CONSISTENCY VERSUS COMPLETENESS IN MEDICAL DECISION MAKING:

EXEMPLAR OF 155 PATIENTS AUTOPSIED AFTER CORONARY ARTERY BYPASS GRAFT SURGERY

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ABSTRACT

Diagnoses made at autopsy are usually yes-no (binary) decisions inferred from clinicopathologic data. A major conceptual problem in determining cause of death is that variables used in classifying some patients may be missing in other patients. A model with too few logical implications will be mathematically incomplete for small data sets; but a model too many implications may be inconsistent with large data sets. We examined the 155 patients autopsied after coronary artery bypass surgery from The Johns Hopkins Hospital autopsy database of 43200 cases. Diagnoses entered on a word processor and transmitted to a minicomputer were solved by the Quine-McCluskey algorithm. Our analysis disclosed that 41% of patients suffered a fatal complication of cardiac surgery; 43% had established surgical complications or unrelated causes of death; and in 17% of cases the cause of death was unexplained. Computerized symbolic logic analysis of medical information is useful in testing the completeness of a proposed set of causes of death.

INTRODUCTION

Diagnoses made at autopsy, like those made during life, are usually yes-no (binary) decisions inferred from a large body of historical, physical examination, laboratory, and pathologic data. These data, in turn, are often either inherently binary or else involve only a few grades of severity which actually enter the decision-making process. Because of the diversity and large amounts of potentially relevant data of this sort, however, we are still faced with the formidable problems of (i) entering large volumes of reasonably error-free medical data, (ii) sifting through large databases in search of a few cases of interest, and (iii) analyzing these relatively heterogenous data in a coherent manner.

Autopsy data, including clinical summaries prepared by the autopsy prosector, include a wealth of information relevant to the cause of death. The Johns Hopkins Hospital autopsy database consists of 43200 autopsy records (beginning in 1889 and typewritten after 1901) and accompanying glass slides, all housed in a single room accessible to authorized personnel. Demographic data (age, race, sex) are in computer-readable form for all cases, and the autopsy report itself (i.e., list of diagnoses) is computer-readable for the most recent 23200 cases (since 1948). All cases are indexed. The computer-readable part of the index employs the actual words used by the attending faculty who signed out the case, rather than the Systematic Nomenclature of Pathology (SNOMED). It is our view that standardized coding is time-consuming, prone to error, and tends to sacrifice desired nuances of meaning. Recovery of cases is done by a Key Word In Context (KWIC) system such as that used in the *Federation Proceedings* index. Investigators select desired cases on the basis of contextual information. Our autopsies are typed routinely on word processors, text is transmitted to a minicomputer, sorted keyword lists are generated by a short MUMPS program, and updated KWIC index books are printed at timely intervals.

Patients autopsied after coronary artery bypass surgery are an appealing group for study from several perspectives. Coronary artery bypass surgery is a very widely used therapy in patients with ischemic heart disease (estimates of up to 100,000 cases yearly in the U.S.A).[1] The procedure has a low operative mortality,[2-5] is usually successful in relieving or eliminating anginal pain,[3,6-9] and may be useful in prolonging life in certain groups of patients, such as those with significant left main coronary artery narrowing,[2,10,11] but the effectiveness of the coronary artery bypass graft procedure in prolonging life in larger groups of patients with significant coronary artery lesions remains controversial.[1,4,8,12] The cause of death among patients dying in the perioperative period is often poorly understood. The complexity of many of these cases make them particularly suitable candidates for computer approaches. We were able to recover all cases in the present study from our computer indexes. We then supplemented the autopsy records with data from the original medical records and from our own review of the preserved heart specimen and postmortem radiographs. Data included descriptive (binary) statements, semiquantitative grades of severity, and quantitative measurements (percent arterial occlusion, infarct size, etc.). We formulated deductions in symbolic logic as we might rationalize our decisions to professional colleagues. Although symbolic logic does not mimic actual human thought

0195-4210/82/0000/0805$00.75

processes, it does resemble the natural language in which we account for our decisions. Using a word processor interactively with a time-shared minicomputer, we refined our arguments until each case was handled by the computer to our satisfaction.

