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Title: Automatic Generation of Dendritic Meshes using Paving Techniques

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Automatic Generation of Dendritic Meshes using Paving Techniques

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Paving

Paving allows all-quadrilateral meshes to be generated for arbitrary geometries [BS91].

- Quadrilaterals are “laid down” along the boundary until the interior of the region is completely filled (Fig. 1)
- Strict control of the flow of the algorithm guarantees high quality quad meshes are generated.

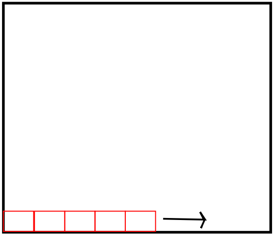

Figure 1: Preliminary Insertion of points and elements.

Paving has enjoyed much success over the past two decades:

- Generates all-quad meshes w/ minimal user input
- Handles arbitrary geometries (holes)
- Good resolution of boundary features.
- Fundamentals of algorithm match geometric intuition.

Effort has been made in extending paving to automatic generation of hexahedral meshes, known as “plastering,” as well as generation of all-quadrilateral surface meshes.

Dendritic Meshes

Dendritic meshes are a class of "quad-like" meshes where elements may have more than one neighbor sharing a straight edge.

The insertion of transition elements allows for the control of element size while respecting the contours of the boundary.

Jean, et. al. generated dendritic meshes by combining unidirectional feathering with transfinite interpolation.

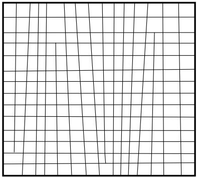

Figure 2: Unidirectional feathering on a rectangle.

Dendritic Paving

Dendritic Paving

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Present progress on the generation of dendritic meshes using paving.

Algorithm follows largely from Blacker and Stephenson's original algorithm.

Detail some of the changes in heuristics to handle dendrites and non-quad elements.

Incorporating Dendrites

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1. Element Size Transition (Tucks and Wedges)
2. Mesh Smoothing
3. Dendritic Seaming (Preliminary)
4. Primitive Insertion (Future Work)
5. Mesh Cleanup (Future Work)

Tucks and Wedges

During the paving algorithm, concave and convex regions of the boundary will cause successive rows of elements to be expand or contract past a desired limit.

This expansion/contraction is counteracted through the introduction of a new element (wedge) or deletion of an existing element (tuck).

Wedges and Tucks are a natural fit for dendrites:

- **Wedge** Insert a dendrite on the paving front.
- **Tuck** Merge two elements on the paving front, creating a dendrite away from the front.

Dendritic Wedge Insertion

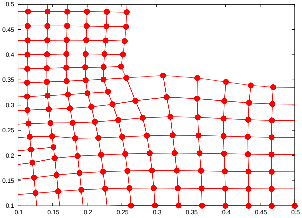


Figure 3: Expansion Detection

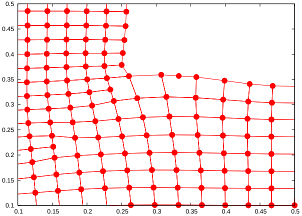


Figure 4: Placement of Dendrite

Near Finalized Region

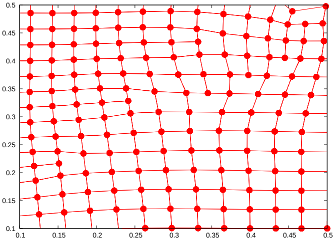


Figure 5: Paving on Dendrites

Dendritic Wedges

Wedges are inserted via insertion of dendritic points on elements whose edge length on the front has expanded compared to the edge length away from the front.

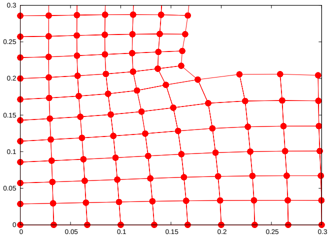
- Determine a consecutive number of elements on front which qualify for dendrite insertion
- Find the angle of rotation between starting edge and ending edge.
- Place a dendrite at approximate midpoint of every $\pi/2$ rotations.

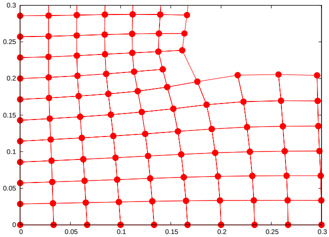
Dendritic Tucks

Tucks are achieved by merging two regular quadrilaterals to form a transition element with dendrite located away from the paving front.

