RADIATION IN ENCLOSURES WITH SOME SPECULARLY REFLECTING SURFACES

- Exchange Factor
- Net Radiation Method
- Curved Specularly Reflecting Surfaces

Scope

optical roughness

long wavelength (infrared): specular fashion

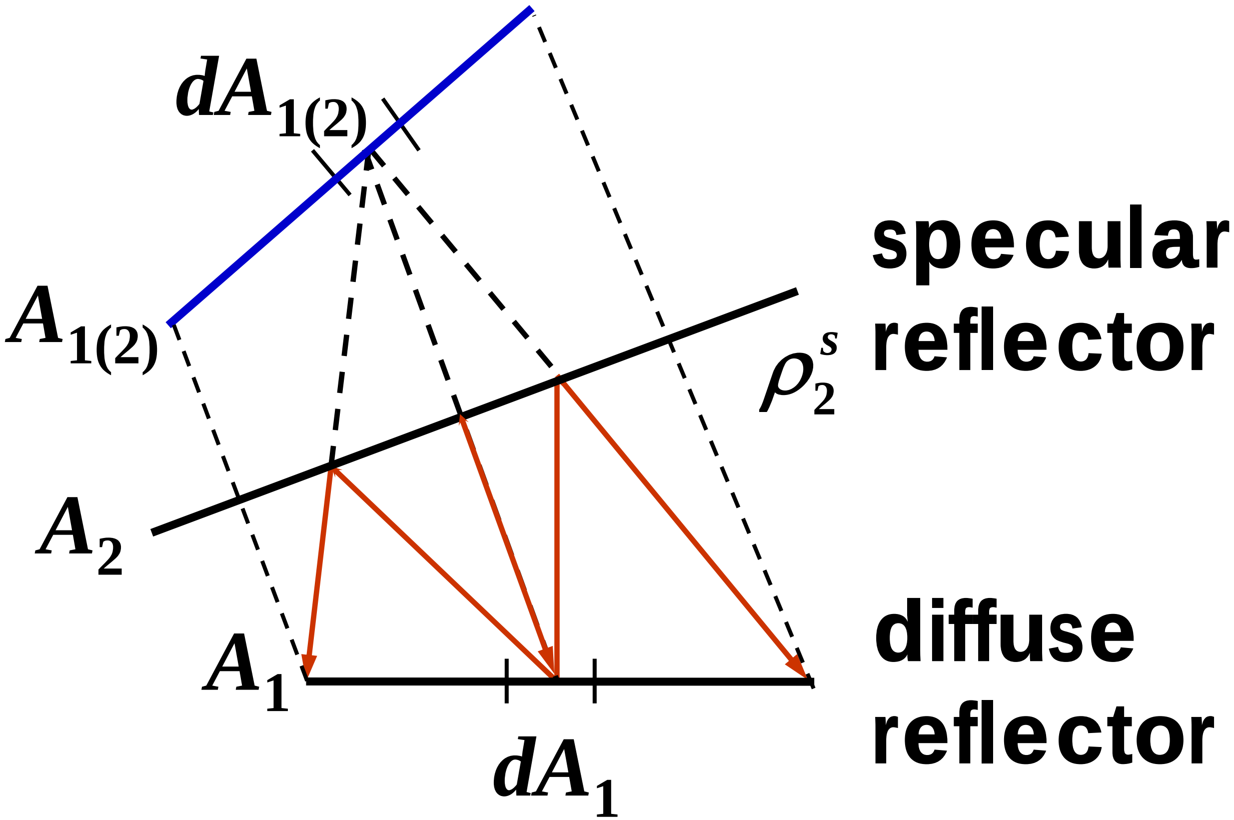
assumptions:

1) **Specular reflectivity is independent of incident angle of radiation.**
2) **All surfaces are gray.**

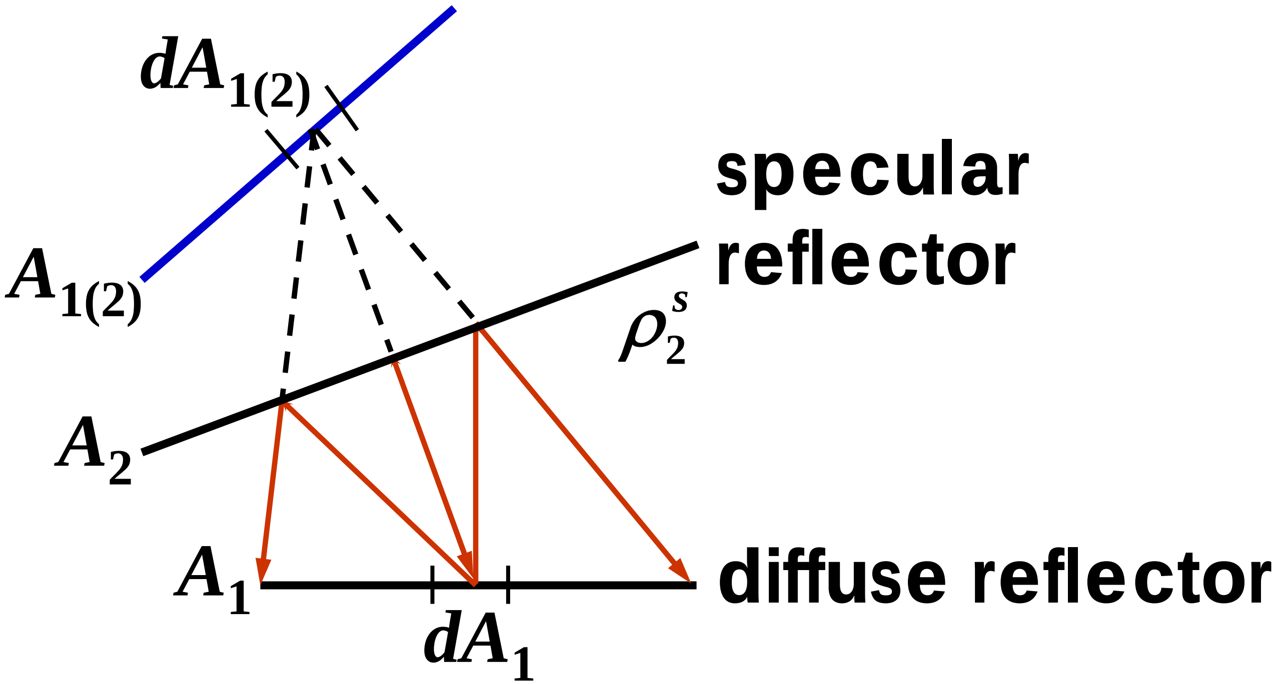
Exchange Factor

Specular view factor

Mirror-image method (Eckert & Sparrow 1965)



