gamma/40$. The statistics of scattered probe photons for a coherent state wave-packet input remains similar to that of a Fock state input, provided that the bandwidth of the coherent state input is significantly narrower than the emitter's line-width. We show this in Fig.4 for a ladder-type 3LE with a large dephasing loss from the state $|3\rangle$ [14]. The nature of second-order coherence changes from bunching (Fig.4(a)) to antibunching (Fig.4(b)) to one for coherent state (Fig.4(c)) as Ω_c is increased from a weak to a strong value.

In conclusion, we have shown that second-order coherence of the scattered probe photons from a Λ - or ladder-type 3LE can be tuned by changing Rabi frequency of the control beam. The transmission coefficient of the probe beam from a ladder-type 3LE at different strength of the control beam has been already measured, and various rudimentary quantum devices, such as a switchable mirror or a single-photon router which can route a single-photon signal from an input port to either of two output ports, have been proposed [14, 19]. These devices might have important applications in building photonic quantum networks for quantum information processing. The second-order coherence in these systems can be measured experimentally using a Hanbury-Brown-Twiss measurement setup [20].

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