However, tucks are restricted by the mesh topology with which elements can merge together.

Dendritic Tucks





Comparison with Blacker/Stephenson

Dendritic Tucks and Wedges have several advantages in comparison with the original paving algorithm.

- Fewer irregular nodes (usually dendrites)
- Boundary contour better preserved in interior.
- Smooth transitions between elements of differing sizes.

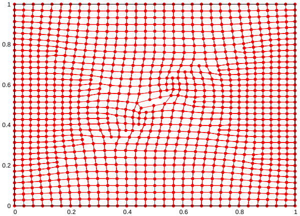


Figure 6: Near finalized mesh.

Mesh Smoothing

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Mesh Smoothing is the most often used subroutine in paving algorithm

- Ensures element quality throughout mesh generation.
- Requires minor modification when encountering dendrites.

Two types of smoothing are used:

1. Interior Smoothing (Modified Laplace Iteration)
2. Boundary Smoothing (Modified Isoparametric)

Interior Smoothing

Algorithm 1 Laplace Iteration with Dendrites

for All non-dendrite nodes N_i **do**
    Initialize vector $\boldsymbol{\Delta}_i = \mathbf{0}$
    for $\forall$ Neighbors N_j **do**
        Compute displacement vector $\boldsymbol{V}_j = \overrightarrow{N_i N_j}$
        if N_j is enslaved by N_i **then**
            $\boldsymbol{V}_j = k\boldsymbol{V}_j$ where $1.5 \leq k \leq 2.0$
        end if
        $\boldsymbol{\Delta}_i \leftarrow \boldsymbol{\Delta}_i + \boldsymbol{V}_j$
    end for
    $\boldsymbol{\Delta}_i \leftarrow \frac{1}{n}\boldsymbol{\Delta}_i$
end for
Displace all nodes N_i by $\boldsymbol{\Delta}_i$

Boundary Smoothing

The paving boundary is frequently smoothed using the modified isoparametric boundary smoother as described in [BS91].

The algorithm used in dendritic paving is nearly the same save for a modification to the length-modifier as described in [BS91].

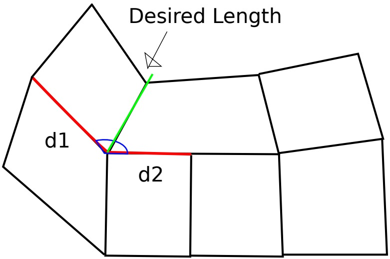


Figure 7: Old Length Modification

Boundary Smoothing: Modification

Old length modification takes into account smaller elements and can lead to a “kink” near a tuck.

Taking into account the entire length of the transition element leads to a desired length which will be more square.

