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Statistical Hot Channel Analysis for the NBSR

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Abstract

A statistical analysis of thermal limits has been carried out for the research reactor (NBSR) at the National Institute of Standards and Technology (NIST). The objective of this analysis was to update the uncertainties of the hot channel factors with respect to previous analysis for both high-enriched uranium (HEU) and low-enriched uranium (LEU) fuels. Although uncertainties in key parameters which enter into the analysis are not yet known for the LEU core, the current analysis uses reasonable approximations instead of conservative estimates based on HEU values.

Cumulative distribution functions (CDFs) were obtained for critical heat flux ratio (CHFR), and onset of flow instability ratio (OFIR). As was done previously, the Sudo-Kaminaga correlation was used for CHF and the Saha-Zuber correlation was used for OFI. Results were obtained for probability levels of 90%, 95%, and 99.9%.

As an example of the analysis, the results for both the existing reactor with HEU fuel and the LEU core show that CHFR would have to be above 1.39 to assure with 95% probability that there is no CHF. For the OFIR, the results show that the ratio should be above 1.40 to assure with a 95% probability that OFI is not reached.

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1. Introduction

Limiting values for thermal limits must be determined with high confidence in order to help assure safe power operation of the NIST research reactor (NBSR). The approach taken for the NBSR is to do this probabilistically by computing the cumulative distribution functions (CDFs) of the critical heat flux ratio (CHFR) and the onset of flow instability ratio (OFIR), two of the thermal limits. The calculations for the CHFR and OFIR entail the determination of the CDF of the coolant bulk temperature rise (ΔT_b) and the local heat flux (q) as well.

The CDFs are calculated for steady-state full power conditions, accounting for uncertainties in various hot channel factors, and are customarily used to establish acceptance criteria for the thermal limits under accident conditions. For CHFR and OFIR, the CDF is used to determine the limiting value with a corresponding probability of not exceeding the critical heat flux or onset of flow instability, respectively. The probabilities considered are 90%, 95%, and 99.9%.

The methodology is the same as previously used [1]. The hot channel factors for HEU have been revised and approximations have been made for the hot channel factors of the LEU fuel.

2. Methodology

In the present analysis, each factor contributing to the CDF in the determination of CHFR and OFIR is considered to be a random variable with a defined (normal) probability distribution. These individual distributions are then combined by Monte-Carlo methods to arrive at the probability distributions of interest, i.e., for CHFR and OFIR. To simplify the manipulation of these individual variables in the analysis, they are converted to dimensionless form and normalized so that their mean value is unity. Generally, this is done by dividing the random value of the variable by its mean or nominal value. The dimensionless normalized variables are the F factors shown in Table 1 (in Section 3), and are assumed to have normal distributions.

To generate the CDF of the parameters of interest (CHFR, OFIR, and others required in their determination), these parameters must be expressed in terms of the individual uncertainty factors presented in Table 1. These relationships are shown below, and their derivation can be found in [1]. The subscripts r refers to the random value of the variable and n refers to the nominal or calculated value.

Channel Velocity (V)

$$\frac{V_r}{V_n} = \frac{F_5 F_6 F_4^{2/3}}{F_3} \qquad (1)$$

Bulk Temperature Rise (ΔT_b)

$$\frac{(\Delta T_b)_r}{(\Delta T_b)_n} = \frac{F_1 F_2 F_8}{F_5 F_6 F_4^{5/3}} \tag{2}$$

Local Heat Flux (q)

$$\frac{q_r}{q_n} = F_1 F_2 F_7 \tag{3}$$

Critical Heat Flux (q_{CHF})

The critical heat flux is calculated with the Sudo-Kaminaga correlation [2], as was done previously [3]. For steady-state conditions, the dimensionless critical heat flux CHF^* is defined as:

$$\mathrm{CHF}^* = 0.005\left|G^*\right|^{0.611}\left(1 + \frac{5000}{\left|G^*\right|}\Delta T^*_{\mathrm{sub,\,o}}\right) \tag{4}$$

where

$$G^* = \frac{G}{\sqrt{\lambda\, g\, \rho_g \left(\rho_l - \rho_g\right)}} \tag{5}$$