MATERIALS AND METHODS

The 155 patients autopsied at The Johns Hopkins Hospital after coronary artery bypass graft surgery were identified from the computerized keyword index. For 150 of the patients the heart had been studied after postmortem coronary arteriography and fixation in distention,[13] and in 154 instances the cardiac surgery had been performed at The Johns Hopkins Hospital. In 67 cases, other procedures besides coronary artery bypass graft surgery had been performed. Clinical and pathologic variables were determined for each patient and were recorded on a standardized form using a Raytheon VT1303 word processor with asynchronous ASCII communications software.[14] Proofread data diskettes were transmitted in "BATCH SEND" mode via a dial-up or direct line to a Digital Equipment Corporation PDP 11/70 minicomputer with MUMPS operating system and language interpreter in the Department of Laboratory Medicine of The Johns Hopkins Medical Institutions. Means, standard deviations, correlation coefficients, and frequency distributions of selected variables were obtained from the Statistical Analysis System (SAS) program package on the IBM 370/3031 computer in the Information Systems Department of The Johns Hopkins Medical Institutions.[15] Symbolic logic calculations were more easily obtained on the PDP 11/70 minicomputer, due to the character string subscripting, dynamic storage, and implicit sort capabilities of the MUMPS language.[16] Symbolic logic solutions were obtained using an established algorithm of Quine[17,18] and McCluskey.[19] This algorithm can be programmed quite compactly in MUMPS, as shown in Table 1, and we obtained solutions for all 155 cases in 94 minutes in a time-sharing environment with 15 competing jobs.

COMPLETENESS VERSUS CONSISTENCY

Our approach to the analysis of a medical database is to propose a hypothesis and then to ask whether the relevant autopsy data are reasonably consistent with this hypothesis. Among patients autopsied after coronary artery bypass surgery, it had been our impression that a large proportion of patients dying in the perioperative period died as a result of myocardial injury or alterations of myocardial geometry related to the cardiac operative procedure. We prepared a list of recognized surgical complications and unrelated events generally regarded as sufficient to cause death, and then worked repeatedly through the remaining cases searching for additional problems which might suffice to cause death. Since it is difficult to apply uniform standards of judgment to many cases with many variables per case, we constructed a system of criteria for each potential cause of death and used symbolic logic to apply these criteria objectively to each case.[20,22]

Ordinary symbolic logic (first order propositional logic) is attractive as a medium for expressing relationships among observed data and hypothetical concepts because (i) it closely resembles declarative assertions written in a natural language and (ii) its mathematical properties and computer solution procedures are well-studied. In our experience the fact that symbolic logic does not handle probabilistic data has not been a particularly compelling objection to its use. Quite the contrary, the many probabilistic models which call either for highly subjective "personal probabilities" or for frequentist probabilities which are difficult to obtain seem more contrived to us than ordinary symbolic logic. A more serious problem in our experience is the difficulty of constructing a complete and consistent system of symbolic logic with respect to "reasonable" medical data sets.

In mathematical logic, a system of axioms is said to be <u>complete</u> if every true theorem is provable. A system is <u>consistent</u> if no theorem in the system is both true and false. In 1931, Gödel published his celebrated proof that systems of logic with the strength of ordinary arithmetic cannot be both complete and consistent.[23] Since inconsistency is intolerable in a mathematical system,[24] we face the somewhat unsettling fact that consistent systems of set theory, algebra, and geometry, are all incomplete--i.e., they all contain true theorems which are unprovable. An intuitively similar (but mathematically less momentous) problem is encountered in the management of medical data. Let us say that a system of symbolic logic is <u>complete</u> if it generates a diagnosis for every set of patient data; and <u>consistent</u> if the system never obtains a diagnosis that is both true and false.

Now suppose we have a system of logic which asserts that a positive barium enema followed by pathologic confirmation suffices to diagnose transverse colon carcinoma. But suppose we also require that a <u>negative</u> barium enema is the only way to <u>rule out</u> transverse colon carcinoma. Since it is not current medical practice to perform a barium enema on all patients regardless of symptoms (and rightly so), the above logical requirement would result in incomplete data sets with respect to the important question of transverse colon carcinoma. On the other hand, suppose we say that an absence of gastrointestinal complaints, negative rectal examination and stool Guaiac, etc., suffice to rule out transverse colon carcinoma. There will then be the occasional patient with a negative history and physical examination who subequently has a laparotomy for unrelated reasons (say, motor vehicle trauma) and is found to have a transverse colon carcinoma. Logically speaking, the diagnosis of transverse colon carcinoma is both true and false in this patient. Various contrivances are available for resolving this dilemma; we favor an extension of symbolic logic based on <u>modal logic</u>.[21,25,26]

