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Title: Identifying Heterogeneities in Subsurface Environment using the Level Set Method

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# Identifying Heterogeneities in Subsurface Environment using the Level Set Method

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August 25, 2016

# Outline

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1. Motivation
2. Level Set Method (LSM)
3. Algorithms
4. Examples
5. Future Work

# 1. Motivation

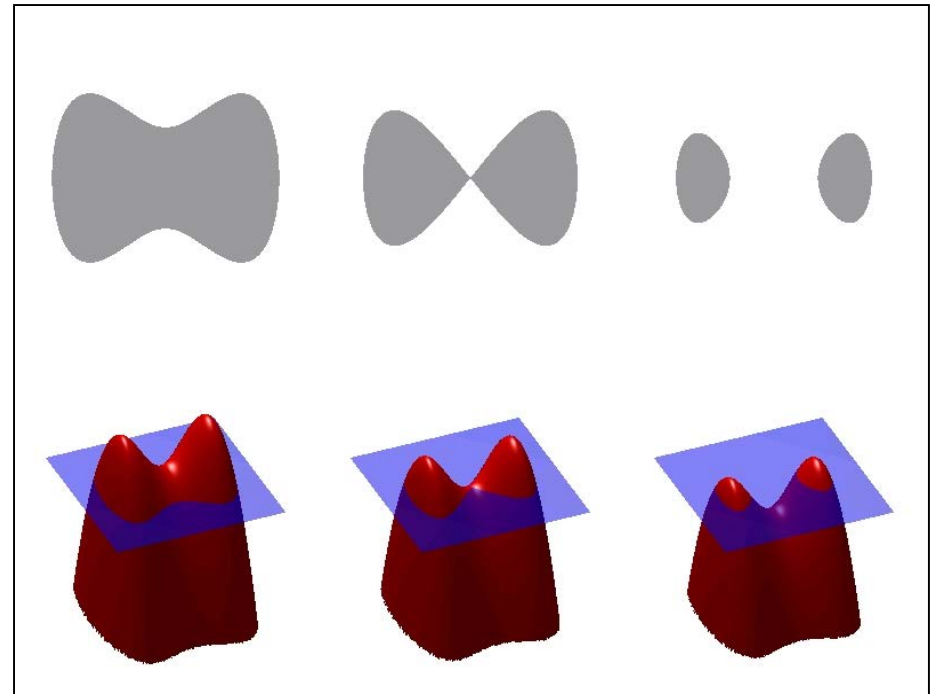
Identifying parameter zonation is **difficult**:

1. **No information** (or little information) regarding the shape, size, and locations of zones.
2. **Existing** inverse methods may not work or too time-consuming.



## 2. Level Set Method (LSM)

- Developed by Osher and Sethian in 1988.
- A great tool for solving problems that involve geometry and geometric evolution.
- Popular in:
  - Image processing,
  - Computer graphics,
  - Computational geometry,
  - Computational fluid dynamics,
  - etc.



## 2. Level Set Method (LSM)

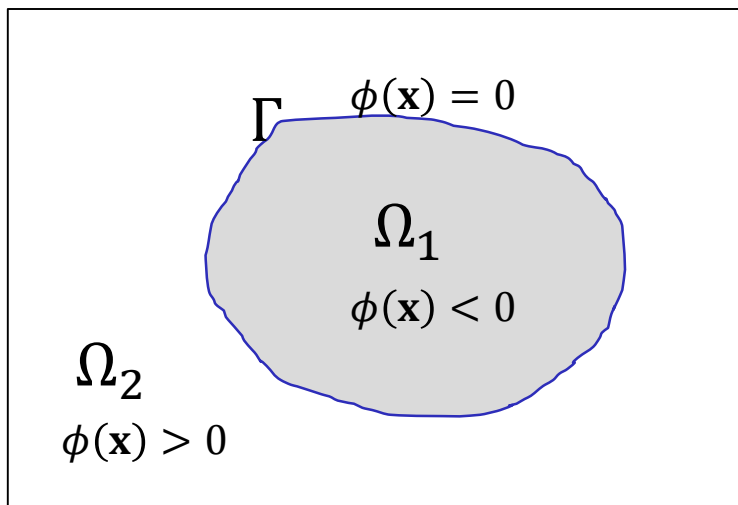
- Geometric boundaries  $\Gamma$  are represented by the **zero** level set of level set function  $\phi$ , which is negative inside  $\Gamma$  and positive outside.