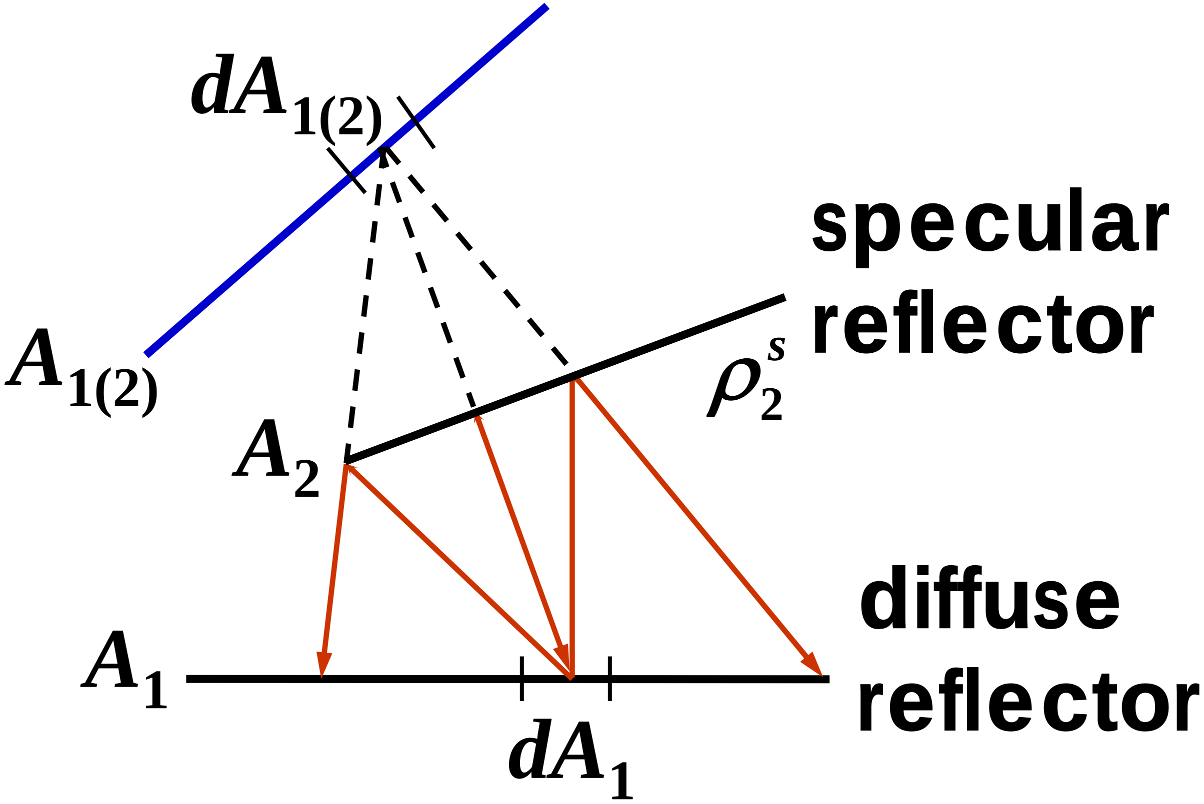
$dA_{1(2)}$: mirror image of dA_1 through A_2



When A_2 is a perfect specular reflector : $\rho_2^s = 1$

the fraction of radiation energy leaving dA_1 that strikes A_1 after one specular reflection at A_2 : $F_{d1(2)-1}$

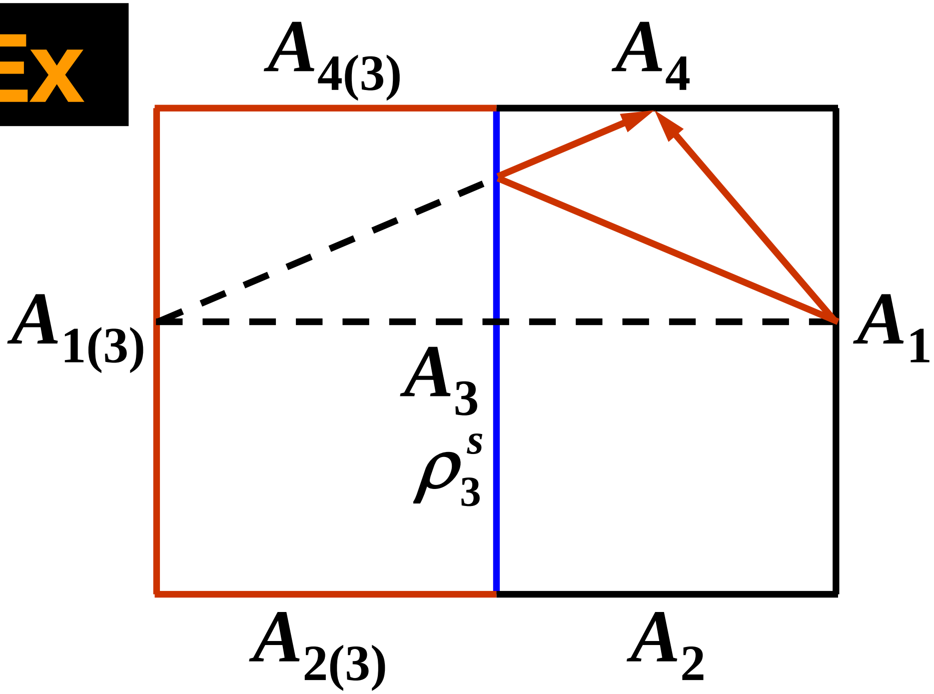
When $\rho_2^s \neq 1$: $\rho_2^s F_{d1(2)-1}$



