

## **Final report for the period of 03/01/2001 to 06/30/2014**

During the period of the grant 57 articles were published listed after the main text. The following review is in inverse chronological order.

My recent work under this grant was mainly focused on establishing the presence of large N phase transitions in SU(N) gauge theory. Some of these transitions were expected and of the bulk type and others were surprising and occurring only in some nonlocal gauge invariant observables as their typical size varied. The articles relevant to this topic are: 1, 4-11, 13-29, 35, 47 and 48. Earlier, my research was mainly focused on chirality and lattice fermions. Various applications of the overlap Dirac operator which I invented were studied. The articles relevant to this topic are: 30-34, 36-43, 45, 46, 49 and 51-57. Two new research lines were started more recently. The first is on lattice radial quantization in 2 and 3 and the second is on entanglement entropy in 12. Two more general reviews were also published, in 44 and 50.

My most impactful work was the one on lattice fermions. The work on large N transitions also had quite an impact and had an important offshoot which are of current interest, which goes under the name of ``smearing''. The central theme of my most recent line of research is on conformal theory in the lattice context. The aims range from developing a new lattice technique for dealing with conformal field theories to a search for a relaxation of the principle of ``fine tuning'' in the context of Particle Physics. Further developments in this direction occurred after the end date of this grant.

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