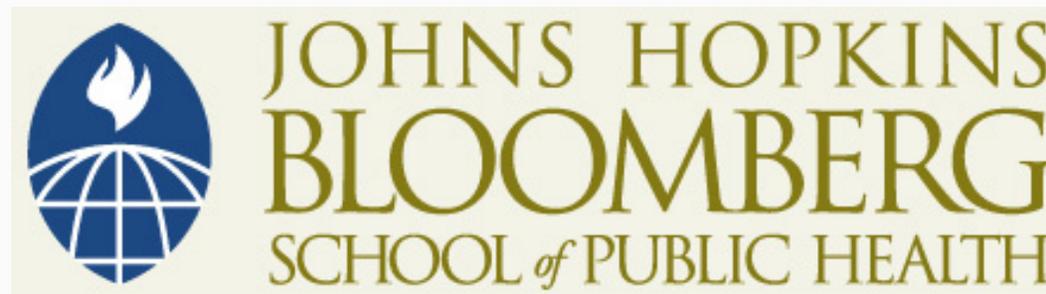


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## Section C

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Two Sample t-test, Approach with Smaller Samples

# Sampling Distribution

- What is sampling distribution of the difference in sample means?
  - If either (or both) sample sizes are less than 60, a t-distribution is used with  $n_1 + n_2 - 2$  degrees of freedom: this is the degrees of freedom for the total sample size from both groups minus two

# Two Sample t-test

- Example
  - In a randomized design, 23 patients with hyperlipidemia were randomized to either take Treatment A or Treatment B for 12 weeks
  - 12 patients assigned to Treatment A
  - 11 patients assigned to Treatment B

# Two Sample t-test

- Example
  - LDL cholesterol levels (mmol/L) measured on each subject at baseline, and 12 weeks after start of study
  - The 12-week change in LDL cholesterol was computed for each subject

# Two Sample t-test

- Summary of results:

	Treatment Group	
	A	B
Number of subjects (n)	12	11
Mean LDL change (mmol/L) Post-trt less pre-trt	-1.41	-0.32
Standard deviation of LDL changes (mmol/L)	0.55	0.65

# Two Sample t-test

- Scientific question
  - Is there a difference in LDL change between the two treatment groups?
- Methods of inference
  - Confidence interval for the difference in mean LDL cholesterol will change between the two groups
  - Statistical hypothesis test

## 95% Confidence Interval for Difference in Means

- The general formula (large samples):

$$(\bar{x}_1 - \bar{x}_2) \pm 2 \times \hat{SE}(\bar{x}_1 - \bar{x}_2)$$

- The general formula (“smaller” samples):

$$(\bar{x}_1 - \bar{x}_2) \pm t_{.95, n_1 + n_2 - 2} \times \hat{SE}(\bar{x}_1 - \bar{x}_2)$$

# Two Sample t-test

- Sample mean difference and estimated standard error:

	Treatment Group	
	A	B
Number of subjects (n)	12	11
Mean LDL change (mmol/L) Post-trt less pre-trt	-1.41	-0.32
Standard deviation of LDL changes (mmol/L)	0.55	0.65

$$\bar{x}_1 - \bar{x}_2 = 1.41 - (-0.32) = -1.09 \text{ mmol/L}$$

$$SE(\bar{X}_1 - \bar{X}_2) = \sqrt{\frac{0.55^2}{12} + \frac{0.65^2}{11}} \approx 0.25$$

## 95% CI for Difference in Means: Hyperlipidemia Ex

- How many standard errors to add and subtract?
  - Since sample sizes are small we will have to add slightly more than two standard errors
- Number we need add and subtract for 95% confidence comes from a t-distribution with  $(12 + 11 - 2 = 21)$  degrees of freedom
  - From t-table this value is 2.08
- So, 95% CI for true mean difference in change in LDL cholesterol, drug A to drug B

$$-1.09 \pm 2.08 \times .25 \rightarrow$$

$$-1.61 \text{ mmol/L to } -0.57 \text{ mmol/L}$$

# Hypothesis Test to Compare Two Independent Groups

- Two-sample (unpaired) t-test: getting a p-value
- Is the change in LDL cholesterol the same in the two treatment groups?
  - $H_0: \mu_1 = \mu_2 \quad \rightarrow \quad H_0: \mu_1 - \mu_2 = 0$
  - $H_A: \mu_1 \neq \mu_2 \quad \rightarrow \quad H_A: \mu_1 - \mu_2 \neq 0$

# Hypothesis Test to Compare Two Independent Groups

- Recall, general “recipe” for hypothesis testing . . .
  1. Start by assuming  $H_0$  true
  2. Measure distance of sample result from  $\mu_0$  (here again its 0)
  3. Compare test statistic (distance) to appropriate distribution to get p-value

$$t = \frac{(\text{observed dif f}) - (\text{null dif f})}{SE \text{ of observed dif f erence}}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