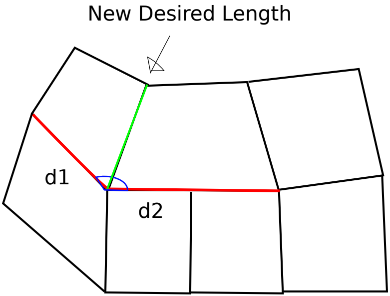


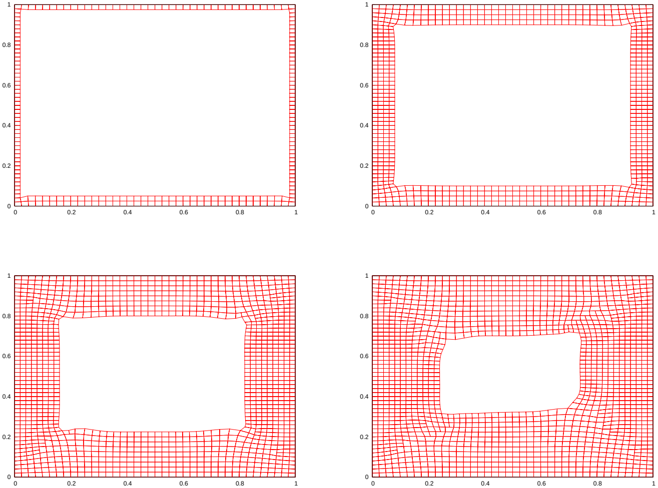
Figure 8: New length Modification

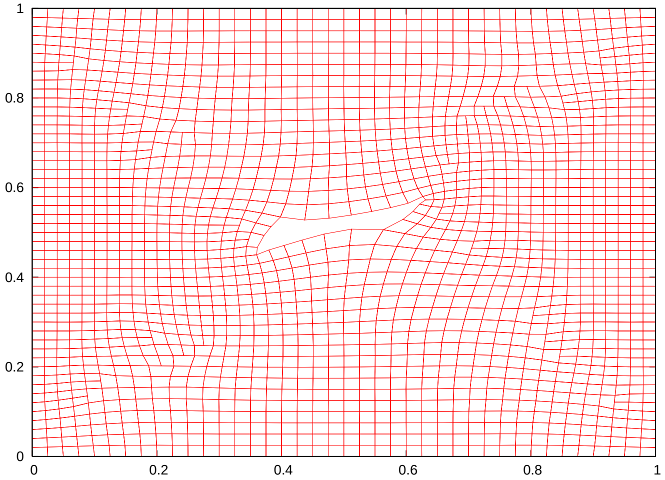
Anisotropic Square

Our first case:

- Unit square: $[0, 1] \times [0, 1]$
- Discretized with nx intervals along top and bottom and ny intervals along left and right sides.
- $nx = 40$, $ny = 50$ Example is shown.

Anisotropic Square Paving: $nx = 40,\ ny = 50$




1
0.8
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
1

Semi-Circle

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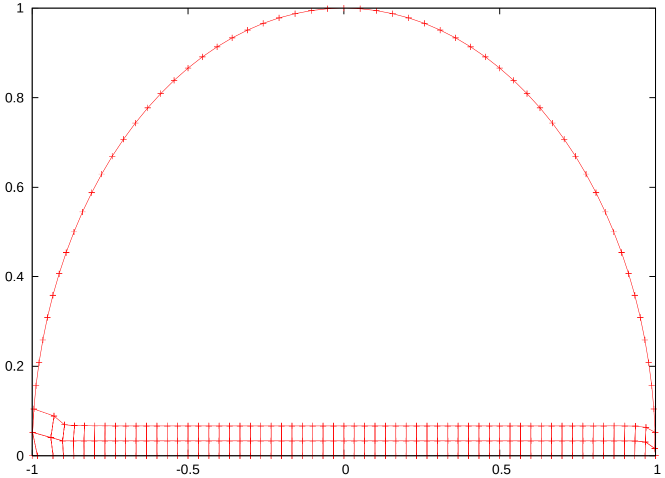
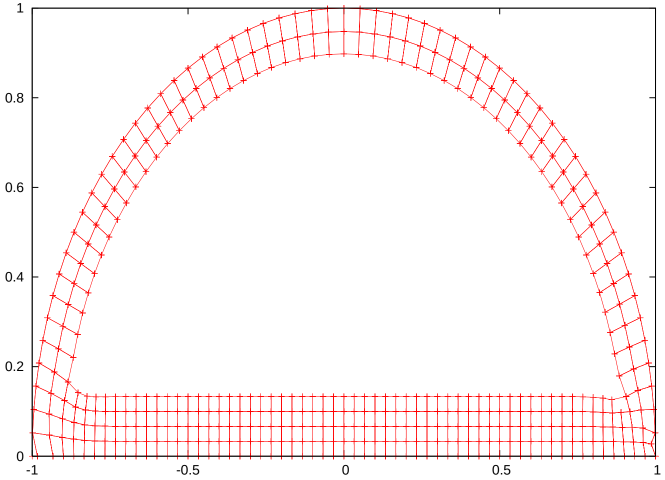


Semi-Circle



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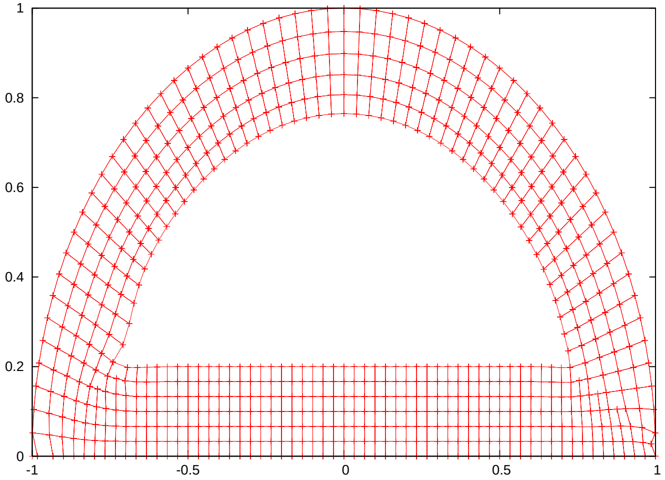