$$\lambda = \left[\sigma \Big/ (\rho_l - \rho_v)\cdot g\right]^{1/2} \tag{6}$$

$$\Delta T^*_{\mathrm{sub}} = \frac{C_{pl}\,\Delta T_{\mathrm{sub}}}{h_{fg}} \tag{7}$$

and

G : mass flux (kg/m^2.s)
σ : surface tension (N/m)
ρ_g, ρ_l : density of gas and liquid (kg/m^3)
g : acceleration of gravity (m/s^2)
C_{pl} : specific heat at constant pressure of the liquid (kJ/kg.K)
h_{fg} : latent heat of evaporation (kJ/kg)
$\Delta T_{\mathrm{sub,\,o}}$ = subcooling at the outlet (K)

It is not possible to express the critical heat flux CHF in terms of F factors only. The Sudo-Kaminaga correlation can be expressed as:

$$CHF = \psi(\Delta T_b, V) \tag{8}$$

since G=V*ρ and $\Delta T_{sub,o} = T_{sat} - T_{out} = T_{sat} - (T_{in} + 1 + \Delta T_b)$

where the inlet temperature T_{in} is not considered a normally distributed variable but rather a constant value (T_{in}) increased by 1 K to conservatively account for the small uncertainty in its measurement.

As previously discussed, F_9 is defined as the ratio of the nominal value of the critical heat flux (CHF, calculated by the Sudo-Kaminaga correlation) to the random value (actual critical heat flux q_{CHF}), for conditions described by $(\Delta T_{b,r}, V_r)$.:

$$F_9 = \frac{CHF_r}{q_{CHF,r}} = \frac{\psi(\Delta T_{b,r}, V_r)}{q_{CHF,r}} = \frac{\psi_r}{q_{CHF,r}} \tag{9}$$

For nominal conditions $(\Delta T_{b,n}, V_n)$, the actual critical heat flux is assumed to be the one calculated by the Sudo-Kaminaga correlation:

$$q_{CHF,n} = CHF_n = \psi\left(\Delta T_{b,n}, V_n\right) = \psi_n \tag{10}$$

which leads to:

$$\frac{q_{CHF,r}}{q_{CHF,n}} = \frac{\psi_r}{\psi_n F_9} \tag{11}$$

Critical Heat Flux Ratio (CHFR)

The CHFR is defined as:

$$CHFR = \frac{q_{CHF}}{q} \tag{12}$$

Normalizing,

$$\frac{CHFR_r}{CHFR_n} = \frac{\psi_r}{\psi_n F_9 F_1 F_2 F_7} \tag{13}$$

Onset of Flow Instability

The NBSR operates in the region of high mass flow (Pe ≈ 170000), where the Saha-Zuber criterion reads [4]:

$$St = \frac{OFI}{GC_{pf}(T_{sat} - T_\lambda)} = 0.0065 \tag{14}$$

where

OFI: heat flux at which OFI occurs (as predicted by Saha-Zuber correlation)
T_{sat}: saturation temperature of liquid
T_λ: temperature of liquid at the point of net vapor generation
G : mass flow
C_{pf} : specific heat at constant pressure of the liquid

The heat flux for OFI predicted by the Saha-Zuber correlation becomes:

$$OFI = 0.0065\ GC_{pf}(T_{sat} - T_\lambda) \tag{15}$$

The specific heat is assumed constant, and OFI is a function of the velocity V (G=V*ρ) and ΔT_b, which is used as surrogate for T_λ ($T_\lambda \sim T_{out} = T_{in} + 1 + \Delta T_b$):

$$OFI = \phi(\Delta T_b, V) \tag{16}$$

Similarly to F_9, F_{10} is defined as the ratio of the nominal value of the heat flux at which OFI occurs (OFI, calculated by the Saha-Zuber correlation) to the random value (actual heat flux for which OFI occurs, q_{OFI}), for conditions described by $(\Delta T_{b,r}, V_r)$:

$$F_{10} = \frac{OFI_r}{q_{OFI,r}} = \frac{\phi(\Delta T_{b,r}, V_r)}{q_{OFI,r}} = \frac{\phi_r}{q_{OFI,r}} \tag{17}$$