For **two** zone case below

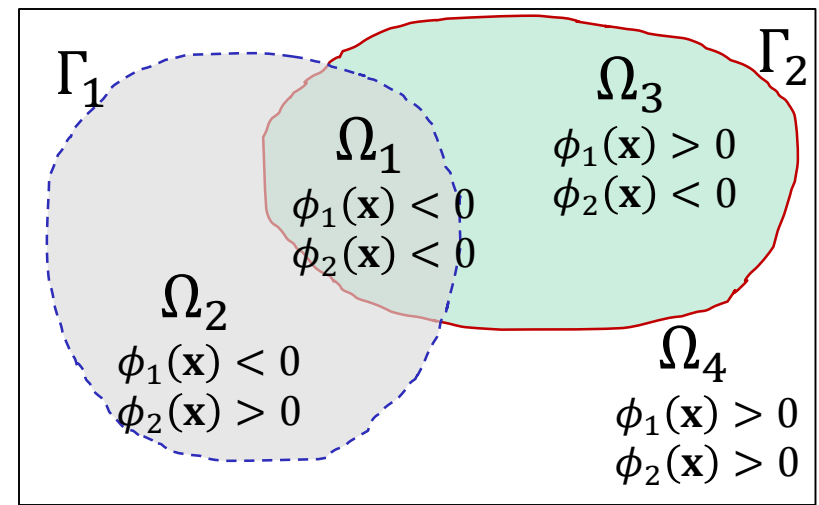
$$\phi(\mathbf{x}) < 0, \text{ for } \mathbf{x} \in \Omega_1,$$

$$\phi(\mathbf{x}) > 0, \text{ for } \mathbf{x} \in \Omega_2,$$

$$\phi(\mathbf{x}) = 0, \text{ for } \mathbf{x} \in \Gamma,$$



**Two** zone case



**Four** zone case

### 3. Algorithms

We try to **minimize** function

$$F(K) = \frac{1}{2} \|\mathbf{h}(K) - \mathbf{h}_m\|$$

where  $\mathbf{h}_m$  is measured head and  $\mathbf{h}(K)$  is simulated head.

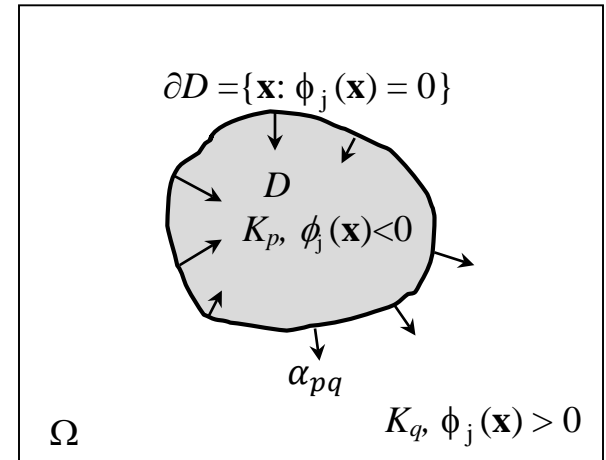
Let the **propagation** of boundary  $\partial D$

$$\alpha_{pq}(\mathbf{x}, t) = \text{sign}(K_q - K_p) \mathbf{J}^T(K) (\mathbf{h}(K) - \mathbf{h}_m)$$

where  $\mathbf{J} = (d\mathbf{h}/dK)$  is the Jacobian matrix.

Then, the variation

$$\delta F(K) = - \sum_{p,q} \int_{\Omega_{pq}} [\alpha_{pq}(\mathbf{x}, t)]^2 d\mathbf{x} < 0$$