fraction : $\rho_2^s F^*_{d1(2)-1}$

partial diffuse view factor

Ex



A_1, A_2, A_4: **diffuse**

A_3: **specular**

$$1 \rightarrow 4: \quad E_{14} = F_{14} + \rho_3^s F_{1(3)-4}$$

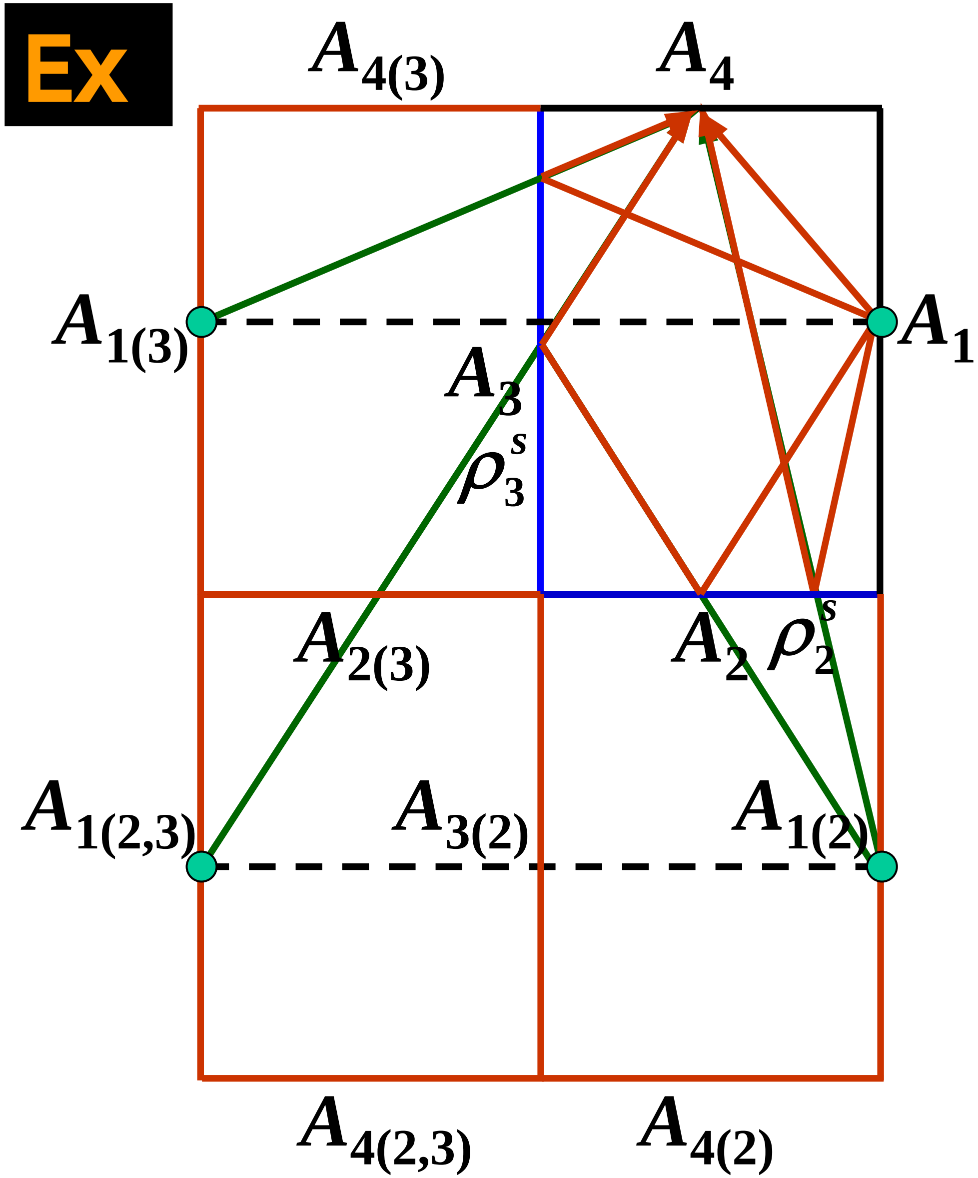
$$2 \rightarrow 4: \quad E_{24} = F_{24} + \rho_3^s F_{2(3)-4}$$

$$1 \rightarrow 1: \quad E_{11} = \rho_3^s F_{1(3)-1}$$

$$1 \rightarrow 2: \quad E_{12} = F_{12} + \rho_3^s F_{1(3)-2}$$

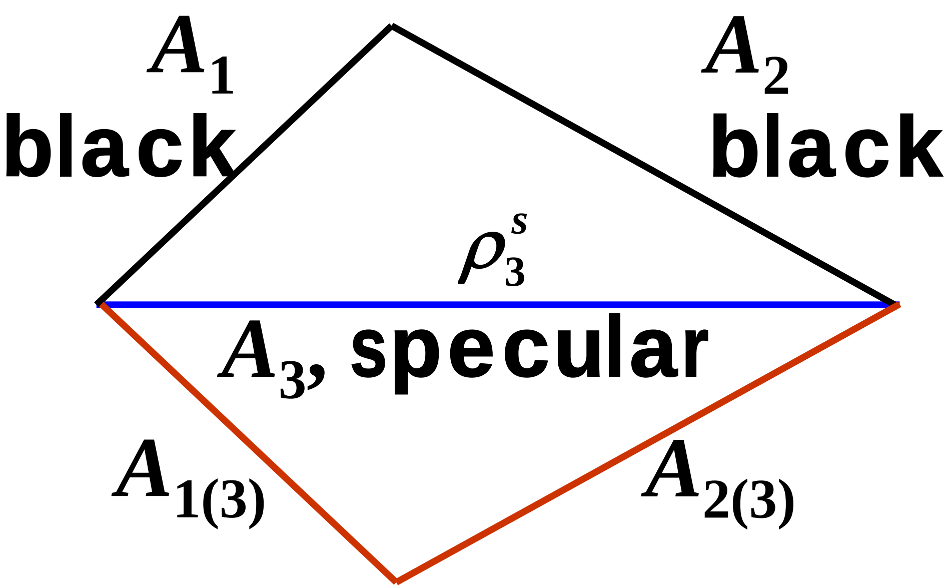
$$2 \rightarrow 1: \quad E_{21} = F_{21} + \rho_3^s F_{2(3)-1}$$