For nominal conditions $(\Delta T_{b,n}, V_n)$, the actual heat flux at which OFI occurs is defined as:

$$q_{OFI,n} = OFI_n = \phi(\Delta T_{b,n}, V_n) = \phi_n$$

which leads to:

$$\frac{q_{OFI,r}}{q_{OFI,n}} = \frac{\phi_r}{\phi_n F_{10}} \tag{18}$$

Onset of Flow Instability ratio (OFIR)

The OFIR is defined as:

$$OFIR = \frac{q_{OFI}}{q} \tag{19}$$

Normalizing,

$$\frac{OFIR_r}{OFIR_n} = \frac{\phi_r}{\phi_n F_{10} F_1 F_2 F_7} \tag{20}$$

3. Hot channel factors

Table 1 summarizes the hot channel factors used in this study and gives the appropriate reference. Table 2 gives the nominal values for the parameters needed for the Sudo-Kaminaga or Saha-Zuber correlations.

A priori, the only difference between the future LEU NBSR and the current reactor is the fuel it will use. All other systems (including instrumentation) and their corresponding F factors will remain the same.

Table 1 Hot Channel Factors for HEU and LEU

Source of Uncertainty	Normalized Variable Identification	HEU		LEU	
		Standard Deviation (*)	Source	Standard Deviation	Source
Reactor Power Measurement	F_1	0.025	Table 3.2-1 of [5]	0.025	Same as HEU
Power Density Calculation	F_2	0.040	Table 3.2-1 of [5] / Engineering judgment	0.040	Same as HEU
Channel Dimensional Tolerance (local)	F_3	0.042	NBSR Dwg # E-04-016 in [8]	0.042	Fuel element assembly assumed same as HEU
Channel Dimensional Tolerance (average)	F_4	0.035	Dwg # E-04-016 in [8]	0.035	Fuel element assembly assumed same as HEU
Velocity Distribution Measurement	F_5	0.025	Table 3.2-1 of [5]	0.025	Same as HEU
Primary Flow Rate Measurement	F_6	0.022	[7]	0.022	Same as HEU
Fuel Loading Tolerance (local)	F_7	0.069	[8]	0.069	Monolithic fuel more uniform; bound by HEU value
Fuel Loading Tolerance (average)	F_8	0.0112	[8]	0.0123	Uncertainties in U10Mo+ enrichment [9]
Critical Heat Flux Correlation	F_9	0.202	Sudo-Kaminaga correlation [2]	0.202	Same as HEU
OFI Heat Flux Correlation	F_{10}	0.153	Saha-Zuber correlation [4]	0.153	Same as HEU

(*)Uncertainty limits represent 1 σ standard deviation assuming a normal distribution. When the referenced uncertainties were given as lower and upper limits, the range was assumed to represent a $\sqrt{12}\sigma$ value.

Table 2 Nominal Values for the Parameters Used in the Study

Variable	Nominal Value
Equivalent diameter	0.0055702 m (0.2193 in.)
Heated diameter	0.0064897 m (0.2555 in.)
Pressure	1.0132x105 Pa (14.7 psia)
Coolant velocity (*)	4.2993 m/s (14.1 ft/s)
Core Inlet temperature	316.48 K (110°F)
Bulk temperature rise	19.4 K (35°F)

(*)Coolant velocity includes a 0.94 multiplier for variation across the width of a channel

Reactor Power Measurement (F_1)

From [5],

$$\sigma(F_1^{HEU}) = \sigma(F_1^{LEU}) = 0.025$$

Power Density Calculation (F_2)

Currently, the power density is calculated via a sophisticated MCNPX model of the NBSR core. The model performs a detailed burn-up analysis where each half-element has a unique fuel inventory and includes the effect of the shim arms for each point in the cycle. The statistical error associated with the model, however, is not representative, in the absence of direct comparison with experimental data. For example, the error associated with the assumption that the fuel compositions are constant within each half-element is not compounded in the statistical error. It is known that the lack of axially variant burnup yields hot spot factors that are artificially high, and the correction for uneven burnup utilized in the 2004 SAR [10] is only accurate to first order.

