

**Final Report**  
**on the project “Theory and Modeling of High-Power Gyrotrons”**  
**(DEFG0295ER54325)**  
**to the project manager,**  
**Dr. Barry Sullivan**

**Principal Investigator**  
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**Abstract:** This report summarized results of the work performed at the Institute for Research in Electronics and Applied Physics of the University of Maryland (College Park, MD) in the framework of the DOE Grant “Theory and Modeling of High-Power Gyrotrons”. The report covers the work performed in 2011-2014. The research work was performed in three directions:

- possibilities of stable gyrotron operation in very high-order modes offering the output power exceeding 1 MW level in long-pulse/continuous-wave regimes,
- effect of small imperfections in gyrotron fabrication and alignment on the gyrotron efficiency and operation,
- some issues in physics of beam-wave interaction in gyrotrons.

### **1. Operation in high-order modes.**

This is a long standing problem in the development of gyrotrons for plasma fusion because it is impossible to provide thermal management of MW-class gyrotrons in the case of operation in low-order modes. Our work in this direction can be subdivided into two categories. First, by using the self-consistent, non-stationary code MAGY developed at the University of Maryland (in collaboration with the Vacuum Electronics Branch of NRL) we studied the excitation of high-order quintets of almost equidistant modes [A1]. The study was started from the modes used in a 1 MW gyrotron developed for ITER in Japan by the group led by K. Sakamoto. That gyrotron was driven by a to kV, 50 A electron beam and operated in the  $TE_{31,12}$  -mode. Correspondingly, the quintet was formed by the modes with the same radial index  $p=12$  and azimuthal indices 29, 30, 31, 32, and 33. Then, the study was expanded in two directions: the increase in the beam current and the increase in the radial index of modes. Simulations were performed for the beam current of 60 and 70A. Also the radial index of competing modes was increased from 12 to 14, 16, 18 and 20. It was found that, when the current is equal to 50 A, even the operation in the  $TE_{31,18}$ -mode can be stable. When the current is increased to 60 A, the oscillations in the desired mode are accompanied by excitation of sidebands and, what is quite interesting, the intensity of distant sidebands exceed that of close sidebands. This

statement is illustrated by Fig. 1 showing the temporal evolution of sideband amplitudes under assumption that, due to a proper start-up scenario, the desired mode is excited first.

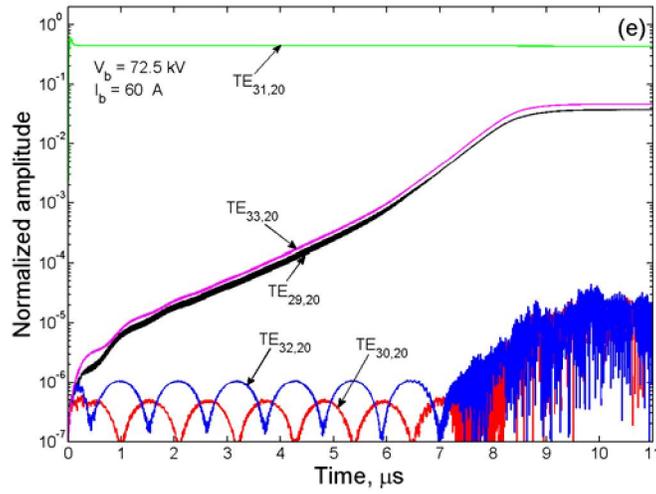


Fig. 1. Gyrotron operation in the  $TE_{31,20}$ -mode is accompanied by excitation of sidebands. The amplitude of distant sidebands (with radial indices 29 and 33) is much higher than the amplitudes of close sidebands with the radial indices 30 and 32.

When the beam current is increased to 70 A, the level of close sidebands significantly increases and they exhibit some counter-phase oscillations indicating a parametric interaction of equidistant modes, as shown in Fig. 2.