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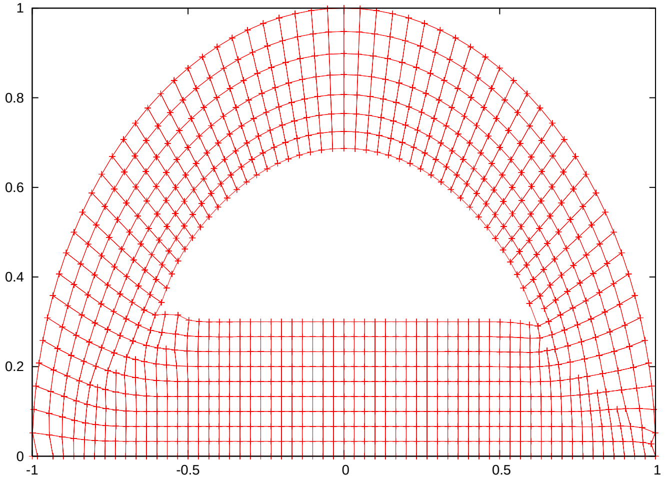
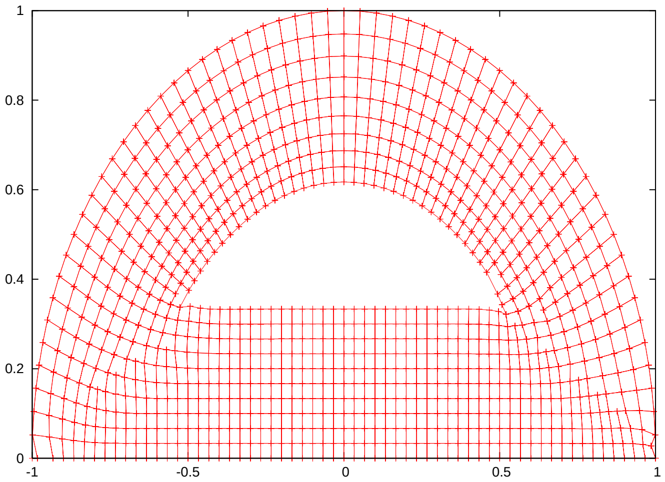


Semi-Circle



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Dendritic Seaming

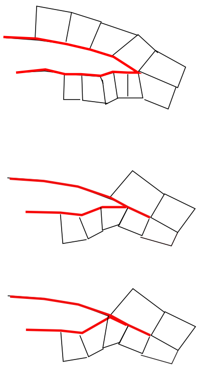
Seaming is an important operations which merges edges on the paving front with very small (even negative) interior angles.

A few ideas on Dendritic Seaming have been proposed:

- Seaming based on relative lengths
- Connectivity improvement

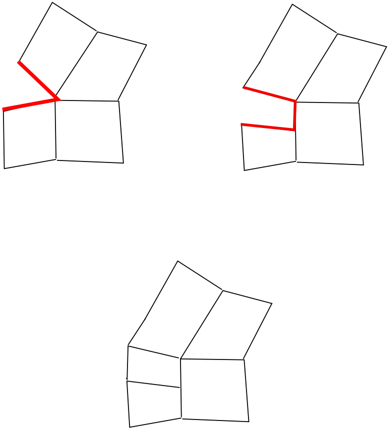
Seaming I

Obvious seaming operation, if angle sufficiently small and adjacent edges on front are close to 2:1, then seam edge and turn one into dendrite.



Seaming II

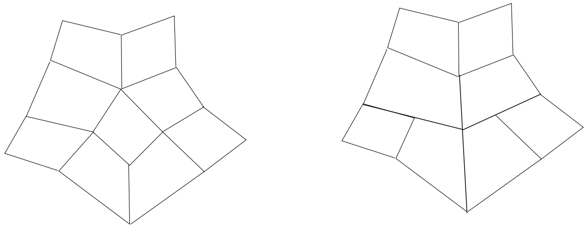
Use dendrites to improve connectivity of point which otherwise would not be seamed.



Mesh Cleanup

Mesh cleanup is aimed at improving regularity of points.

Dendrites can give extra flexibility in improving point connectivity in some cases.



Primitives with Dendrites

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Near the end of the paving process, predetermined “primitives” close the mesh with good quality elements.

Unfortunately require even number of points in current loop to produce all quads.

We often do not satisfy this requirement at the end, so could require development of dendritic primitives which we can use to finalize mesh generation.

References

T. D. Blacker and M. B. Stephenson, *Paving: A new approach to automated quadrilateral mesh generation*, Int. J. Num. Meth. Engng **32** (1991), 811–847.