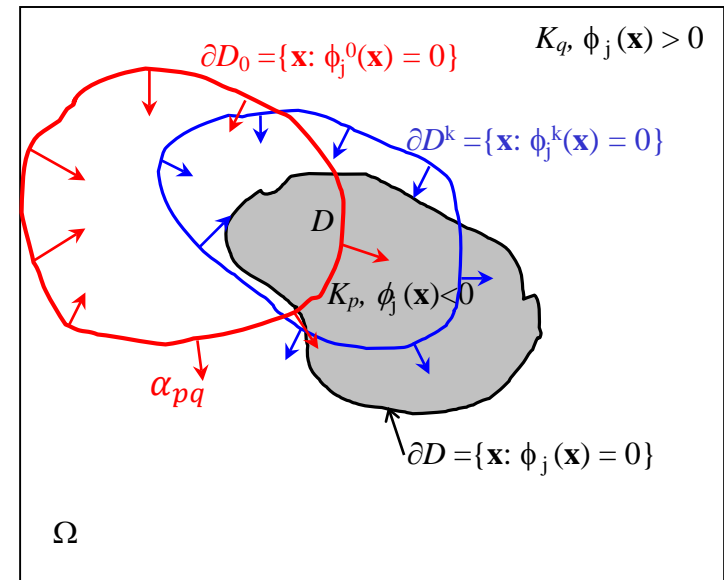
### 3. Algorithms

Level set function can be **updated** by solving

$$\frac{\partial \phi_j(\mathbf{x}, t)}{\partial t} + \alpha_{pq}(\mathbf{x}, t) |\nabla \phi_j(\mathbf{x}, t)| = 0$$