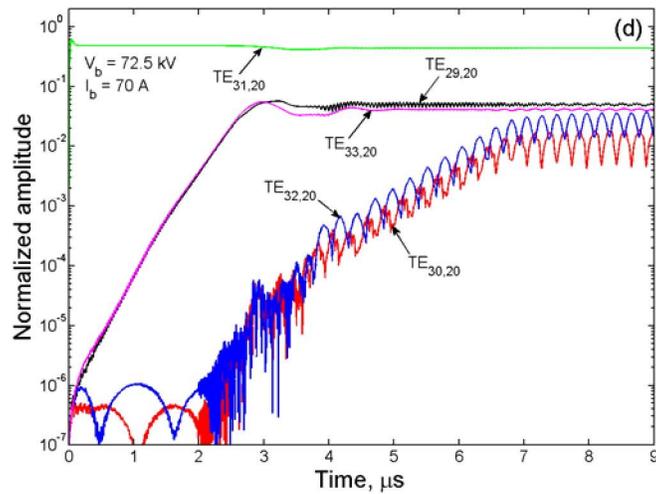


Fig. 2. The same for the beam current 70A.

Note that the beam power in the last case exceeds 5 MW. So for a typical gyrotron efficiency above 30% the power of outgoing radiation should be larger than 1.5 MW.

The second direction of our theoretical studies was the analysis of the stability of gyrotron operation in high-order modes with very large azimuthal indices. These studies

(performed in collaboration with Dr. O. Dumbrajs from the University of Latvia) were based on the wave envelope approach originally formulated in references [B1 and B2]. In the framework of this approach, the electric field of the wave rotating in a cylindrical resonator is represented as  $E \propto f(z, t, \varphi) \exp\{i(\omega_0 t - m_0 \varphi)\}$  (where  $\omega_0$  and  $m_0$  are the carrier frequency and azimuthal index of the wave, respectively). Taking into account the condition of azimuthal periodicity in a cylindrical waveguide, such field can be represented as a series of azimuthal harmonics. Hence, each term in this series can be treated as an eigenmodes with a specific azimuthal index.

It was found [A2] that, as the azimuthal index of the operating mode  $m_0$  increases, at relatively low beam current the operation in the desired mode becomes unstable, while at higher currents it remains stable. This statement is illustrated by Fig. 3.

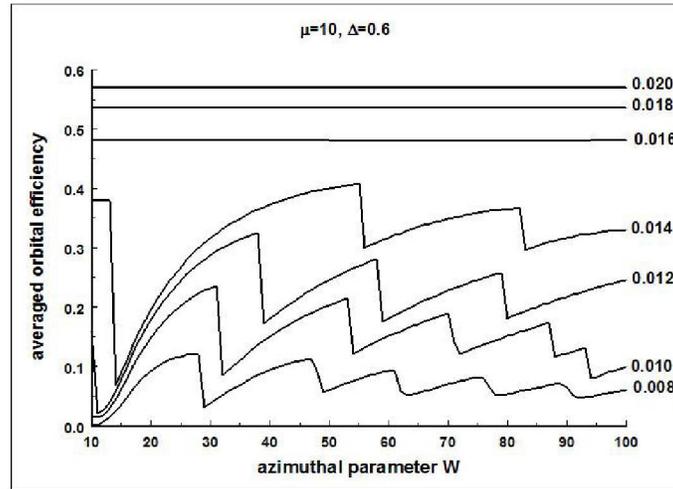


Fig. 3. Dependence of the gyrotron orbital efficiency on the azimuthal parameter  $W = \pi\beta_{\perp}^2 m_0$  (here  $\beta_{\perp}$  is the electron orbital velocity normalized to the speed of light). Different curves correspond to different values of the beam current parameter shown on the right.

The sawtooth-like dependence of the efficiency on the azimuthal parameter  $W = \pi\beta_{\perp}^2 m_0$  shown in Fig. 3 can be interpreted as it was explained in Ref. [A3] by relatively simple arguments. As known [B3], the high efficiency operation corresponds to a relatively large cyclotron resonance mismatch  $\omega - s\Omega$  between the wave frequency and corresponding resonant harmonic of the electron cyclotron frequency. In the process of electron deceleration by the wave, the relativistic cyclotron frequency  $\Omega \propto B_0 / \gamma$  (here  $\gamma$

