Restoration of the Sphere-Cortex Homeomorphism

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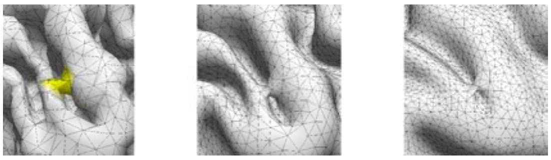
Abstract. The proposed algorithm has been developed as a pre-processing tool for inflating cortical surface meshes, which have been created using segmentation and subsequent triangulation of magnetic resonance images (MRI) [1]. It works directly on the triangulated surface and is therefore completely independent from the underlying segmentation. It needs no other information than the triangle mesh itself, which makes it generally applicable for the removal of topological noise. The homeomorphism between cortical surface and sphere is re-established by removing handles and opening connections. Moreover, the presented approach guarantees a manifold mesh by locally examining connectivity in the neighbourhood of each vertex and removing non-manifold components. It will be embedded into the source reconstruction software package CURRY (Compumedics Neuroscan, El Paso, TX, USA).

1 Introduction

CURRY is a multimodality neuroimaging tool, which allows the integration of EEG, MEG or ECoG with other functional or anatomical imaging modalities, such as MRI, CT, fMRI, SPECT and PET. Sources of brain activity are computed from these data. The source reconstruction results are, besides other visualization techniques, directly mapped onto the triangulation of the cerebral cortex. Due to the intrinsic complex structure and the folding pattern of the cortical surface the visual inspection of neuronal activity, which is mapped directly onto the mesh, is difficult. Functional foci might be hidden inside sulci and in addition, functionally widely separated foci on opposite walls of a sulcus might appear to be close together. Hence, for visualization purposes, deformation techniques like flattening, inflation or spherical inflation (see e.g. [2]) to unfold the cortical surface are widely used in the neuroscience community. These methods are based on a well known fact from Riemannian geometry: any 2-manifold, such as the cerebral cortex (if the brain stem is artificially closed), without topological artefacts can be mapped conformally onto a sphere and local parts thereof to a disk in $\mathbb{R}^2$ [3].

Fig. 1. Handle connecting two potentially different gyri within three iteration steps of the unfolding process, where (a) is the initial surface and (c) illustrates an almost completely smoothed surface.



(a) (b) (c)

However, due to imaging noise, image inhomogeneities and the partial volume effect connections across potentially disparate parts of the cortex are produced during segmentation or triangulation, respectively. These erroneous and anatomically incorrect connections, so-called handles, lead to distortions in the unfolded representation of the cortical surface (see fig. 1).

Thus, it is necessary to remove them in order to re-establish the homeomorphism between sphere and brain. During the last few years extensive research has been aimed at producing topologically correct representations of the cortex. Besides manual editing (see e.g. [2]), different automatic approaches for the topologically correct reconstruction of cortical surfaces from MRI are presented in the literature. They can be divided into three groups: there are (*i*) methods that enforce topology constraints during segmentation (see e.g. [4]), (*ii*) methods that correct the segmentation results directly on volumetric data (see e.g. [5], [6]) and (*iii*) methods that work on triangulated surfaces (see [7], [8]). The approach presented here belongs to group (*iii*).

2 Materials and Methods

It is desired to find a method that leaves the surface geometry essentially unchanged, except in regions where topological artefacts are detected. The method presented here is based on a *wave propagation* concept, which was first introduced by Guskov and Wood [9]; errors are identified by analysing the genus of local patches.

2.1 Preliminaries

Before introducing the algorithm for topological noise removal the combinatorial structure of a *manifold triangulation* needs to be defined. Mathematically, an n-dimensional manifold is a so called *Hausdorff Space*, which is locally isomorphic to a Euclidiean n-space. Thus, a compact connected *2-manifold* is a topological space, where every point has a neighbourhood being topologically equivalent to an open disk in $\mathbb{R}^2$ [10]. The combinatorial structure of such a triangulated 2-manifold $M\,(K, x)$ can be represented through an *abstract simplicial complex* K, where $x : V \rightarrow \mathbb{R}^3$ denotes the coordinate function for each vertex $v \in V$. It embeds K in $\mathbb{R}^3$. $V = \{v_1, v_2, \ldots, v_n\}$ is the set of *vertices* in the mesh. Formally, a q-dimensional simplicial complex denotes a finite collection K of i-dimensional $(i = 0, 1, \ldots, q)$ simplices σ. Each simplex $\sigma \in K$ is a convex hull of a set of $i+1$ affine independent points - the so called vertices.

2.2 Non-Manifold Components

In subsection 2.1 it was stated that any point of a manifold triangulation has a neighbourhood, which is topologically equivalent to an open disk in $\mathbb{R}^2$. Thus, in order to guarantee a manifold triangulation, which is an essential underlying principle for the presented approach, all non-manifold components need to be identified and removed in a first step. This can be done quite easily by locally examining the connectivity in the neighbourhood of each vertex: M is called a *manifold mesh*, when (i) the three corners of a triangle refer to different vertices (no zero area), (ii) each edge bounds exactly two triangles and (iii) the star $St\,(\sigma) = \{\tau \in K : \tau \subseteq \sigma\}$ of each vertex v forms a single cone. The *star* $St\,(\sigma)$ of a simplex σ is a subcomplex of K and defined as all simplices τ containing σ [11]. In particular, it is the union of all edges and triangles incident upon v.