It is assumed that the uncertainty of the power density calculation reported in Table 3.2-1 of reference [5], though stemming from a calculation utilizing different methods, represents a bound for the uncertainty of the methods presently used, thus:

$$\sigma(F_2^{HEU}) = \sigma(F_2^{LEU}) = 0.040$$

Channel Dimensional Tolerance (local) (F_3)

From drawing # E-04-016 [8], the thickness of the gap at centerline is 0.116 ± 0.007 in. The minimum gap (at sideplate) is 0.106 in.

$$\sigma({F_3}^{HEU}) = \sigma({F_3}^{LEU}) = \frac{0.116 + 0.007 - 0.106}{0.116 * \sqrt{12}} = 0.042$$

The fuel plates for the LEU fuel will be manufactured differently than the HEU fuel plates, but the final dimensions of the fuel plates will be the same. It is assumed that the LEU fuel elements will be assembled in a similar manner to the HEU elements, and will be subjected to the same requirements as the HEU fuel. Consequently, the channel dimensional tolerances (local and average) are kept the same for the HEU and LEU fuels.

Channel Dimensional Tolerance (average) (F_4)

F_4 is used as surrogate for flow area. The fuel plates are curved, as described in drawing # E-04-016 [8] and shown in Figure 1. The outer plate (grey, top) has a radius of curvature of 5.50 in, while the inner plate (grey, bottom), taking into account the 0.05 in thickness of the plate, has a radius of curvature of 5.55 in. The flow area is shown in light blue. d_{gap} is the thickness of the gap between fuel plates at the centerline, i.e. 0.116 ± 0.007. The flow area (A) is calculated by:

$$A = \int_{-1.312}^{1.312} (y_2 - y_1)dx = \int_{-1.312}^{1.312} \left(\sqrt{5.5^2 - x^2} + d_{gap} + 0.05 - \sqrt{5.55^2 - x^2}\right) dx$$

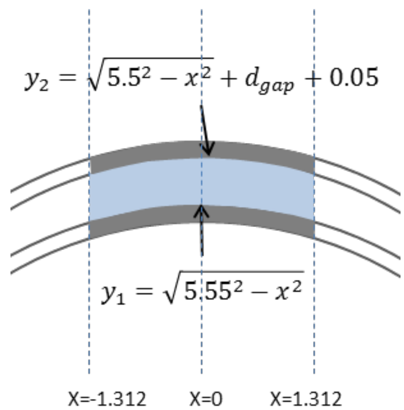

Figure 1 – Calculation of the Flow Area Between Curved Plates

The standard deviation is defined as:

$$\sigma\left({F_4}^{HEU}\right) = \sigma\left({F_4}^{LEU}\right) = \frac{1}{\sqrt{12}} \frac{A_{max} - A_{min}}{A_{avg}}$$

Where A_{max}, A_{min} and A_{avg} are the maximum flow area (d_{gap}=0.123 in), the minimum flow area (d_{gap}=0.109 in) and the average flow area (d_{gap}=0.116 in), respectively. $\sigma(F_4)$ becomes:

$$\sigma\left({F_4}^{HEU}\right) = \sigma\left({F_4}^{LEU}\right) = \frac{1}{\sqrt{12}} \frac{\int_{-1.312}^{1.312}(0.123 - 0.109)dx}{\int_{-1.312}^{1.312}\left(\sqrt{5.5^2 - x^2} + 0.166 - \sqrt{5.55^2 - x^2}\right)dx} = 0.035$$

Velocity Distribution Measurement (F_5)

The reduction of coolant velocity across a channel near the narrow sides of the channel is not a random variable. This effect has been incorporated in the analysis by assuming that the coolant velocity is 0.94 of the value that would have been calculated using the nominal primary flow rate. Thus the uncertainty in the velocity distribution measurement is for the variation of velocity from channel to channel only. The flow was determined to be uniform from channel to channel within the accuracy of the measurements. From [5],

$$\sigma\left({F_5}^{HEU}\right) = \sigma\left({F_5}^{LEU}\right) = 0.025$$

Primary Flow Rate Measurement (F_6)

The uncertainty in the primary flow rate measurement is the combination of the flow transmitter accuracy and the indicator accuracy [7]:

$$\sigma\left({F_6}^{HEU}\right) = \sigma\left({F_6}^{LEU}\right) = \sqrt{0.01^2 + 0.02^2} = 0.022$$

Fuel Loading Tolerance (local) (F_7)