is the relativistic Lorentz factor, i.e. the ratio of the electron energy to the rest energy) increases, so the mismatch decreases and electrons stay in resonance with the wave. At the same time, in a gyrotron with a dense spectrum of eigenmodes there can be some parasitic modes with frequencies close to the center of the zone of the coherent cyclotron radiation. In a non-excited gyrotron, such modes have the lowest start current and, when the beam current is small enough and, correspondingly, the amplitude of oscillations of the operating mode is small, such modes can still be self-excited. However, at high currents, the operating mode suppresses these parasitic oscillations. So, with the densification of the mode spectrum that corresponds to increasing the azimuthal parameter  $W = \pi\beta_{\perp}^2 m_0$  in Fig. 3, first, the closest parasitic modes enter this dangerous region. Then, more distant parasitic modes become the most dangerous and so on. Now, we can compare results shown in Fig. 3 with those in Fig. 1 and conclude that the case shown in Fig. 1 corresponds to the second “tooth” of the curves shown in Fig. 3 for small values of the current parameter. For considering this case, indeed, it is sufficient to take into account 5 competing modes (i.e. to consider a quintet, as we did), while in the case of operation in modes with larger azimuthal indices corresponding to the next “tooth” of the dependence shown in Fig. 3, at least, 7 modes should be taken into consideration.

Some results of these theoretical studies were confirmed in the experiments carried out at MIT which were supported by MAGY simulations performed by our group in collaboration with MIT researchers [A4].

## 2. Effect of small imperfections

With the frequency increase (the wavelength shortening) the gyrotron operation becomes more and more sensitive to such small imperfections as electron beam misalignment, the presence of electron spread in velocities and guiding center radii, the presence of ions etc. We considered some of these issues in a series of papers. First, it was shown [A5] how to analyze the trade-off between two effects deteriorating the gyrotron efficiency, namely: the spread in electron velocities and the spread in the guiding center radii. This trade-off is caused by the fact that it is possible to decrease the velocity spread by forming a more laminar (quasi-laminar) electron beam. This laminarity can be improved by increasing the angle between the emitter surface and the

magnetic force line. However, this increase also enlarges the electron beam thickness that makes the effect of the radial non-uniformity of the wave field on the beam-wave interaction stronger. So this trade-off was analyzed and corresponding recommendations were formulated. In particular, there was formulated the limitation on the angle between the emitter and the magnetic force line. This limitation depends on a certain combination of such parameters as the magnetic compression of the beam, the wavelength, the total beam current and the cathode loading, and the azimuthal index of the operating mode.

Taking into account a certain interest in developing sub-THz gyrotrons for active plasma diagnostics (collective Thomson scattering), which do not require operation at the MW level, we studied some peculiarities of gyrotron design in the case of operation at low voltages. This study was performed in collaboration with researchers from the Russian Institute of Applied Physics (N. Novgorod). Results presented in [A6] allow gyrotron designers to optimize the efficiency of gyrotrons operating at the fundamental and higher cyclotron harmonics. Examples of the designs of first, second and third harmonic gyrotrons are presented in Ref. [A6].

In Ref. [A7] the effect of electron beam misalignment on the gyrotron efficiency was analyzed. Namely, there were considered two kinds of misalignment: the tilt and the parallel shift of the beam axis with respect to the axis of the microwave circuit. It was shown that these effects lead not only to the efficiency deterioration, but may also cause the excitation of the counter-rotating waves. In collaboration with the researchers from the Center of Far-Infrared Technology (Fukui, Japan), this effect of this misalignment on the operation of the Fukui gyrotron FU IV A was carried out. The results were described in Ref. [A8].

Lastly, we can add to this group of studies of various imperfections the study of the role of ions on gyrotron operation because very often it is assumed that the beam-wave interaction takes place in an idea vacuum, while in real gyrotrons there is always a technical vacuum, i.e. some ions and molecules of the residual gas are present. The most important effect of ions is the compensation of the voltage depression in the electron beam caused by the beam space charge. Results of our study performed in collaboration with Russian researchers from the Institute of Applied Physics are described in Ref. [A9]. It is shown that the ion compensation of the beam space charge may cause significant

changes in the efficiency of gyrotron operation and, in some cases of operation in the hard-excitation regime, even result in the break of oscillations.