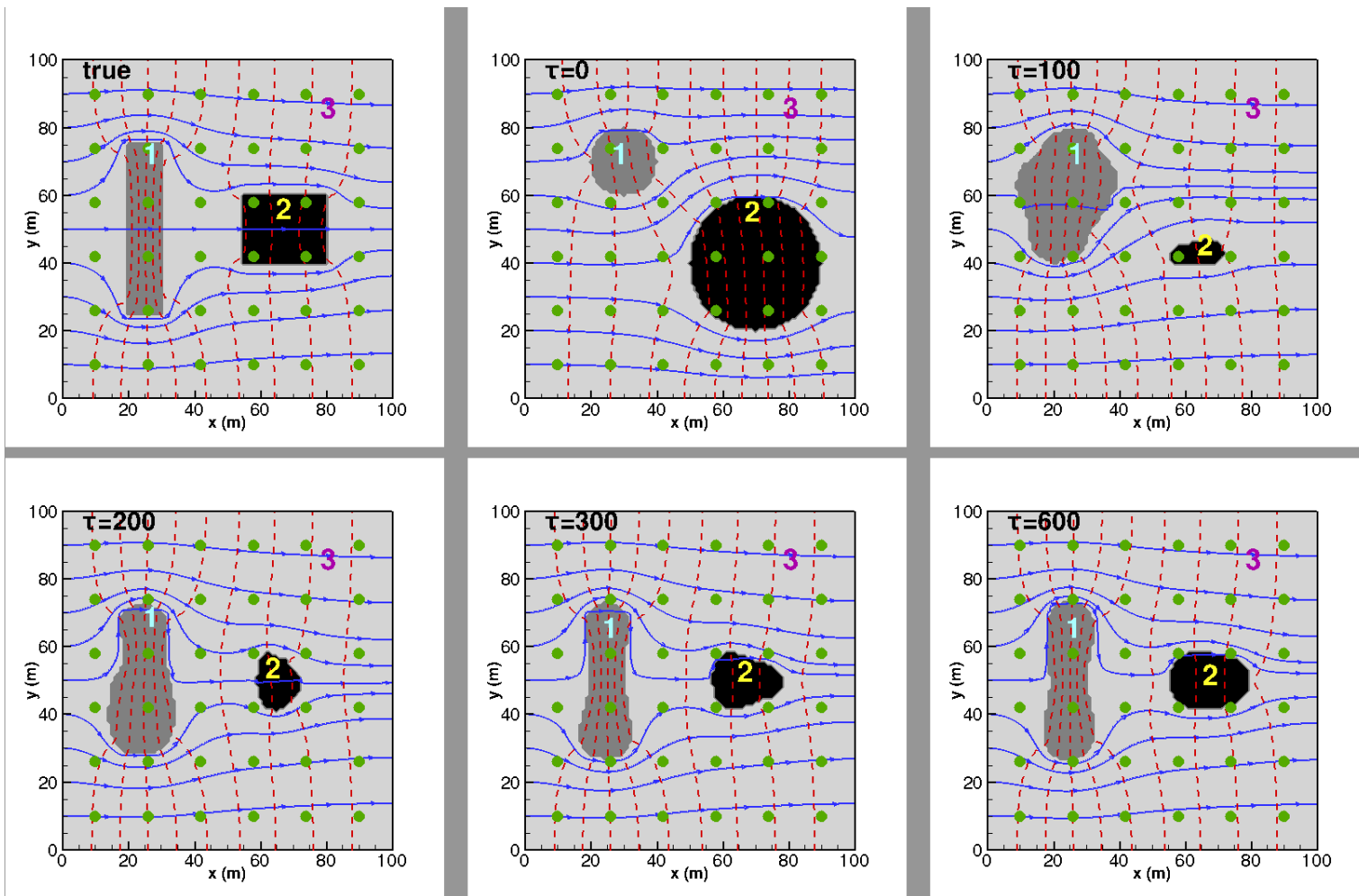
subject to initial condition

$$\phi_j(\mathbf{x}, 0) = \phi_j^0(\mathbf{x})$$

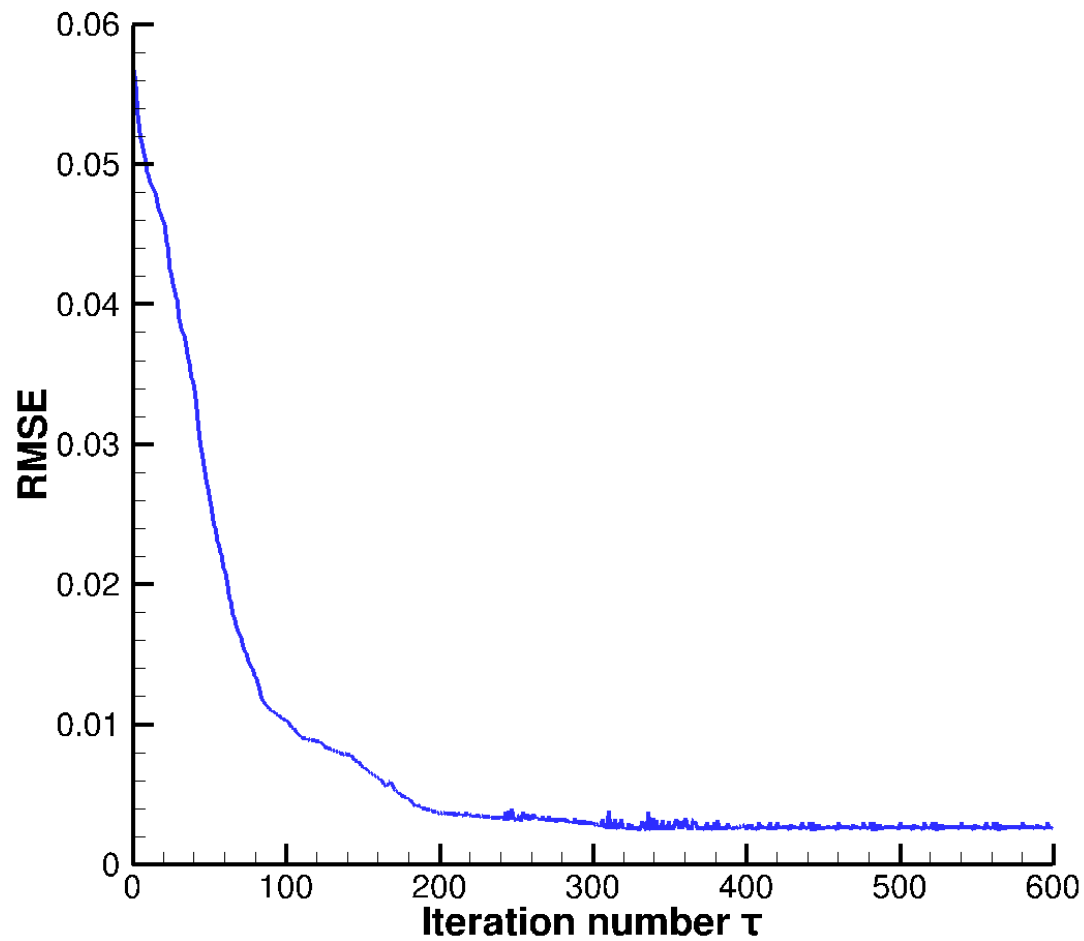


The set  $\{\mathbf{x}: \phi_j(\mathbf{x}, t) = 0\}$  defines the boundary of  $D$  at time  $t$ .