The HEU fuel specifications [8] require that the “average surface density within a 0.080 inch x 1.0 inch band along the plate within the maximum core shall be less than or equal to +12% as compared to a standard”. Converting from a uniform distribution to a Gaussian distribution,

$$\sigma({F_7}^{HEU}) = \frac{2x0.12}{\sqrt{12}} = 0.069$$

While the manufacturing process of the LEU fuel has not yet been finalized, it is reasonable to assume that a monolithic fuel is more likely to be more uniform than a dispersion fuel, and that the LEU local fuel loading tolerance will be bound by the HEU value, i.e.:

$$\sigma({F_7}^{LEU}) = \sigma({F_7}^{HEU}) = 0.069$$

Fuel Loading Tolerance (average) (F_8)

For HEU, from [6], “each fuel plate shall contain 10.294 ± .20 grams of ^{235}U”.

$$\sigma({F_8}^{HEU}) = \frac{2 * 0.2}{10.294 * \sqrt{12}} = 0.0112$$

For LEU, the molybdenum content in U10Mo can vary by up to 10%, and the enrichment is 19.75 ± 0.2 % [9]. Thus the uncertainty in the ^{235}U mass is:

$$\sigma\left({F_8}^{LEU}\right) = \frac{1}{\sqrt{12}} \frac{m_{U235}^{max} - m_{U235}^{min}}{m_{U235}^{avg}} = \frac{1}{\sqrt{12}} \frac{0.91 * 0.1995 - 0.89 * 0.1955}{0.9 * 0.1975} = 0.0123$$

Sudo-Kaminaga Correlation (F_9)

From [3], the experimental CHF data has a deviation of -33% from the proposed CHF correlations. The -33% deviation was estimated to be -1.63 times the standard deviation implying a confidence level of about 90% assuming the CHF data has a normal distribution and the CHF correlation represents the mean. The standard deviation assigned to the Sudo-Kaminaga correlation is thus:

$$\sigma\left({F_9}^{HEU}\right) = \sigma\left({F_9}^{LEU}\right) = \frac{0.33}{1.63} = 0.202$$

Saha-Zuber Criterion (F_{10})

From [4], approximately 90% of the experimental data at high Péclet numbers fall within ±25% of the correlation, thus:

$$\sigma(F_{10}^{HEU}) = \sigma(F_{10}^{LEU}) = \frac{0.25}{1.63} = 0.153$$

4. Results

Table 3 shows the results based on the CHFR and OFIR CDFs for HEU fuel. The numbers for CHFR and OFIR are for not exceeding either CHF or OFI with the given probability. The limiting values at 95% probability level for CHFR and OFIR are 1.391 and 1.403 respectively. Table 4 shows the results obtained for LEU fuel for different probability levels. The limiting values at 95% probability for CHFR and OFIR are 1.391 and 1.403, respectively. It should be noted that the values obtained for HEU and LEU are identical, which is explained by noting that the only difference in the F factors between HEU and LEU is F_8, and that F_8 only contributes to the determination of $\Delta T_{b,r}$. While $\Delta T_{b,r}$ is needed to calculate ψ_r and ϕ_r, its final contribution to CHFR and OFIR is negligible.

Table 3 Statistical Analysis Results for HEU Fuel

Hot Channel Variable	**Probability Level**		
	90%	95%	99.9%
CHFR	1.301	1.391	1.778
OFIR	1.310	1.403	1.828

Table 4 Statistical Analysis Results for LEU Fuel

Hot Channel Variable	**Probability Level**		
	90%	95%	99.9%
CHFR	1.301	1.391	1.778
OFIR	1.310	1.403	1.828

5. Conclusions

The limiting values for CHFR and OFIR have been determined at different probability levels. The hot channel factors for HEU have been revised and more meaningful approximations have been made for the hot channel factors of the LEU fuel. The recommended limiting values for CHFR and OFIR are independent of the fuel type (HEU or LEU).

The Sudo-Kaminaga and Saha-Zuber correlations used in this analysis are valid for nominal operating conditions. While the correlations might not be applicable to accident conditions (e.g. LOCAs), it is customary to use the limiting values of CHFR and OFIR as thermal limits for both nominal operating conditions and accident conditions.

6. References

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