Ex



$$E_{14} = F_{14} + \rho_2^2 F_{1(2)-4}$$
$$+ \rho_3^2 F_{1(3)-4}$$
$$+ \rho_2^2 \rho_3^2 F_{1(2,3)-4}$$

Reciprocity for specular surfaces



$q_{1\to2}$: 1) $A_1 \to A_2$,
2) $A_1 \to A_3 \to A_2$

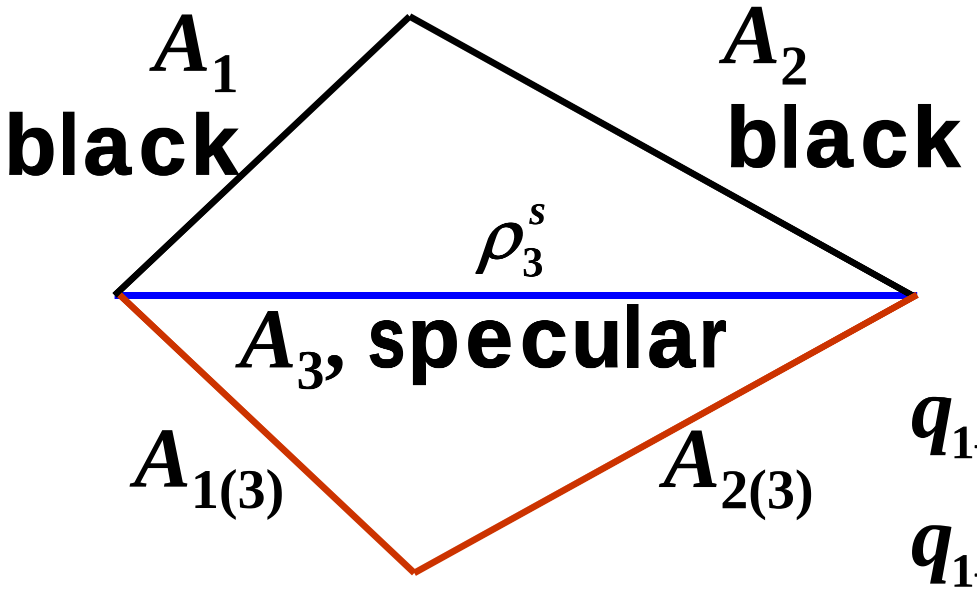
$q_{2\to1}$: 1) $A_2 \to A_1$,
2) $A_2 \to A_3 \to A_1$

$$q_{1\to2} = \sigma T_1^4 A_1 \left(F_{12} + \rho_3^s F_{1(3)-2} \right)$$

$$q_{2\to1} = \sigma T_2^4 A_2 \left(F_{21} + \rho_3^s F_{2(3)-1} \right)$$

isothermal enclosure $\quad q_{1\to2} = q_{2\to1}$

since $A_1 F_{12} = A_2 F_{21} \to A_1 F_{1(3)-2} = A_2 F_{2(3)-1}$



$$q_{1\to 2} + q_{3\to 2} = q_{2\to 1} + q_{2\to 3}$$

$$q_{1\to 2} - q_{2\to 1} = q_{2\to 3} - q_{3\to 2}$$

$$q_{2\to 3} = \varepsilon_3 \sigma T_2^4 A_2 F_{23}$$

$$q_{3\to 2} = \varepsilon_3 \sigma T_3^4 A_3 F_{32}$$

isothermal enclosure

$$q_{2\to 3} = \varepsilon_3 \sigma T^4 A_2 F_{23}$$

$$q_{3\to 2} = \varepsilon_3 \sigma T^4 A_2 F_{23}$$

$$q_{1\to 2} = q_{2\to 1}$$

Net Radiation Method

enclosure with n plane surfaces

- d: diffuse reflecting surfaces
- n - d: specularly reflecting surfaces

for diffuse surfaces: $1 \le k \le d$

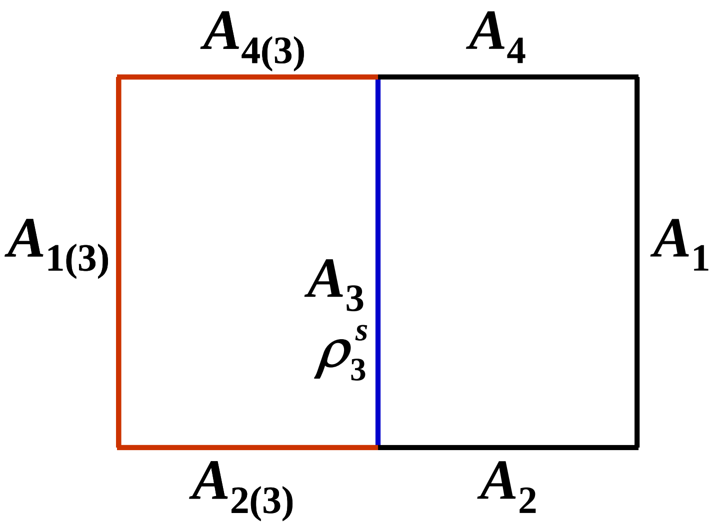
$$q_k'' = J_k - G_k \quad \text{or} \quad q_k'' = \frac{\varepsilon_k}{1-\varepsilon_k}\left(\sigma T_k^4 - J_k\right)$$

$$J_k = \varepsilon_k \sigma T_k^4 + \left(1-\varepsilon_k\right) G_k$$

for specular surfaces: $d+1 \le k \le n$

$$q_k'' = \varepsilon_k\left(\sigma T_k^4 - G_k\right)$$

irradiation $$G_k = \sum_{i=1}^{d} J_i E_{ki} + \sigma \sum_{i=d+1}^{n} \varepsilon_i T_i^4 E_{ki}$$



$$G_k = \sum_{i=1}^{d} J_i E_{ki} + \sigma \sum_{i=d+1}^{n} \varepsilon_i T_i^4 E_{ki}$$

$$G_1 = J_1 E_{11} + J_2 E_{12} + J_4 E_{14} + \varepsilon_3 \sigma T_3^4 E_{13}$$

$$\begin{aligned} G_1 A_1 = {} & J_1 A_1 \rho_3^s F_{1(3)-1} \\ & + J_2 A_2 \left(F_{21} + \rho_3^s F_{2(3)-1} \right) \\ & + J_4 A_4 \left(F_{41} + \rho_3^s F_{4(3)-1} \right) \\ & + \varepsilon_3 \sigma T_3^4 A_3 F_{31} \end{aligned}$$