## Diet Type and Weight Loss Study

- In the diet types and weight loss study, recall:

$$\bar{x}_1 - \bar{x}_2 = -1.09 \text{ mmol} / L$$

$$\hat{SE}(\bar{x}_1 - \bar{x}_2) = 0.25 \text{ mmol} / L$$

- So in this study:

$$t = \frac{-1.09}{.25} \approx -4.4$$

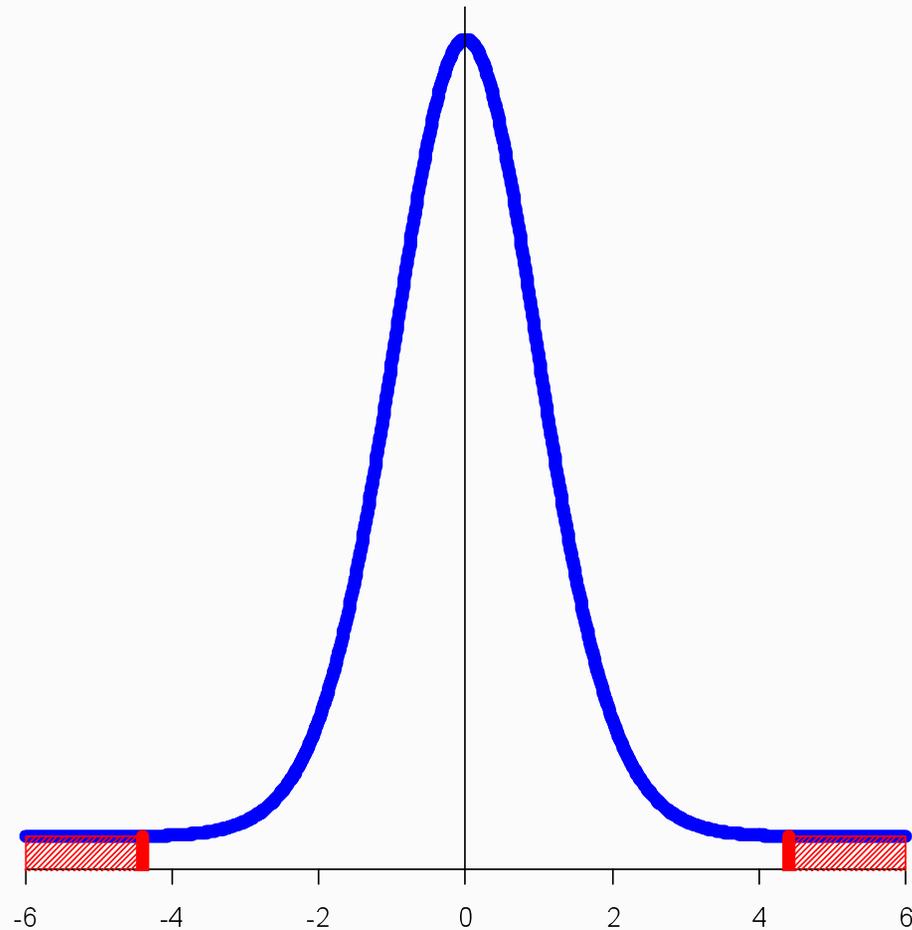
- So this study result was 4.4 standard errors below the null mean of 0 (i.e., 4.4 standard errors from the less expected mean difference in cholesterol change between the two treatments if null was true)

## How Are p-values Calculated?

- Is a result 4.4 standard errors below 0 unusual?
  - It depends on what kind of distribution we are dealing with
- The p-value is the probability of getting a test statistic (distance) as or more extreme than what you observed (-4.4) by chance if it was true
- The p-value comes from the sampling distribution of the difference in two sample means
- What is the sampling distribution of the difference in sample means?
  - t-distribution with  $12 + 1 - 2 = 21$  degrees of freedom

# Hyperlipidemia Example

- To compute a p-value, we would need to compute the probability of being 4.4 or more standard errors away from 0 on a t-distribution with 21 degrees of freedom



# Using Stata

- Command syntax:
  - `ttesti  $n_1$   $\bar{x}_1$   $s_1$   $n_2$   $\bar{x}_2$   $s_2$ , unequal`

```
. ttesti 11 -1.41 .55 12 -.32 .65, unequal
```

```
Two-sample t test with unequal variances
```

	Obs	Mean	Std. Err.	Std. Dev.	[ 95% Conf. Interval]	
x	11	-1.41	.1658312	.55	-1.779495	-1.040505
y	12	-.32	.1876388	.65	-.7329903	.0929903
combined	23	-.8413043	.1692296	.8115967	-1.192265	-.4903436
diff		-1.09	.2504163		-1.61095	-.5690505

```
diff = mean(x) - mean(y)                                t = -4.3528  
Ho: diff = 0                                           Satterthwaite's degrees of freedom = 20.8813
```

```
Ha: diff < 0  
Pr(T < t) = 0.0001
```

```
Ha: diff != 0  
Pr(|T| > |t|) = 0.0003
```

```
Ha: diff > 0  
Pr(T > t) = 0.9999
```

# Using Stata

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Two-sample t test with unequal variances

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Pr(T < t) = 0.0001

Ha: diff != 0  
Pr(|T| > |t|) = 0.0003

Ha: diff > 0  
Pr(T > t) = 0.9999

## Summary: Weight Loss Example

- **Statistical method**

- Twenty-three patients with hyperlipidemia were randomly assigned to one of two treatment groups: Treatment A or Treatment B
- 12 patients were assigned to receive Treatment A
- 11 patients were assigned to receive Treatment B

## Summary: Weight Loss Example

### ■ **Statistical method**

- Baseline LDL cholesterol measurements were taken on each subject, and LDL was again measured after 12 weeks of treatment
- The change in LDL cholesterol was computed for each subject
- The mean LDL changes in the two treatment groups were compared using an unpaired t-test and a 95% confidence interval was constructed for the difference in mean LDL changes

## Summary: Weight Loss Example

### ■ Result

- Patients on treatment A showed a decrease in LDL cholesterol of 1.41 mmol/L and subjects on treatment B showed a decrease of .32 mmol/L (a difference of 1.09 mmol/L, 95% CI .57 to 1.61 mmol/L)
- The difference in LDL changes was statistically significant ( $p < .001$ )