### 3. Physics of beam-wave interaction

One study that can be related to this group was motivated by the interest in the process of field formation in gyrotron cavities because the axial profile of the resonator field plays an important role in the beam-wave interaction and determines the gyrotron efficiency. Typically, the axial field structure is determined in the framework of the self-consistent theory, i.e. with the account for the effect of the beam-wave interaction. At the same time, it is assumed that in the cold-cavity approximation, i.e. when the effect of an electron beam is ignored, the problem of defining the axial structure of the resonator field is trivial because this function  $f(z)$  describing this structure obeys the non-uniform string equation

$$\frac{d^2 f_s}{dz^2} + k_{s,z}^2(\omega_s, z) f_s = 0 \quad (1)$$

with corresponding boundary conditions (Zommerfeld conditions) at both ends: the exponential decay of the field on the left (cathode) end and the condition for outgoing radiation on the right. In (1),  $k_{s,z}(z) = \sqrt{(\omega/c)^2 - k_{s,\perp}^2(z)}$  is the axial wave number. Equation (1) has the same form as the stationary Schrodinger equation describing a particle in the 1D potential well with the potential  $U(z) = k_{s,\perp}^2(z)$ . Therefore, it is typically assumed that the solution of this equation is trivial as discussed in many papers starting from [B4]. This statement is, however, correct only in the cases when the output from the regular part of a non-uniform waveguide forming a resonator contains a cutoff narrowing neck. Present-day, MW-class gyrotrons do not have such necks increasing the diffractive Q. Therefore, the potential well corresponding to this profile looks like it is shown in Fig. 4b; here Fig. 4a shows the profile of such resonator.

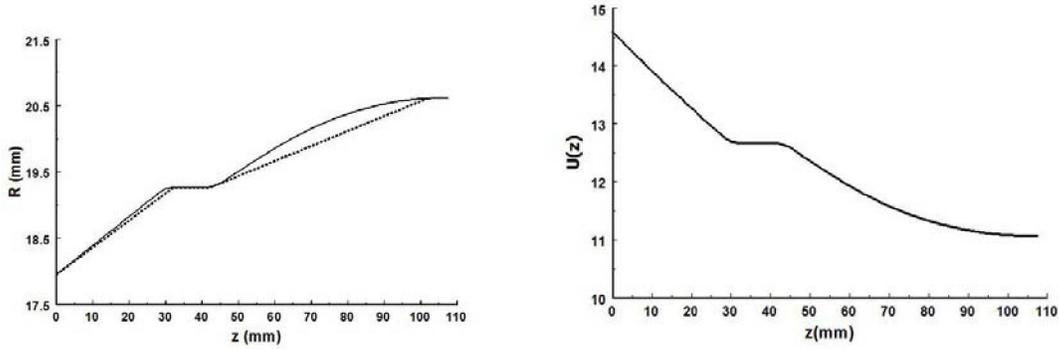


Fig. 4. Left: resonator profile of the 1 MW European gyrotron for ITER with different uptapers; right: potential well corresponding to the profile shown on the left by solid line.

So, the potential well shown in Fig. 4 on the right (this ‘well’ occupies the region between 30 and 40 mm) practically does not have the wall on the right. Therefore in defining corresponding eigen-functions and diffractive quality factors it becomes difficult to use the analogies with the studies done in the framework of the non-relativistic quantum theory. Instead, strictly speaking, one should take into account a certain wave diffusion resulting in the reflection of some wave power from any irregularity of the waveguide surface [B5]. Our study of this problem is described in Ref. [A10].

The role of the difference in axial structures on the mode competition was analyzed in the joint study of our group with FIR Center of Fukui [A11]. The study was done for the gyrotron operating in the  $TE_{22,6}$ -mode which is the mode of choice in 110 GHz US gyrotrons developed for Doublet IID experiments. Both, cold-cavity and self-consistent approaches were used and it was shown that the influence of the electron beam plays a very important role in the mode competition, especially when some modes can be excited in the region of backward-wave excitation as modes with many axial variations.

In conclusion let us mention that after completion of the grant we showed some possibilities of stable gyrotron operation at the second cyclotron harmonic in high-order modes. These studies described in [A12] and [A13] may pave the way to developing high-power second harmonic gyrotrons which is especially important in view of the interest of the international plasma community in developing MW-class gyrotrons for DEMO at frequencies 220 GHz and above.

Results briefly described above were also presented in a series of plenary, invited and contributed talks at numerous international conferences and workshops.

In the framework of this grant, two graduate students, D. G. Kashyn and R. Pu were partially supported who successfully graduated in 2015.

In 2012, Gregory S. Nusinovich was awarded with the Kenneth J. Button Prize for fundamental contributions to the theory and development of gyrotron oscillators and amplifiers with frequencies up to the terahertz range.

In 2015 Gregory S. Nusinovich got the Distinguished Scientist Award from the College of Computer, Mathematical and Natural Sciences of the University of Maryland.

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