SYMBOLIC LOGIC ANALYSIS

Ordinary symbolic logic (first order propositional logic) consists of descriptive statements (operands) related to one another by operators such as "not", "and", "or", "implies". The most popular notation for symbolic logic is the so-called Principia notation (used in Whitehead and Russell's Principia Mathematica).[27] Examples: "it is true that a" is written "⊤ a"; "a implies b" is written "a ⇒ b"; "a and b implies c" is written "(a & b) ⇒ c". The key feature of Principia notation is that the operators and statements are written in the same order as they appear in most European languages. Some mathematical arguments are facilitated by the use of Lukasiewicz notation (or Polish notation), in which each two-operand operator is placed before (rather than between) the two operands which it modifies. Thus "it is true that a" is still written "⊤ a"; but "a implies b" is written "⇒ a b"; and "a and b implies c" is written "⇒ & a b c".[27] In the present report, we use nullity notation, introduced by Quine,[28] because of the ease with which this notation lends itself to computer solution procedures, especially the Quine-McCluskey algorithm.[17-19] Translation of Principia or Polish notation into nullity notation is easily accomplished by Gentzen's algorithm.[27,29] A nullity is any set of statements which contains no statement and its negation. Using "+" to denote "it is true that" and "-" to denote "it is false that", {+a}, {-a}, {+a, -b}, and {+a, +b, -c} are all nullities; but {+a, -a}, {+a, +b, -b}, and {+a, +b, +c, -c} are not nullities, because each contains a statement and its negation. A study nullity is any nullity containing at least one statement which is false for the study. For example, "it is true (for the study) that a" corresponds to the study nullity, {-a}. In the sentence "a implies b", it cannot simultaneously be the case that both a is true and b is false; therefore, {+a, -b} is a study nullity. In the sentence "a and b implies c", it cannot be the case that a and b are true but c is false; therefore, {+a, +b, -c} is a study nullity. The importance of nullities as a computing convenience is the fact that two study nullities can be "null-added" (denoted ●) to form a new study nullity or null sum. Thus

$$\begin{array}{l} \{-a\} \\ \underline{\bullet\ \{+a,\ -b\}} \\ \quad\ \{-b\} \end{array}$$

$$\begin{array}{l} \{-a\} \\ \underline{\bullet\ \{+a,\ +b\ \ -c\}} \\ \quad\ \{+b,\ -c\} \end{array}$$

$$\begin{array}{l} \{-b\} \\ \underline{\bullet\ \{+b,\ -c\}} \\ \quad\ \{-c\} \end{array}$$

are all valid null additions, because in each case a single statement variable is cancelled to obtain the solution. Mathematically it has been proved that exhaustive null additions (the MUMPS program in Table 1) suffice to determine all and only the study nullities in the system.[20,21] A system is consistent if and only if { } (the empty set, ∅) is not a study nullity; and complete with respect to diagnosis a if either {+a} or {-a} is always present in the set of study nullities.

Quine's proof that the Quine-McCluskey algorithm obtains all and only valid study nullities from the input study nullities is so simple and elegant that it is worth sketching here. Given the list, $a_1, a_2, \ldots, a_n$, of n binary statements under consideration, we define a maximal nullity as any complete set of these statements, each of which is prefixed by "+" or "-"; there are 2^n maximal nullities. Every nullity is a subset of one or more maximal nullities, and in fact every nullity, A, may be characterized in terms of the maximal nullities of which A is a subset. It is intuitively clear that if A and B are nullites, A is a study nullity, and A is a subset of B ($A \subseteq B$), then B is also a study nullity; because if A contains at least one statement that is false for the study and B contains every statement of A, then B must contain at least one statement that is false for the study. The proof that exhaustive null additions find all and only valid study nullities has two parts. To show that null addition, $B \bullet C = D$, obtains only a valid study nullity, D, we may demonstrate that for every maximal nullity, E, of which D is a subset, either $B \subseteq E$ or $C \subseteq E$. Let $B = \{a_1, \ldots, a_i, +a_{i+1}\}$ and $C = \{-a_{i+1}, a_{i+2}, \ldots, a_j\}$; then by definition of null sum, $D = \{a_1, \ldots, a_i, a_{i+2}, \ldots, a_j\}$. By definition of maximal nullity, either $+a_{i+1} \in E$ or $-a_{i+1} \in E$. Suppose that $+a_{i+1} \in E$; then $B \subseteq D \cup \{+a_{i+1}\} \subseteq E$. Otherwise, $C \subseteq D \cup \{-a_{i+1}\} \subseteq E$.