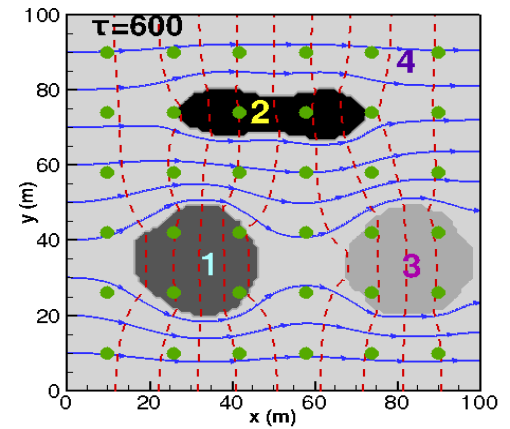
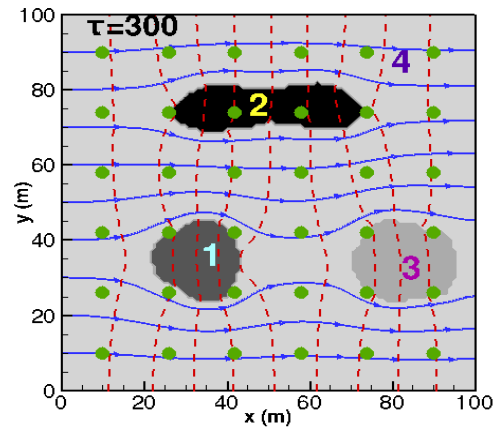
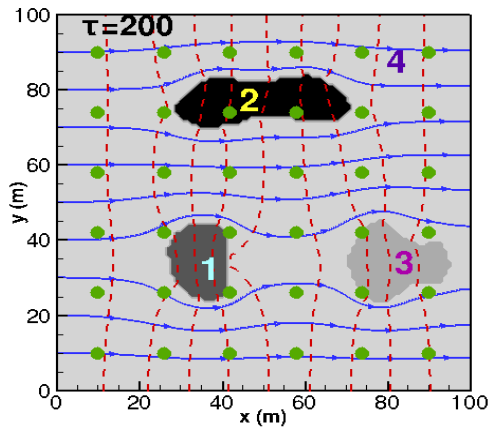
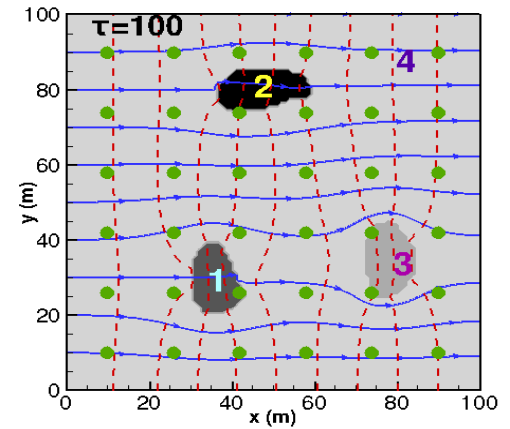
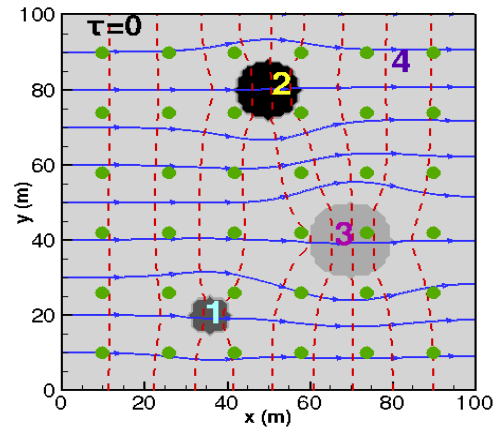
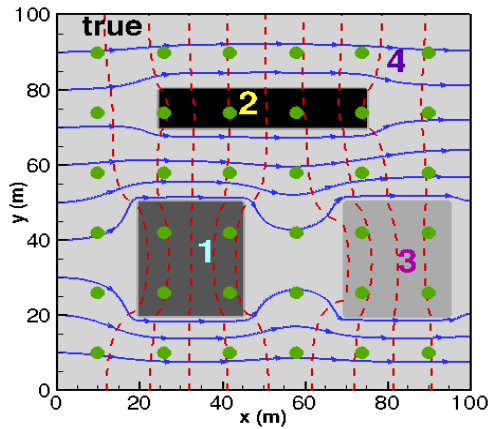
## 4. Example