2.3 Topological Artefacts

Having removed all non-manifold components by introducing new vertices to the mesh and thus, breaking all non-manifold components, the next step is to locate and remove topological artefacts. The presence of these errors can be detected by computing the Euler number

$$\chi\,(M) \equiv |V| - |E| + |F| = 2 - 2g\,(M)\,,$$

where $|V|$, $|E|$ and $|F|$ denote the number of vertices, edges and faces of M, respectively; $g\,(M)$ is the genus of the surface M. It is an invariant property of a manifold and denotes the largest number of non-intersecting simple closed curves that can be drawn on a surface, without separating it [9]. Cutting the surface along such a curve does not yield disconnected components of the surface. For a sphere $\mathcal{S}^2$ the genus $g = 0$ and accordingly, the Euler number $\chi = 2$. Hence, the cerebral cortex also represents a so called *genus zero surface*.

However, as both χ and g are global scalar values, neither the Euler number nor the genus include information about the location or the size of a topological defect. Thus, in order to identify topological errors, a local measure of χ or g is needed instead of a global one. Accordingly, patches $P(T, K)$ iteratively are grown around each vertex $v \in V$ within a defined neighbourhood, where $P(T, K)$ denotes a subcomplex of K, given by a set of triangles T. Starting from a seed vertex v_s, triangles $T_i \notin P\,(T, K)$ are added iteratively from the star $St\,(v_i)$ of each vertex v_i at the border of $P(T, K)$. Doubling all components of $P(T, K)$ besides the edges and vertices at its border, i.e. gluing $P(T, K)$ to its copy, will result in a 2-manifold representation $\tilde{\mathcal{P}}(T, K)$ of the patch $P(T, K)$. Thus, if no handle is located inside the manipulated subcomplex $\tilde{\mathcal{P}}(T, K)$ its genus

Table 1. Removing topological artefacts on real brain data. #T and #V denotes the number of triangles and vertices in the tessellation, respectively. n is the number of non manifold components and g is the genus of the triangulated cortical surface. The given timings are for an AMD Athlon 1.5 GHz with 512 MB RAM.

Before				**After**						
#T	#V	g	n	#T	#V	g	n	triangle size	time	time per vertex
85460	42628	44	34	85644	42771	9	0	2.0 mm	223 sec	5.23 msec
74410	37126	32	43	74484	37228	2	0	2.0 mm	242 sec	6.52 msec
40084	19957	38	30	40238	20092	2	0	2.5 mm	88 sec	4.41 msec
144052	71955	34	11	144222	72085	5	0	1.0 mm	338 sec	4.70 msec

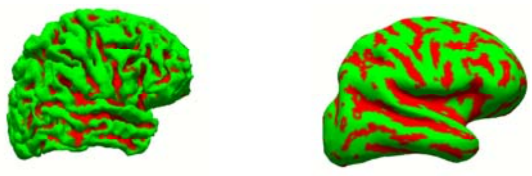
$g\left(\tilde{\mathcal{P}}(T,K)\right) = 0$. Otherwise, a topological artefact is detected. In contrast to the approach presented in [10], each topological defect is not corrected directly after its detection. Instead, the smallest correcting cuts are identified first. Each vertex v is encoded with a number $\eta\,(v)$ denoting the iteration step on which the artefact was detected. Subsequently, connected vertices carrying values $\eta\,(v)$ smaller than their neighbor's denote candidates for topological correction. After identifying areas with small values $\eta\,(v)$ a non-intersecting cut is found on one side of the isolated area surrounding the handle or the tunnel. Having assured that the cut will not result in separated parts of the cortical surface, vertices along the cut are doubled. Consequently, the corresponding handle or the tunnel, respectively, is broken into two parts . Afterwards, the produced holes inside the triangle mesh are sealed and consequently, the topological defects are removed.

3 Results and Discussion

We have presented an approach to repair triangulated surfaces of the cerebral cortex, which is completely independent from the underlying segmentation. It needs no other information than the triangle mesh itself, which makes it generally applicable for the reduction of topological artefacts. Thus, it might be applied to other surface models besides triangulated representations of the cerebral cortex. The presented approach repairs local topological artefacts of triangle meshes with a time-complexity of $O(N)$. It is implemented in C++ and will become part of the source reconstruction software CURRY (Compumedics Neuroscan, El Paso, TX, USA). First results can be seen in tab. 1.

The algorithm removes any non-manifold components produced by the triangulation. Moreover, nearly all topological artefacts present on the cortical surface are eliminated. For the removal of the topological noise the patch radius $\eta\,(v)$ was set to approximately 15 mm. Unfortunately, the algorithm is not capable of removing all handles or holes (see tab. 1). The decision of filling the associated tunnel or cutting the handle is dependent on the diameter of the hole or the handle, respectively. In some cases the perimeter of the handle or the cavity is larger than the predefined maximal radius $\eta\,(v)$ of the patch (15 mm in our case).

Fig. 2. Comparision between (a) initial and (b) inflated representation of the cortical surface. The colouring illustrates the curvature; sulci are coloured red and gyri are coloured green.



(a) (b)

Thus, the artefact is not detected. However, having removed most topological artefacts, the surface can be inflated properly for visual inspection of functional foci (see fig. 2).

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