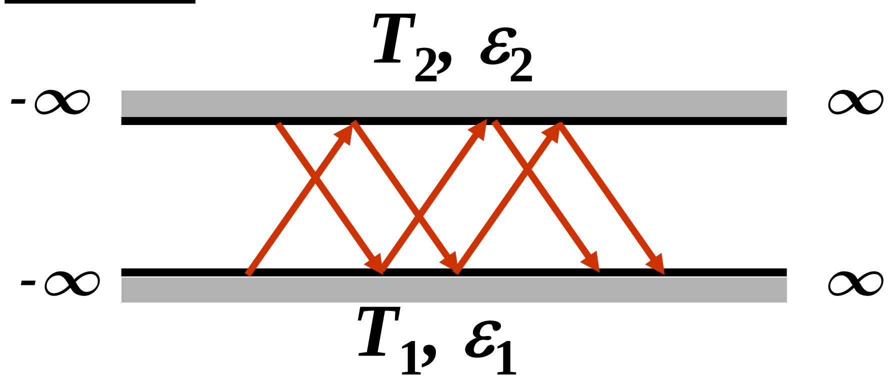
$$A_2 F_{21} = A_1 F_{12},$$

$$A_2 F_{2(3)-1} = A_1 F_{1(3)-2},$$

$$A_4 F_{4(3)-1} = A_1 F_{1(3)-4}, \; A_4 F_{41} = A_1 F_{14}, \; A_3 F_{31} = A_1 F_{13}$$

$$\begin{aligned} G_1 = {} & J_1 \rho_3^s F_{1(3)-1} + J_2 \left(F_{12} + \rho_3^s F_{1(3)-2} \right) \\ & + J_4 \left(F_{14} + \rho_3^s F_{1(3)-4} \right) + \varepsilon_3 \sigma T_3^4 F_{13} \\ = {} & J_1 E_{11} + J_2 E_{12} + J_4 E_{14} + \varepsilon_3 \sigma T_3^4 E_{13} \end{aligned}$$

Ex parallel plates (both specular)



$$q''_k = \varepsilon_k\left(\sigma T_k^4 - G_k\right)$$

$$G_k = \sum_{i=1}^{d} J_i E_{ki} + \sigma \sum_{i=d+1}^{n} \varepsilon_i T_i^4 E_{ki}$$

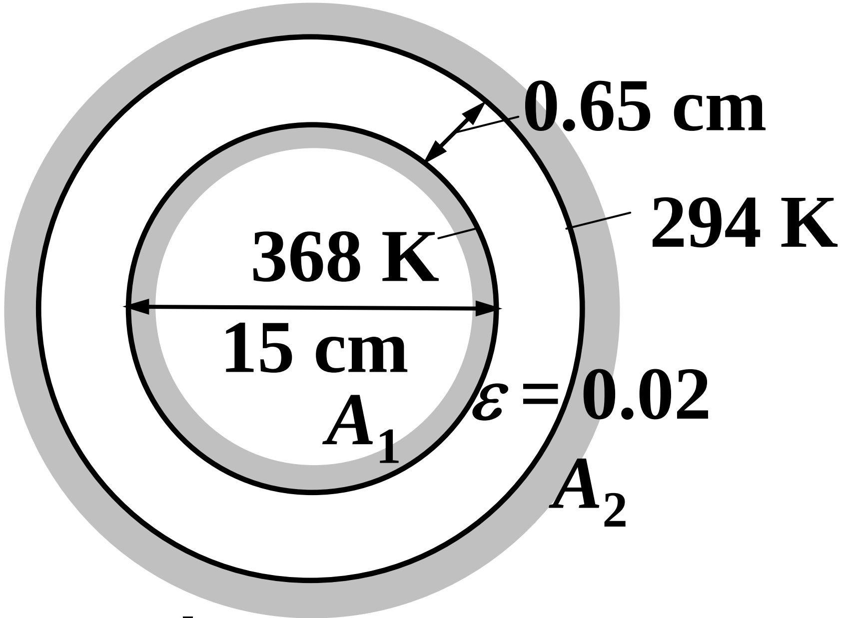
$$q''_1 = \varepsilon_1\left(\sigma T_1^4 - G_1\right), \quad G_1 = \sigma\left[\varepsilon_1 T_1^4 E_{11} + \varepsilon_2 T_2^4 E_{12}\right]$$

$$E_{11} = \rho_2^s F_{1(2)-1} + (\rho_2^s)^2(\rho_1^s) F_{1(2,1,2)-1} + (\rho_2^s)^3(\rho_1^s)^2 F_{1(2,1,2,1,2)-1} + \cdots = \frac{\rho_2^s}{1-\rho_1^s\rho_2^s}$$

$$E_{12} = F_{12} + \rho_1^s\rho_2^s F_{1(1,2)-2} + (\rho_1^s)^2(\rho_2^s)^2 F_{1(1,2,1,2)-2} + \cdots = \frac{1}{1-\rho_1^s\rho_2^s}$$

$$q''_1 = \frac{\sigma\left(T_1^4 - T_2^4\right)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$$

Ex 9-1 spherical vacuum bottle



specular: $q_1 = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1}$

diffuse: $q_1 = \dfrac{A_1\sigma(T_1^4 - T_2^4)}{\dfrac{1}{\varepsilon_1} + \dfrac{A_1}{A_2}\left(\dfrac{1}{\varepsilon_2} - 1\right)}$

specular:

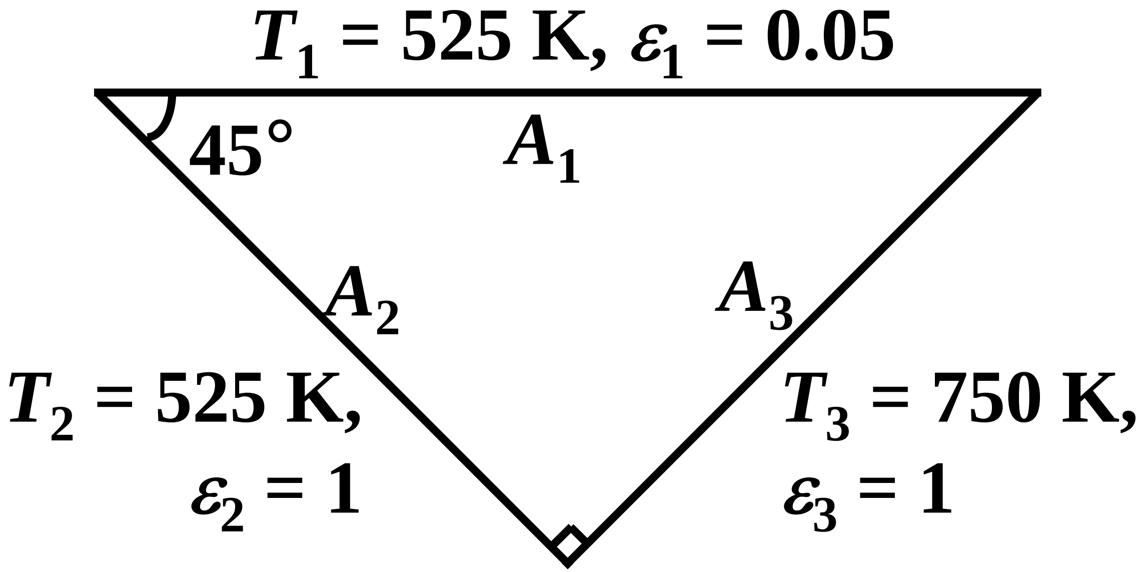
$$q_1 = \frac{\pi(0.15)^2 \times 5.671 \times 10^{-8}(368^4 - 294^4)}{\dfrac{1}{0.02} + \dfrac{1}{0.02} - 1} = 0.440\ \mathrm{W}$$

diffuse:

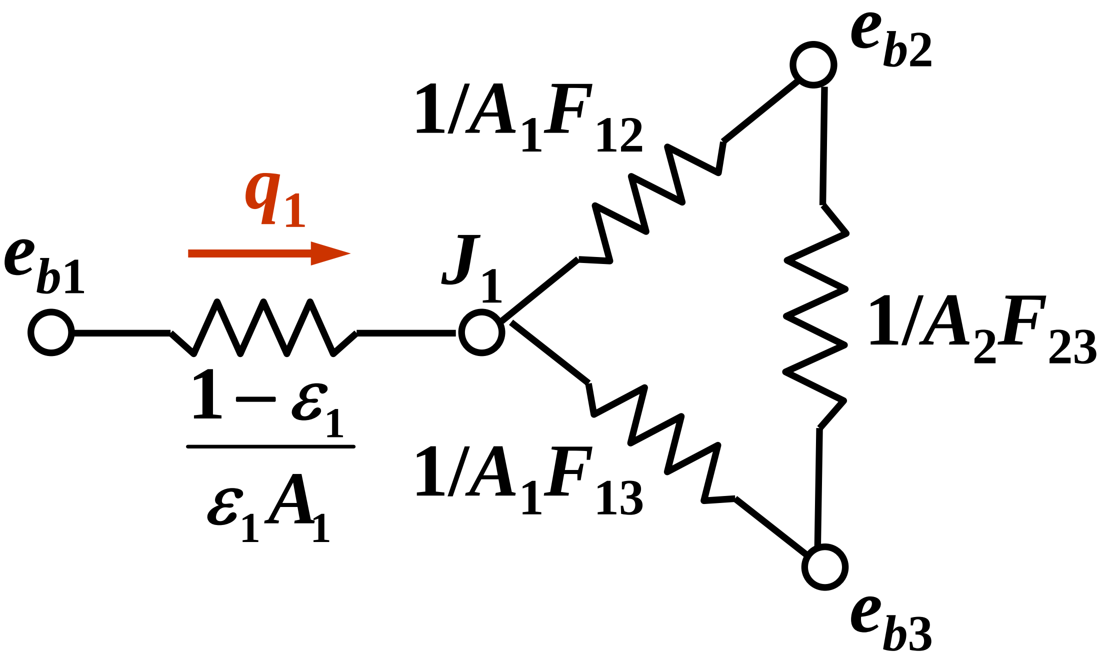
$$q_1 = \frac{\pi(0.15)^2 \times 5.671 \times 10^{-8}(368^4 - 294^4)}{\dfrac{1}{0.02} + \left(\dfrac{15}{16.3}\right)^3\left(\dfrac{1}{0.02} - 1\right)} = 0.476\ \mathrm{W}$$