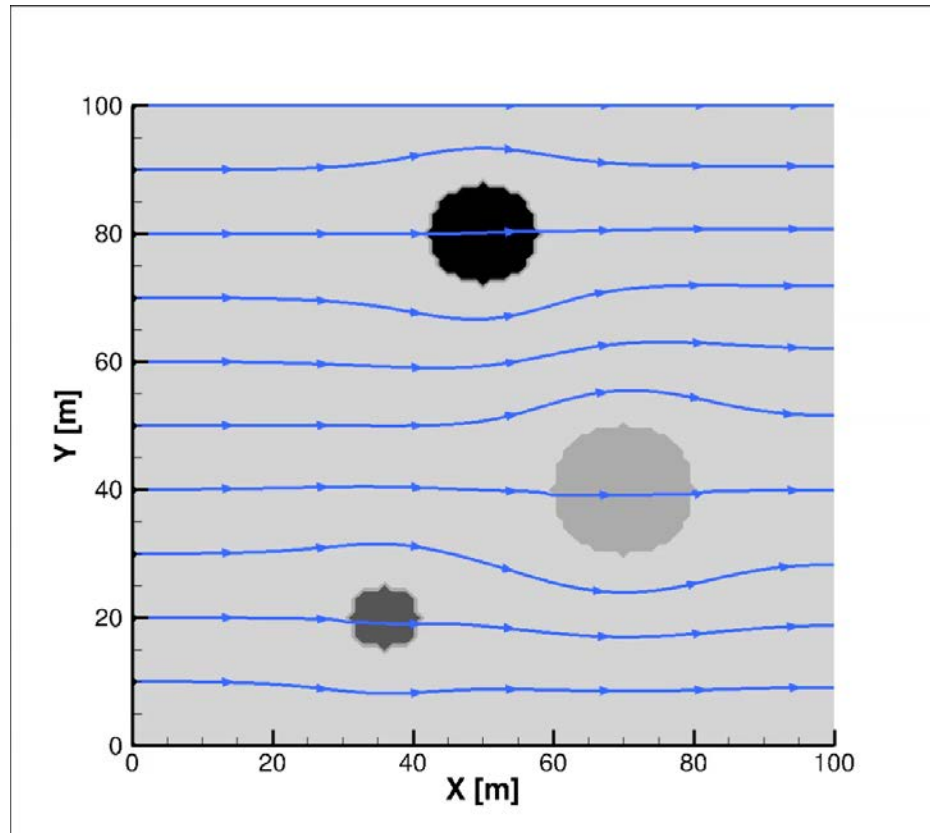
## 4. Example



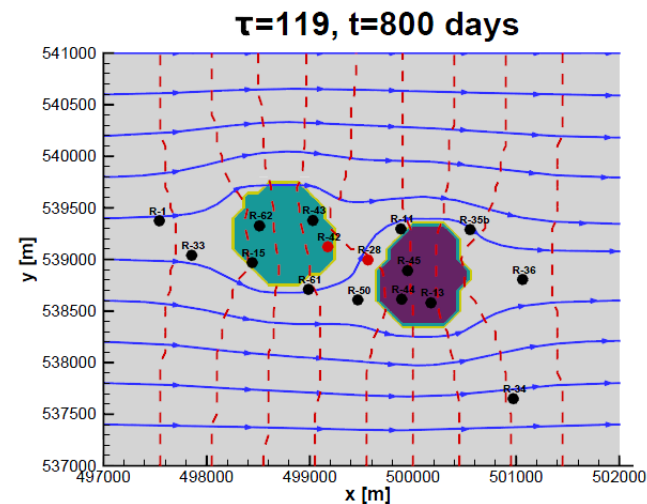
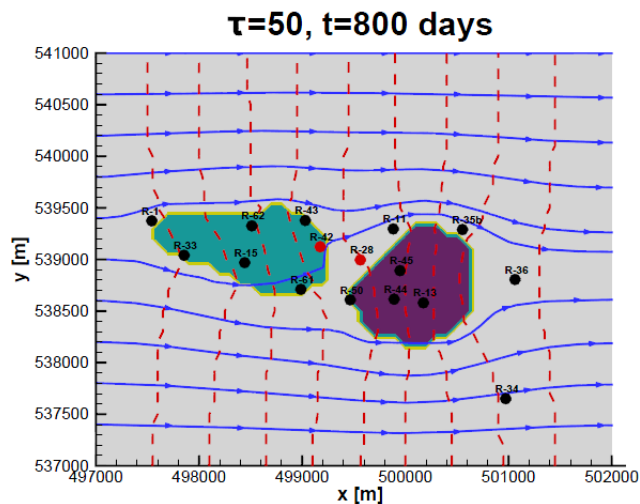
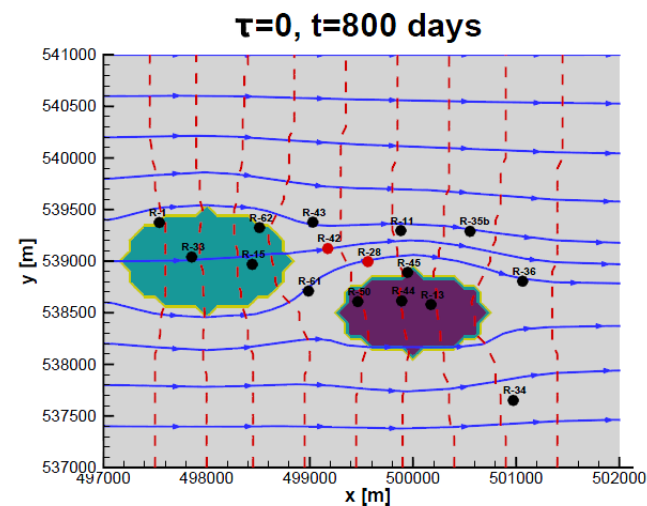
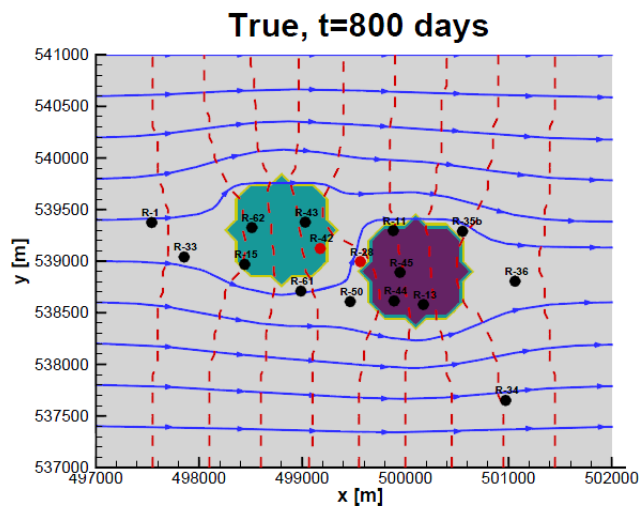
## 4. Example