To show that exhaustive null addition obtains all valid study nullities, suppose that there is a most inclusive, valid study nullity, $A = \{a_1, a_2, \ldots, a_j\}$, which is not obtained after exhaustive null additions. This means that $B = A \cup \{+a_{j+1}\}$ and $C = A \cup \{-a_{j+1}\}$ are obtained by exhaustive null additions, or else A would not be "most inclusive". But $B \bullet C = A$, so that exhaustive null additions could not have been performed. End of proof.

Our approach to the dilemma exemplified above by the diagnosis of transverse colon carcinoma employs a "certainty operator", denoted "\$" (read, "certain whether or not"). The \$-operator has a formal relationship to the modal logic □-operator ("necessarily") by the formula, \$a = (□a | □-a).[24,25] We use the formula, -\$a ⇒ a ("if not certain whether or not a, then a"), as a default procedure for forcing the value for a when classical symbolic logic has failed. Just as "a ⇒ b" corresponds to the study nullity {a,-b}, likewise "-\$a ⇒ a" corresponds to the study nullity {-\$a,-a}. The Quine-McCluskey algorithm is executed in two steps: first excluding all study nullities which contain certainty operators; then a second time, including all study nullities.[21,22] Using a few, simple rules to prevent the certainty operator nullities from "bumping into" classical symbolic logic, one can guarantee consistency and completeness for a large, relatively heterogeneous body of binary data.[30] In the present application, we employ the study nullity, {-$unx, -unx}, where unx = "unexplained cause of death after coronary artery bypass surgery". This study nullity translates as, "if it is uncertain whether the cause of death is explained, then it is unexplained".

CORONARY ARTERY BYPASS PATIENTS

The 155 patients on whom an autopsy was performed after a coronary artery bypass operation ranged in age from 35 to 81 years (average, 57 years), and included 109 males and 144 whites. In our initial analysis of coronary artery bypass patients, we identified the recognized surgical complications (sur) and unrelated causes of death (unr) which could account for a fatal outcome in these patients. These findings are listed in Table 2, and account for 66 patients in the series. If an agreed-upon list of causes (here, sur and unr) are exclusively responsible for a given effect (here, death), then it should be true that one or more of the causes is present. However, this naive study nullity, {+death, -sur, -unr, -unx}, resulted in 89 unexplained cases in the initial analysis. In each of these patients, there was not a surgical complication regarded as sufficient to cause death (study nullity {+sur}) and there was not an unrelated event sufficient to cause death (study nullity {+unr}). Since all patients in an autopsy series are dead, {-death} is a study nullity. But by null addition, the presence of study nullities {+death, -sur, -unr, -unx}, {-death}, {+sur}, and {+unr} implies that {-unx} is a study nullity and the system is unexplained for that patient.