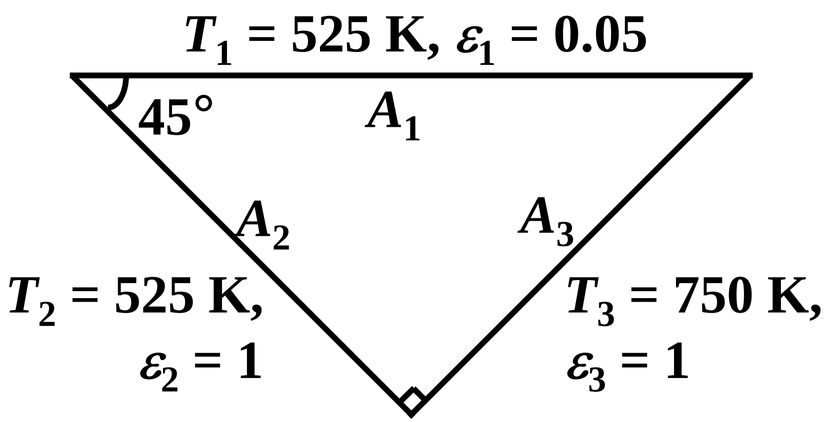
Ex 9-6

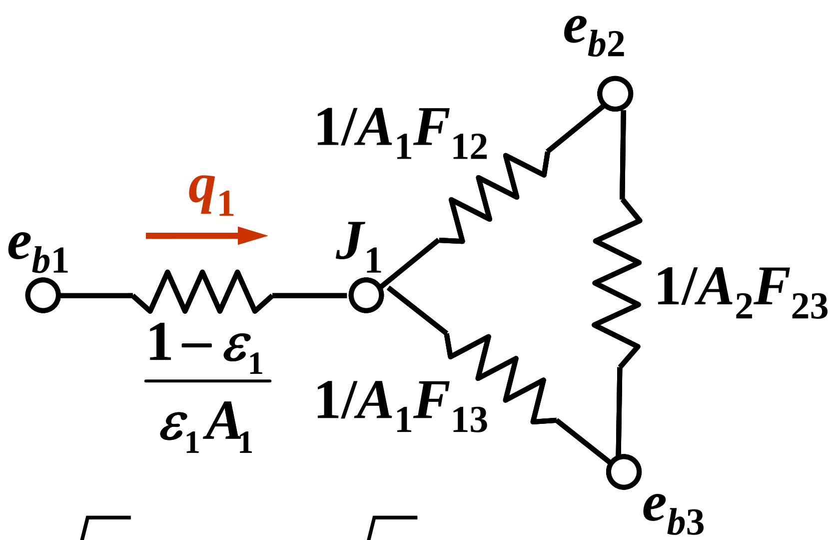
$q_1, q_2, q_3 = ?$



a) surface 1 : diffuse reflector





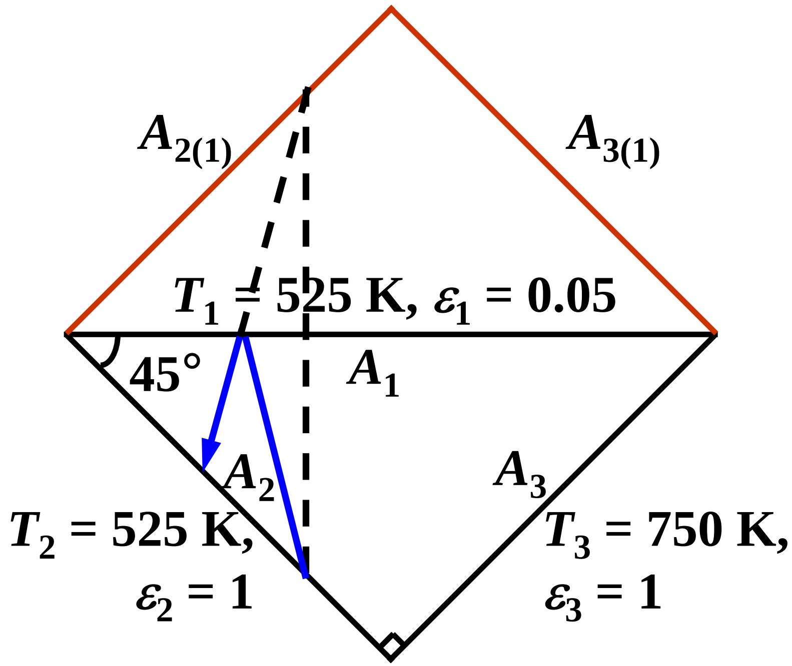


$$F_{12} = F_{13} = \frac{1}{2}, \quad F_{23} = \frac{1+1-\sqrt{2}}{2} = \frac{2-\sqrt{2}}{2}$$

$$q_1 = \frac{\sigma T_1^4 - J_1}{\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_1 - \sigma T_2^4}{\dfrac{1}{A_1 F_{12}}} + \frac{J_1 - \sigma T_1^4}{\dfrac{1}{A_1 F_{13}}}$$

$$q_1 = -144.6 \text{ W}, \ q_2 = -2571.8 \text{ W}, \ q_3 = 2716.4 \text{ W}$$

b) surface 1 : specular



$$q_1 = \varepsilon_1 A_1 \left(\sigma T_1^4 - G_1\right)$$

$$G_1 = J_2 E_{12} + J_3 E_{13} + \varepsilon_1 \sigma T_1^4 E_{11}$$

$$E_{12} = F_{12}, \quad E_{13} = F_{13}, \quad E_{11} = 0$$

$$J_2 = \sigma T_2^4, \quad J_3 = \sigma T_3^4$$

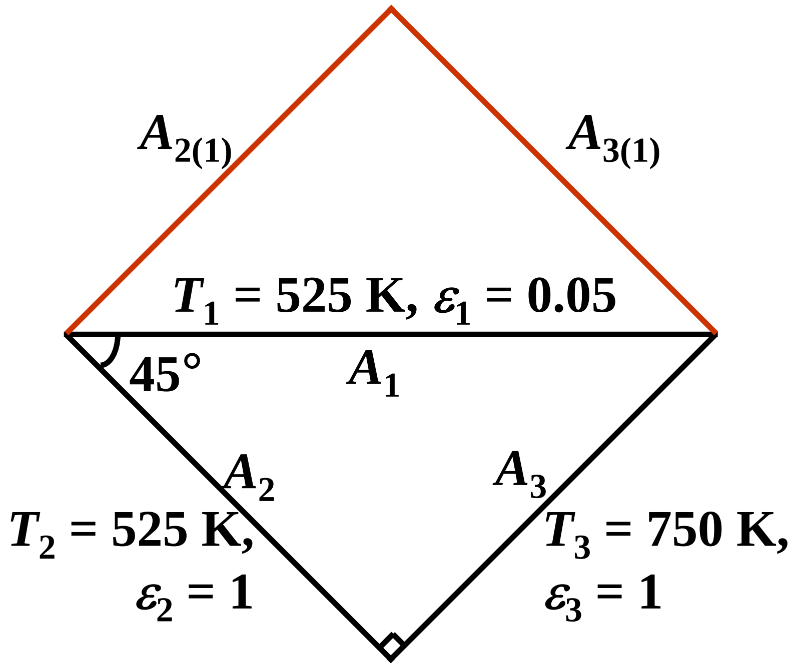
$$q_1 = \varepsilon_1 A_1 \left(\sigma T_1^4 - \sigma T_2^4 F_{12} - \sigma T_3^4 F_{13}\right) = -144.6 \text{ W}$$