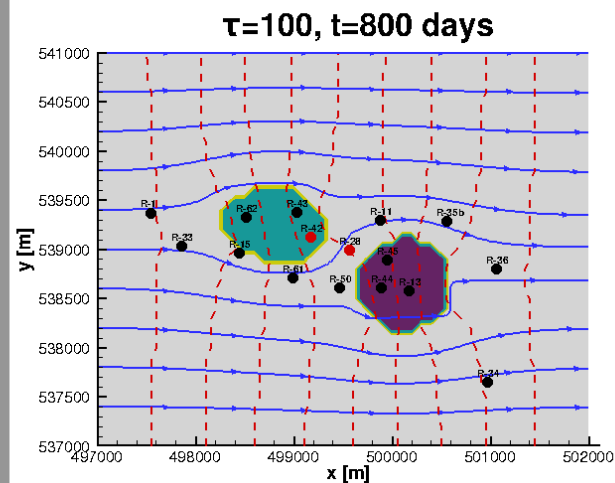
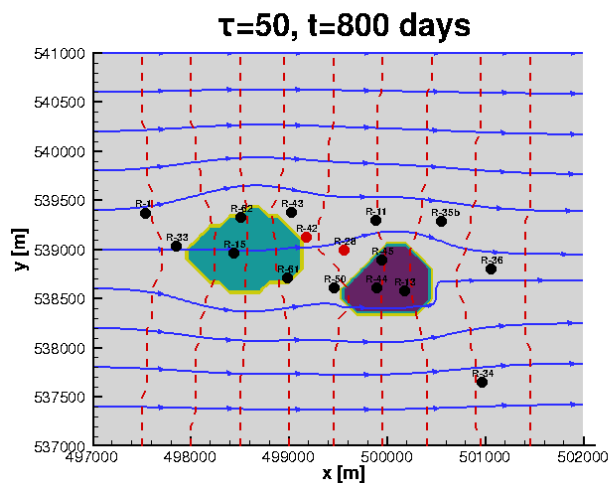
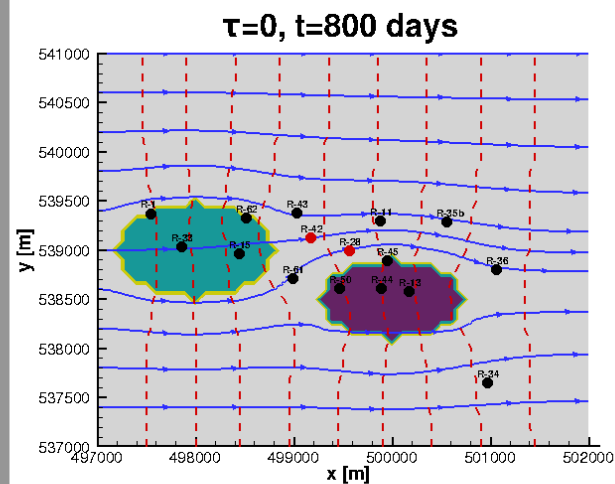
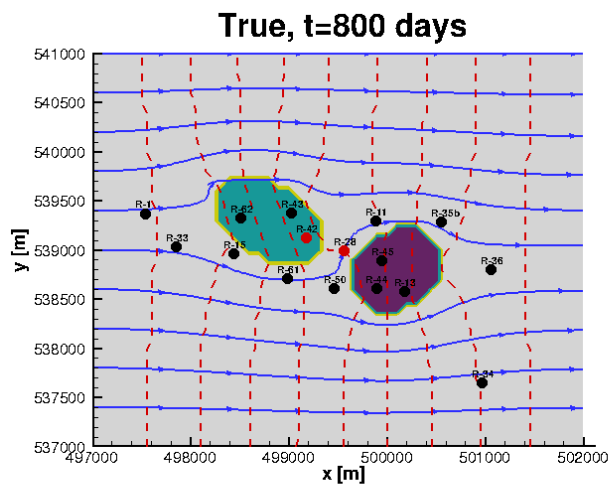
## 4. Example