Many of these initially unexplained patients suffered from perioperative problems which have recently become regarded as sufficient to cause death in patients receiving coronary artery bypass graft surgery. In 13 patients, the coronary artery bypass graft procedure was performed after a preoperative catastrophe (pre), such as a preoperative cardiac arrest or an acute myocardial infarct. In these patients, who survived less than one day postoperatively on the average, death occurred with immediate postoperative pump failure and massive reflow injury, or contraction band necrosis, in the myocardium. Contraction band necrosis is a myocardial lesion recognizable grossly as an area of hemorrhage and microscopically as characteristic, thick, transverse, eosinophilic bands in the cardiac myocytes. Contraction band necrosis is typically seen in association with open coronary artery grafts, and is probably due to excess calcium ion influx into myocytes following the reflow of blood after a period of no flow to the myocardium.[31] In 17 patients, immediate pump failure and death was due to massive contraction band necrosis, probably sustained near the end of the operative procedure (opc). These patients lived an average of about one day after operation, and their postoperative course was marked by immediate, severe shock. This pattern of myocardial injury has been seen significantly less frequently in more recently autopsied patients who received improved myocardial support techniques at operation.[14] In other patients, a clinical course of immediate pump failure or an arrhythmia led to death in the operative or early postoperative period, but the hearts did not have substantial contraction band necrosis. Five of these patients had operative graft occlusion (opg) usually from operative compression of the anastomosis. These patients presumably suffered a fatal myocardial infarct as a result of operative graft occlusion: in two patients the infarct was demonstrable histologically, but in three patients the time between operation and death was too brief for the usual histologic lesion of myocardial infarction to appear. With these additional three causes of death (pre, opc, opg), one can now account for death in 101 patients. This expanded study nullity:

{+death, -pre, -opc, -opg, -sur, -unr, -unx}

resulted in 54 unexplained cases.

Two additional patterns emerged in the remaining 54 patients whose death was as yet unexplained by our analysis. There were 15 patients receiving a concomitant aneurysmectomy (anu) who suffered immediate pump failure unexplained by operative myocardial injury or graft occlusion. Among the 29 patients receiving aneurysmectomy in this series, 18 (62%) suffered a similar postoperative course of immediate pump failure (as compared to 43 (34%) in 126 non-aneurysmectomy patients), suggesting that patients appear to suffer a substantial additional risk of immediate postoperative shock associated with the aneurysmectomy ($p<0.01$). It has been suggested that rearrangements of the geometry of the ventricular segments created by the operative procedure could account for the impaired ventricular function, that is, could operatively produce flattening of the interventricular septum and the left ventricular segments.[32] Such flattening of segments would be predicted to reduce the effectiveness of myocardial tension producing intracavitary pressure.[33] Thus we considered this group of patients as showing a pattern of events which might be sufficient to account for a total outcome.

Finally, in 13 patients early postoperative arrhythmias or cardiac arrests were associated with graft thrombosis (gth). These were all patients who had successfully come off cardiopulmonary bypass without immediate shock, and as a rule these patients showed only trivial operative injury. Graft thrombosis in these patients probably occurs in association with the postoperative reversal of anticoagulation. In the 22 patients who had sustained graft thrombosis, 17 (77%) suffered either immediate pump failure or early postoperative arrhythmias or cardiac arrests ($p<0.01$). Again, this coincidence of events suggested a pattern which might account for death in these 13 otherwise unexplained cases. The final study nullity:

$$\{+death, -pre, -opc, -opg, -anu,$$
$$-gth, -sur, -unr, -unx\}$$

accounted for the clinicopathologic findings in all but 26 patients. In these patients no cause of death was established, and the diagnosis "+unx" was obtained by the use of default nullity {-$unx,-unx}. Results are summarized in Table 3.

DISCUSSION

Medical decision making is generally regarded as prospective process in which observed data on a particular patient and the physician's accumulated experience are combined to render diagnoses and choose among therapies. While this experience may be a good predictor in well-established areas of medicine, its value in predicting frequencies of medical events is less certain in areas such as cardiology and oncology where our concepts are in a state of flux. Methods such as decision theory, which depend heavily upon a priori probabilities of disease processes, are difficult to apply to rapidly changing areas of medical practice. The "limiting frequency" interpretation of probability presupposes an experience based upon many cases, which is often unavailable; but the "personal probability" interpretation is subjective and may does not have a demonstratable relationship to actual observations on patients. The retrospective medical decisions of the autopsy pathologist have an important role in documenting these changing patterns of diagnosis and treatment. Classical autopsy studies, in which observations are systematically collected and the numbers of cases having different features are organized in tables, still command the attention of practicing physicians. It is reasonable to explore the extent to which classical autopsy studies can be streamlined by modern computer methods.