$$q_2 = A_2 \left(\sigma T_2^4 - G_2\right)$$

$$G_2 = J_2 E_{22} + J_3 E_{23} + \varepsilon_1 \sigma T_1^4 E_{21}$$

$$E_{22} = \rho_1^s F_{2(1)-2}, \quad E_{23} = F_{23} + \rho_1^S F_{2(1)-3}, \quad E_{21} = F_{21}$$

$$F_{2(1)-2} = \frac{2-\sqrt{2}}{2}, \quad F_{2(1)-3} = \sqrt{2}-1$$



$$q_2 = A_2\left[\sigma T_2^4 - \sigma T_2^4 \rho_1^S F_{2(1)-2} - \sigma T_3^4\left(F_{23} + \rho_1^S F_{2(1)-3}\right) - \varepsilon_1 \sigma T_1^4 F_{21}\right] = -2807.5 \text{ W}$$

$$q_1 + q_2 + q_3 = 0 \;\rightarrow\; q_3 = 2952.1 \text{ W}$$

$$q_1 = -144.6 \text{ W}, \;\; q_2 = -2807.5 \text{ W}, \;\; q_3 = 2952.1\text{W}$$

$$q_1 = -144.6 \text{ W}, \;\; q_2 = -2571.8 \text{ W}, \;\; q_3 = 2716.4 \text{ W}$$

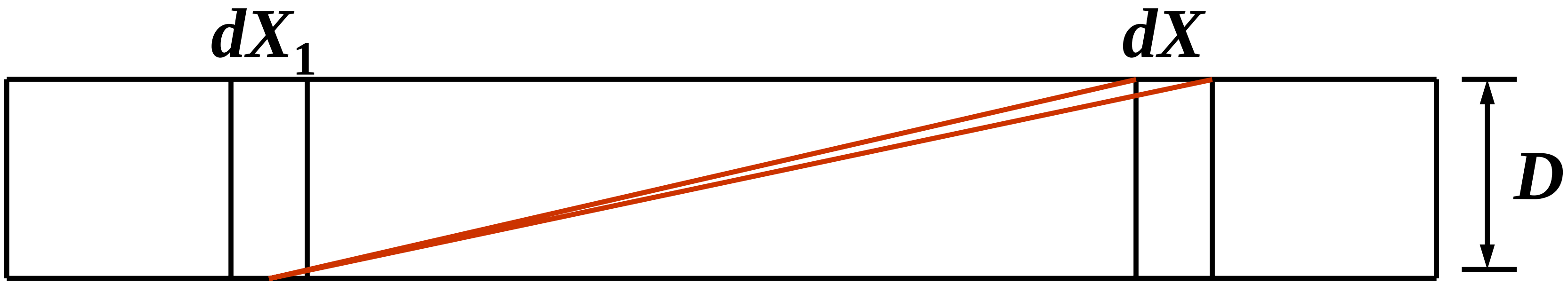
Curved Specularly Reflecting Surface

Ex specular tube

$$K(\xi,\eta)=1-\frac{|\eta-\xi|^3+\frac{3}{2}|\eta-\xi|}{\left[(\eta-\xi)^2+1\right]^{3/2}}$$

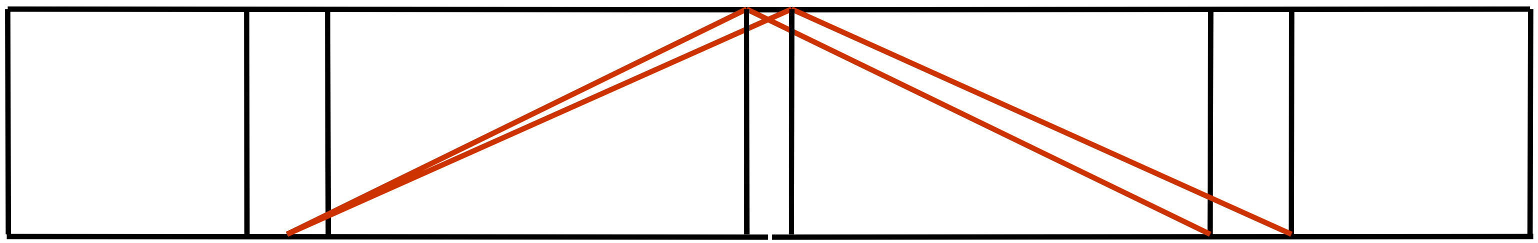
Let $X/D=|\eta-\xi|$

1) direct:



$$dF_{dX_1-dX}=\left\{1-\frac{\left(X/D\right)^3+3X/2D}{\left[\left(X/D\right)^2+1\right]^{3/2}}\right\}dX$$

2) one-reflection:



$$dF_{dX_1-\frac{dX}{2}} = \left\{1-\frac{\left(X/2D\right)^3+3X/4D}{\left[\left(X/2D\right)^2+1\right]^{3/2}}\right\}\frac{dX}{2}$$

3) *n* reflection:

$$dF_{dX_1-\frac{dX}{n+1}} = \left\{1-\frac{\left[X/(n+1)D\right]^3+3X/2(n+1)D}{\left\{\left(X/(n+1)D\right)^2+1\right\}^{3/2}}\right\}\frac{dX}{n+1}$$

$$dE_{dX_1-dX} = \sum_{n=0}^{\infty}(\rho^s)^n dF_{dX_1-dX/(n+1)}$$