## 4. Example (Transient)

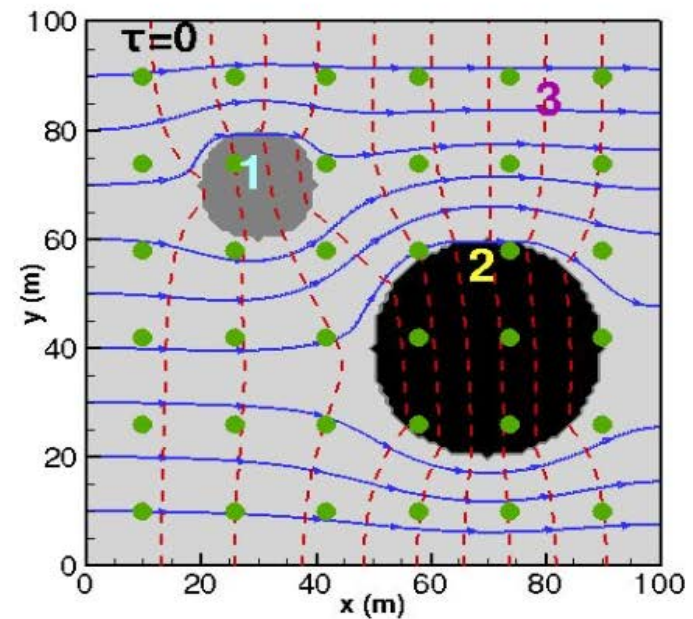
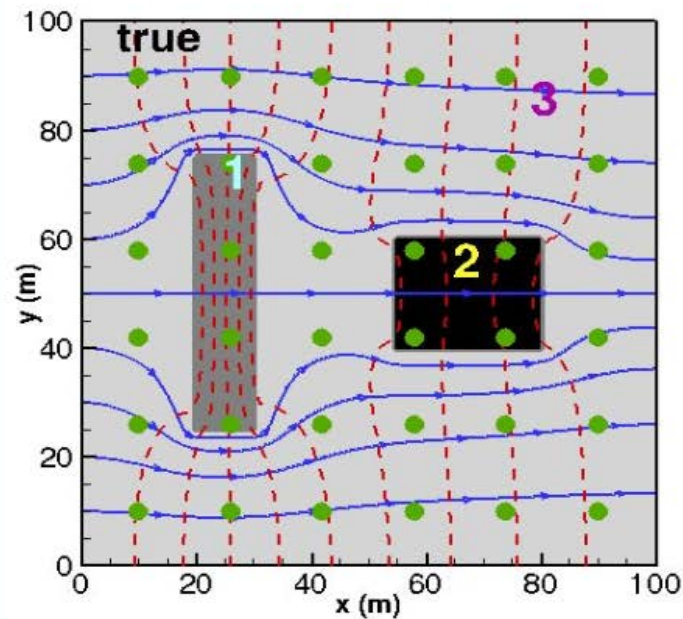


## 4. Example (Transient)



## 5. Future Work

1. Random initial material locations;
2. More initial materials;
3. Transient Flow.





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Thanks!

Questions?