Case identification is often a very burdensome task. Some autopsy services have excellent case material which is virtually inaccessible due to poor indexing. SNOMED coding may not be practical because it involves a significant outlay of the pathologist's time or hiring of additional skilled personnel. Retrospective SNOMED coding of old autopsy material is usually a hopeless task, so that one then ends up with two indexing systems--a "pre-SNOMED" and a "post-SNOMED" index--resulting in further confusion. On the other hand, most pathology departments routinely type all their records, and word processing is being brought into many pathology departments for reasons of office efficiency, quite independent of indexing considerations. Autopsy reports saved on floppy disks can readily be converted into Key Word Index Context (KWIC) index books using a minicomputer with the character manipulation, dynamic storage, and implicit sort capabilities of MUMPS. Elimination of the tedious SNOMED coding step also makes it more feasible to enter older records on the word processor, as we have done with our autopsy records since 1948. The outstanding flaw of KWIC index books in case identification is created by multiple synonyms for the same disease entity. In our experience, however, the interested investigator usually knows the synonyms, and he may have more confidence in his own ability to recognize disease names in context than in the coder's ability to have classified the diseases according to the investigator's desires.

After the appropriate cases are assembled and supplementary data are entered on the word processor, it is often very instructive to examine the means, standard deviations, and all possible pairwise correlations. Unexpected high correlations may point out errors or conceptual misunderstandings in the database; anticipated high correlation values serve as reassurance that the core data are sound. Not every statistically significant result has its most obvious biological meaning. We once found a significant correlation between the severity of peripheral vascular disease and short stature in a series of autopsied diabetic patients.[32] The "short statured" patients were actually bilateral amputees!

The final step in the analysis is to construct a comprehensive explanation of the findings as one might rationalize them to a professional colleague. This explanation can usually be translated into symbolic logic without much trouble, as shown in the present example of coronary artery bypass patients. The word processor is then employed interactively with a minicomputer to refine the explanation and test its coherence with respect to the entire series of cases. Two important criteria for the "coherence" of an explanation are completeness and consistency. We define completeness as the ability of the logical system to obtain a true or false status for all diagnoses of interest (here, causes of death after coronary artery bypass graft surgery). Consistency is the absence of any diagnosis which is both true and false. A few cases may remain unexplained by this method, but these same cases may not make much sense when examined by intuitive methods either. The power of the above approach is its objectivity combined with its resemblance to established reasoning processes in pathology--the computer serves as an extension of the classical methods of counting and comparing rather than as an enforcer of new, unfamiliar assumptions.

The computer revolution in medicine which began over two decades ago is just now being felt in departments of anatomical pathology, where word processors are replacing typewriters as the means of producing autopsy documents. This new source of machine-readable medical information can be used to answer detailed questions of disease natural history and treatment complications, including many ordinary bookkeeping questions which are easily answered with the character manipulation and implicit sort capabilities of computer languages such as MUMPS. We hope to see a greater interest in autopsy pathology applications of word processing and computing in future years.

