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Multiscale Mathematics For Plasma Kinetics Spanning Multiple Collisionality Regimes

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The goal of this project is the development of accelerated simulation methods for plasma kinetics and the application of these methods to problems of interest for plasma fusion. Specifically, this project is developing methods for simulation of Coulomb collisions between charged particles. Collisional effects can be significant in the edge plasma of a fusion device, but correct simulation of the collisional effects is a computational bottleneck for current simulation methods. We developed two approaches to acceleration of simulation for Coulomb collisions.

The first approach uses Monte Carlo simulation for pairwise collisions that represent the effects of Coulomb interactions. Our approach to acceleration of Coulomb collisions is based on a hybrid method that combines Monte Carlo particle dynamics with a coarse-grained hydrodynamic description, following a similar method in rarefied gas dynamics by Pareschi and Caflisch. The velocity distribution $f = f(v)$ is decomposed as a combination $f = M + k$ of a "thernal" component $M = M(v)$ and a "kinetic" component $k = k(v)$. A hydrodynamic description applies to the evolution of M, a portion of the particle distribution function that has been thermalized, so that it is a Maxwellian equilibrium. The kinetic component k is represented by a collection of particles and is evolved in terms of particle dynamics.

The critical part of this method is the interaction between the kinetic and thermal components of the distribution. We developed a general strategy for this interaction, involving two steps: first, collisions between the thermal and kinetic components are simulatd by sampling particles from the Maxwellian thermal component and colliding them with particles from

k. Second, a thermalization/dethermalization (T/D) method is applied to determine that some paricles in the kinetic component have become thermalized and are moved to the thermal component and that some particles from the thermal component have been dethermalized and should be moved to the kinetic component. We developed three specific T/D criteria: The first uses thermalization and dethermalization probabilities, chosen to satisfy a detailed balance criterion; the second approach uses flux rates to determine the amount of thermalization and dethermalization; and the third uses a relative entropy method to determine whether a particle should be thermalized or dethermalized. The latter two methods perform about the same on many problems and are much better than the first method. We are currently applying these methods to a number of important plasma problems.

The second approach uses an interpretation of the nonlinear Landau-Fokker-Planck equation as collisions between field particles and test particles. In the simplest case, the field particles are in a Maxwellian distribution. The velocity of the test particles evolves according to a stochastic differential equation (SDE) of the form

$$dv = c(v)dt + a(v)dW \tag{1}$$

in which $W = W(t)$ is Brownian motion. The simplest quadrature method for stochastic differential equations (SDEs) is the Euler method, and the Milstein method is a higher order quadrature method. An accelerated numerical method for SDEs using using either Euler or Milstein and a multilevel scheme was first introduced by Mike Giles (Oxford) in the context of financial mathematics. Giles' Multi-Level Monte Carlo (MLMC) method uses successive refinement as a control variate. When used with Milstein differencing or related methods that avoid some of the problems of Milstein, it provides even more acceleration than its use with standard Euler differencing.

For numerical solution of the SDE, accuracy of the method is measured in the "weak" sense, in terms of the error in averages involving the solution. Denote

$$\begin{aligned} \varepsilon &= \text{error in solution } v \\ M &= \text{computational time to solve for } v. \end{aligned}$$

For the standard Euler or Milstein methods (without acceleration), the computational effort is

$$M = O(\varepsilon^{-3}). \tag{2}$$

Using the Euler method in the MLMC, Giles shows that

$$M = O(\varepsilon^{-2}(\log \varepsilon)^2). \tag{3}$$

Using the Milstein method (or related methods) in the MLMC, Giles shows that

$$M = O(\varepsilon^{-2}). \tag{4}$$

We have applied MLMC to Coulomb collisions, and achieved equivalent acceleration, in spite of difficult features in the SDEs corresponding to Coulomb collisions.

In a separate study, we also developed a new approach to the solution for the following equations that describe planar flow of an electron beam in a diode (space charge limiting flows):

$$\partial_t \rho + \partial_x(\rho v) = 0 \tag{5}$$

$$\partial_t v + v\partial_x v = -\partial_x \phi \tag{6}$$

$$\partial_x^2 \phi = -\rho \tag{7}$$

in which ρ is electron density, v is electron velocity and ϕ is the electrostatic potential. Characteristic (i.e., particle path) variables for this problem are s and τ in which

$$\begin{aligned} \partial_s x &= v \\ t &= s + \tau. \end{aligned} \tag{8}$$

In these coordinates, we have found that we can directly solve for x, v and the (scaled) electric field $\psi = -\partial_x \phi$.

This project is a collaboration between applied mathematicians (Caflisch and co-workers) at UCLA and plasma physicists (Andris Dimits and Bruce Cohen) at Lawrence Livermore National Laboratory. This includes work at UCLA performed by postdocs Richard Wang, Mark Rosin and Bokai Yann and a graduate student Lee Ricketson. Wang is now working in the financial industry, Rosin is an Assistant Professor at Pratt Institute, Yann is currently on the job market, and Ricketson is a postoc at the Courant Institute (NYU). This report is mainly concerned with the research performed at UCLA, but much of that work was in collaboration with LLNL.

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