LITERATURE

1. McIntosh, H.D., Garcia, J.A.: The first decade of aortocoronary bypass grafting, 1967-1977: A review. Circulation 57:405-431, 1978.
2. Murphy, M.L., Hultgren, H.N., Detre, K., et al., and the participants of the Veterans Administration Cooperative Study. Treatment of chronic stable angina: A preliminary report of survial data of the randomized Veterans Administration Cooperative Study. N. Engl. J. Med. 297: 621-627, 1977.
3. Lawrie, G.M., Morris, G.C., Jr., Howell, J.F., et al: Results of coronary bypass more than five years after operation in 434 patients. Am. J. Cardiol. 40:665-672, 1977.
4. Reul, G.J., Jr., Cooley, D.A., Wukasch, D.C., et al: Long-term survival following coronary artery bypass: Analysis of 4,522 consecutive patients. Arch. Surg. 110:1419-1424, 1975.
5. Assad-Morell, J.L., Connolly, D.C., Brandenburg, R.O., et al: Aorto-coronary artery saphenous vein bypass grafts. J. Thorac. Cardiovasc. Surg. 69: 841-850, 1975.
6. Selden, R., Neill, W.A., Ritzmann, L.W., et al: Medical versus surgical therapy for acute coronary insufficiency. A randomized study. N. Engl. J. Med. 293:1329-1333, 1975.
7. Gott, V.L.: Outlook for patients after coronary artery revascularization. Am. J. Cardiol. 33: 431-437, 1974.
8. Ross, R.S.: Ischemic heart disease. An overview. Am. J. Cardiol. 36: 496-505, 1975.
9. Favaloro, R.G., Effler, D.B., Groves, L.K., et al: Severe segmental obstruction of the left main coronary artery and its divisions. Surgical treatment by the saphenous vein graft technique. J. Thorac. Cardiovasc. Surg. 60: 469-482, 1970.
10. Cohen, M.V., Gorlin, R.: Main left coronary artery disease. Clinical experience from 1964-1974. Circulation 52:275-285, 1975.
11. Zeft, H.J., Manley, J.C., Huston, J.H., et al: Left main coronary artery stenosis: Results of coronary bypass surgery. Circulation 49: 68-76, 1974.
12. Hurst, J.W., King, S.B., III, Logue, R.B., et al: Value of coronary bypass surgery. Controversies in cardiology. Part I. Am. J. Cardiol. 42:308-329, 1978.
13. Hutchins, G.M., Anaya, O.A.: Measurements of cardiac size, chamber volumes and valve orifices at autopsy. Johns Hopkins Med. J. 133:96-106, 1973.
14. Moore, G.W., Hutchins, G.M.: Coronary artery bypass grafts in 109 autopsied patients: Statistical analysis of graft and anastomosis patency and regional myocardial injury. JAMA 246:1785-1789, 1981.
15. SAS User's Guide - 1979 Edition. SAS Institute, Inc., Raleigh, 1979.
16. Bowie, J. and Barnett, G.O.: MUMPS - An economical and efficient time-sharing system for information management. Comput. Programs Biomed. 6:11-22, 1976.
17. Quine, W.V.: The problem of simplifying truth functions. Am. Math. Monthly 59:521-531, 1952.
18. Quine, W.V.: A way to simplify truth functions. Am. Math. Monthly 62: 627-631, 1955.
19. McCluskey, E.J., Jr.: Minimization of Boolean Functions. Bell System Tech. J. 36:1417-1444, 1956.
20. Moore, G.W., Riede, U.N., Sandritter, W.: Application of Quine's nullities to a quantitative organelle pathology. J. Theor. Biol. 65:633-651, 1977.
21. Moore, G.W., Hutchins, G.M., Bulkley, B.H.: Certainty levels in the nullity method of symbolic logic: Application to the pathogenesis of congenital heart malformations. J. Theor. Biol. 76:53-81, 1979.
22. Moore, G.W., Hutchins, G.M.: Symbolic logic analysis of cause of death in humans: Application to the 108 patients after coronary artery bypass graft surgery. J. Theor. Biol. 92:267-291, 1981.
23. Gödel, K.: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme: I. Monatsch. Mathem. Physik 38: 173-198, 1931.
24. Ulam, S.M.: Adventures of a Mathematician, Charles Scribner's Sons, New York, 1976, pp 280-281.
25. Moore, G.W., Hutchins, G.M.: Effort and demand logic in medical decision making. Metamedicine 1:277-304, 1980.
26. Moore, G.W., Hutchins, G.M.: A Hintikka possible worlds model for certainty levels in medical decision making. Synthese 48:87-119, 1981.
27. Snyder, D.P.: Modal Logic and Its Applications. Von Nostrand Reinhold Company, New York, 1971.
28. Quine, W.V.: Theory of Deduction, Parts I-IV. Harvard Cooperative Society, Cambridge, Mass., 1948, pp 54-81.
29. Anderson, J.M. and Johnstone, H.W., Jr.: Natural Deduction. The Logical Basis of Axiom Systems. Wadsworth Publishing Company, Inc., Belmont, California, 1962.
30. Moore, G.W., Haupt, H.M., Hutchins, G.M.: A hypothesis test for causal explanations in human pathology: Evaluation of pulmonary edema in 181 autopsied patients with leukemia. Math. Biosc. (in press).
31. Hutchins, G.M., Bulkley, B.H.: Correlation of myocardial contraction band necrosis and vascular patency: A study of coronary artery bypass graft anastomoses at branch points. Lab. Invest. 36:642-648, 1977.
32. Hutchins, G.M., Brawley, R.K.: The influence of cardiac geometry on the results of ventricular aneurysm repair. Am. J. Pathol. 99: 221-230, 1980.
33. Hutchins, G.M., Bulkley, B.H., Moore, G.W., et al: Shape of the human cardiac ventricles. Am. J. Cardiol. 41:646-654, 1978.
34. Vigorita, V.J., Moore, G.W., Hutchins, G.M.: Absence of correlation between coronary artery atherosclerosis and severity or duration of adult onset diabetes mellitus. Am. J. Cardiol. 46:535-542, 1980.

Supported by NIH Grants LM-03651 from the National Library of Medicine and HL-17655 from the National Heart, Lung and Blood Institute.

TABLE 1. Mumps Program to Perform the Quine-McCluskey Algorithm

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QUINE ; GWMOORE - QUINE MCCLUSKEY ALGORITHM ;
ENTRY K  K VTR K VTS S AU=-1 S IX=-1 S SW=0 S NX=0 S FL=-1 S FQ=-1 S MX=-1
RDA   S AU=$O(^DATA(AU)) G:AU="" EXIT S FF=-1 K VTR K VTS
      W !!!!,"AUTOPSY:  ",AU
      S IX=-1 S SW=0 S NX=0 S FL=-1 S FQ=-1 S MX=-1 K STV
RDL   S FF=$O(^DATA(AU,FF)) G:FF="" LOG
      S NX=NX+1 S VTS(NX)=FF S STV(FF)=NX W !,FF
      S SF=$E(FF,1,1) S:SF="-" SG="+" S:SF="+" SG="-" S FN=SG_$E(FF,2,120)
      S FC=FF G:$D(STV(FN))>0 CNT G RDL
LOG   S FL=$O(^VTR(FL)) G:FL="" STM S SL=-1
LGN   S SL=$O(^VTR(FL,SL)) G:SL="" LOG S:MX<SL MX=SL S VTR(FL,SL)="" G LGN
STM   S IX=$O(VTS(IX)) G:IX="" RPT S FX=VTS(IX)
      S SF=$E(FX,1,1) S:SF="-" SG="+" S:SF="+" SG="-"
      S FI=SG_$E(FX,2,120) S NO=-1
NXL   S NO=$O(^VTR(FI,NO)) G:NO="" STM S FB=-1 K VTP
BKI   S FB=$O(VTR(FB)) G:FB="" NWI G:$D(VTR(FB,NO))=0 BKI S VTP(FB)="" G BKI
NWI   K VTP(FI) S FC=-1
SLN   S FC=-1 S FC=$O(VTP(FC)) G:FC="" CNT S FD=$O(VTP(FC))
      S FR=-1 G:FD="" ONE G MLT
ONE   G:$D(STV(FC))>0 NXL W !,FC S SW=1 S NX=NX+1
      S VTS(NX)=FC S STV(FC)=NX S SX =$E(FC,1,1)
      S:SX="-" SG="+" S:SX="+" SG="-"
      S FN=SG_$E(FC,2,25) G:$D(STV(FN))>0 CNT G NXL
MLT   S MX=MX+1 S FR=-1
NWL   S FR=$O(VTP(FR)) G:FR="" NXL S VTR(FR,MX)="" G NWL
CNT   W !!!,"CONTRADICTION AT:  ",FC,"   ",FN W !! G RDA
RPT   I SW=1 S SW=0 S IX=-1 G STM
      G RDA
EXIT  W !!!,"EXECUTION COMPLETE"
```

TABLE 2. Cause of Death in 66 Patients with Recognized Complications or Unrelated Causes of Death

Surgical Complications		Unrelated Causes	
Hemorrhage	10	Pulmonary embolism	6
Infection	8	Acute myocardial infarct	5
Postoperative cerebrovascular accident	3	Sudden death	7
Operative coronary obstruction	2	Sepsis	3
Multisystem failure	3	Other cardiac	5
Other	4	Other non-cardiac	10
TOTAL	30		36

TABLE 3. Causes of Death after Coronary Artery Bypass Graft Surgery

Patient Group		
I	Preoperative catastrophe (pre)	13 (8%)
II	Operative contraction band necrosis (opc)	17 (11%)
III	Operative graft occlusion (opg)	5 (3%)
IV	Aneurysmectomy (anu)	15 (10%)
V	Graft thrombosis (gth)	13 (8%)
VI	Other surgical and unrelated (sur, unr)	66 (43%)
VII	Unexplained (unx)	26 (17%)
	TOTAL	155